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21 August, 1988

Mike Burka,
Peter Saulson,
Andrej Cadez

re: Scattering in
Fabry-Perot's

Dear Mike, Peter, and Andrej,

Leaning somewhat on the calculation for reflection from baffles by Mike and Peter and the general calculation of Andrej, I have extended my May 1987 calculation of scattering for a Michelson in the LIGO to scattering for a Fabry-Perot. My final conclusions are these. (They surely need checking: I've done this hurriedly because of my impending departure for Moscow, and, moreover, I have set to unity all factors of 2 and π.)

1. Reflection off baffles, while perhaps more important in overall amplitude than diffraction (I have not checked), is less important in terms of sensitivity to baffle motion. The ratio of reflection effect to diffraction effect, in terms of equivalent gravitational-wave amplitude h, is

$$4\pi \frac{\sqrt{D\rho}}{L} \sim 10^{-5} \qquad (1)$$

where $D \simeq 100\,\mathrm{cm}$ is the diameter of the vacuum pipe, $L = 4$ km is its length, and $\rho \sim 10$ microns is the radius of curvature of the baffle's edge. See page 4 of my enclosed calculations.

2. The two dominant effects, it seems to me, are these: (i) Scattering off the end mirror to the edges of the moving baffles, followed by diffraction from the baffles to the corner mirror, followed by transmission through the corner mirror and beam splitter onto the photodiode; I call this the *direct* effect. (ii) Scattering off one of the mirrors to the edges of the moving baffles, followed by diffraction from the baffles to the other mirror, followed by scattering from that mirror back into the main beam in the Fabry-Perot cavity; followed by transmission of that augmented main beam through the corner mirror and beam splitter and onto the photodiode; I call this the *double scattered*, or simply *scattered* effect.

3. I estimate that the double scattered effect is smaller than the direct effect by a factor of order 50; item 7 below.

4. The direct effect for a Fabry-Perot is the same as that for a Michelson (with the number of Michelson bounces B replaced by the Fabry-Perot Finesse Q), to within factors of order unity, with one exception: because the Michelson has B spots on the end mirror from which scattering can take place, its noise (in terms of equivalent gravity-wave amplitude h) is larger by a factor $\sqrt{B}$; see page 7 of attached notes.

5. If $\xi(f)$ is the square root of the spectral density of displacement of a typical point on a typical baffle, then the corresponding square root of the spectral density of the equivalent gravity-wave noise $\tilde{h}(f)$ due to the direct effect is

$$\tilde{h}^D(f) = \sqrt{\beta(1-R_E)}\frac{1}{Q}\frac{\lambda}{L}\left[\frac{\sqrt{\lambda L}}{d}\right]^{1/2}(1-\overline{\eta})\frac{\xi(f)}{D} \sim 10^{-18}\frac{\xi(f)}{1\mathrm{cm}} . \tag{2}$$

Here $\beta(1-R_E) \simeq 10^{-6}$ is the coefficient in the scattering law for the end mirror,

$$\frac{dP}{d\Omega} = \frac{\beta(1-R_E)}{\theta^2} \tag{3}$$

with θ the scattering angle and $1-R_E \simeq 10^{-4}$ the transmissivity of the end mirror; $Q \sim 100$ is the Fabry-Perot finesse; $\lambda \simeq 4\times10^{-5}$cm is the wavelength of the light; $L \simeq 4\times10^{+5}$cm is the length of an arm; $D \simeq 100$ cm is the diameter of the beam tube; $d \sim 1$ cm is the height of a baffle; and $\overline{\eta}$ is the efficiency of the photodiode averaged over the spot that the incoming beam makes on it.

6. The above formula for the direct effect assumes that the light from the various baffles superposes incoherently, and assumes a "Cornu-spiral-type" cancellation of the contributions of all the Fresnel zones on a baffle edge except the largest one, and assumes a "$1/\sqrt{N}$" averaging away of contributions from different Fresnel zones on the photodiode. If, somehow, these cancellations do not occur; i.e. if all the baffles and all Fresnel zones somehow were to contribute coherently, the h noise would be larger than Eq. (2) by a factor

$$\left[\frac{D}{d}\frac{D^3}{(\lambda L)^{3/2}}\right]^{1/2} \sim 10^3 . \tag{4}$$

7. The ratio of the double scattering effect to the direct effect is

$$\frac{\tilde{h}^S}{\tilde{h}^D} \simeq \left[\beta(1-R_E)\frac{\sqrt{\lambda L}}{D}Q\frac{1}{(1-\overline{\eta})^2}\right]^{1/2} \sim \frac{1}{50} . \tag{5}$$

8. In the most pessimistic case [Eq. (2) multiplied by Eq. (4), so $\tilde{h}(f) \sim 10^{-15}\xi(f)/1\,\mathrm{cm}$] the goal of ultimately achieving $\tilde{h} \simeq 10^{-23}/\sqrt{\mathrm{Hz}}$ at frequencies of order 100 Hz will require that the wall motion be kept smaller than

$$\xi \lesssim 10^{-8}\mathrm{cm}/\sqrt{\mathrm{Hz}} . \tag{9a}$$

For comparison, for my most pessimistic estimate in the case of a Michelson was

$$\xi \lesssim 10^{-10}\mathrm{cm}/\sqrt{\mathrm{Hz}} \tag{9b}$$

(see my 1987 Michelson calculations). The best guess formula (2) is probably not too bad; it corresponds to requiring

$$\xi \lesssim 10^{-5}\mathrm{cm}/\sqrt{\mathrm{Hz}} \tag{9c}$$

for a Fabry-Perot. The corresponding best guess for a Michelson was

$$\xi \lesssim 10^{-6}\mathrm{cm}/\sqrt{\mathrm{Hz}} \tag{9d}$$

(see my 1987 notes). None of these limits is accurate to any better than a factor ten; recall, especially, that in the Fabry–Perot calculation I made no attempt to keep factors of 2 and π.

10. The above calculations and limits are somewhat shaky because they are based on a non–recycled model of the detector. The calculation should be generalized to include recycling.

I am off to Moscow for three weeks this coming Wednesday. I hope to learn, when I return, whether you agree with my analysis and conclusions or not.

Best wishes

Kip

Kip S. Thorne

cc: R. Vogt

Kip Thorne

Madison, WI
8/21/88

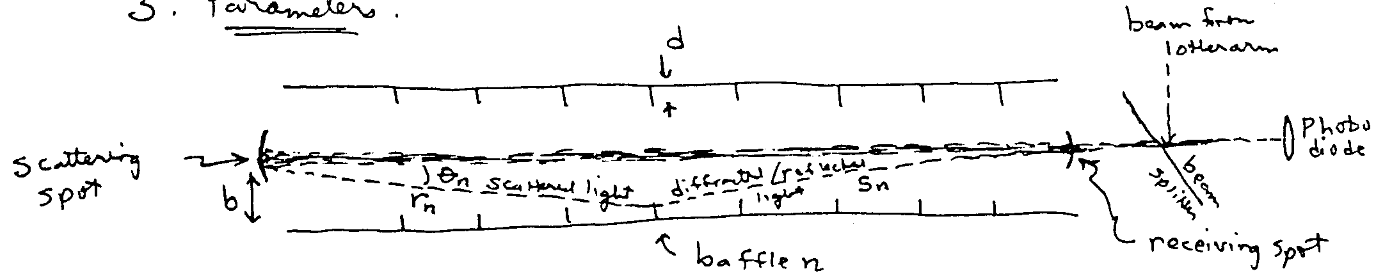
Scattering, & Diffraction/Reflection off Baffles for Fabry-Perot

I. The Model

1. Straightforward extension, to a Fabry-Perot, of my Michelson calculations labeled MIT 5/7/87.

2. Description: Simple (non-recycled) Fabry-Perot
Light is scattered from the spot on the end mirror ("scattering spot") toward baffle, where it is diffracted & deflected to the corner mirror ("receiving spot"). There the light either passes through the corner mirror and interferes with beam from other arm at the beam splitter, or it scatters back into the Gaussian beam inside the Fabry-Perot Cavity.

3. Parameters:



L = length of arm = 4 km

D = diameter of vacuum pipe ≃ 100 cm

d = height of baffles ≃ 3 cm

λ = wavelength of light ≃ 4×10^{-5} cm

$\omega_0 = \sqrt{\lambda L}$ = size of main beam in interferometer ≃ 4 cm

ξ = radial displacement of a typical point on a typical baffle ... $\xi = \xi(t)$

$x_n(\varphi, t)$ = radial displacement of n'th baffle at angular location φ

$\underline{m}(\varphi)$ = unit vector pointing radially (perpendicular to tube wall) at location φ

Q = Finesse of Cavity ≃ $\frac{1}{1-R_c}$, R_c = reflectivity of central mirror

≃ 100

R_E = Reflectivity of end mirror ≃ (the highest possible) ≃ $1-10^{-4}$

$\theta_n = \left(\begin{array}{c}\text{angle of scattering to}\\ \text{reach baffle } n \text{ ... at angular}\\ \text{location } \varphi \text{ on the baffle}\end{array}\right) \simeq \frac{D}{2L} \simeq 10^{-4}$ radians

$$\left(\frac{dP}{d\Omega}\right)_n = \left(\text{probability, per unit solid angle, of a photon ~~from~~ hitting the scattering spot, being scattered onto the edge of the n'th baffle at location } \varphi\right)$$

$$= \beta(\theta_n)\ \frac{1-R_E}{\theta_n{}^2}$$

Note: Michell's measurements on poor mirrors, extrapolated from $\theta_n \sim 30'$ to $\theta_n \sim 10^{-4}\,\text{rad} \sim 0.3'$, give $\beta \sim 3\times10^{-3}$. Čadež infers from Elson & Bennett's data on Supermirrors that $\beta\cdot(1-R_E) \simeq 1.5\times10^{-6}$ which, with $1-R_E \simeq 10^{-4} \Rightarrow \beta \simeq 10^{-2}$ in good agreement with Michell.

I'll use $\beta \simeq 10^{-2}$.

$\mathcal{E}$ = amplitude of light, so defined that $\vec{E} = \sqrt{8\pi}\,\mathcal{E}\,e^{-i\omega t} \times$(polarization vector),

so (energy flux) $= |\mathcal{E}|^2$. Polarization effects will be ignored.

$k = \frac{\omega}{c} = \frac{2\pi}{\lambda}$ = wave number

l_n = distance of baffle n from end mirror

$b(\varphi)$ = (distance of scattering spot from wall of vacuum pipe at location φ) − (baffle height, d)

$r_n = \sqrt{l_n^2 + (b-x_n)^2} \simeq \sqrt{l_n^2+b^2} - \frac{b\,x_n}{\sqrt{l_n^2+b^2}}$ = distance of position φ on baffle n from scattering spot (depends on location in spot)

$S_n = \sqrt{(L-l_n)^2+(b-x_n)^2} \simeq \sqrt{(L-l_n)^2+b^2} - \frac{b\,x_n}{\sqrt{(L-l_n)^2+b^2}}$ = distance of position φ on baffle n from receiving spot (depends on location in spot)

$\bar{r}_n, \bar{S}_n$: values of r_n & S_n @ centers of spot

I_0 = ~~Intensity~~ Power (energy per unit time) of main beam inside the cavity $\simeq I_L Q$

I_L = Laser power [with losses in the optics between laser & cavity neglected]

N = number of baffles $\simeq 3\frac{D}{d}$ for safety $) \simeq 100$

ρ = radius of curvature of tip of baffle $\simeq$ 10 microns $\simeq 10^{-3}$ cm

$\underset{\sim}{x}$ = transverse position inside receiving spot or on photodiode

$\eta(\underset{\sim}{x})$ = efficiency of photodiode at location $\underset{\sim}{x}$

$\bar{\eta}$ = average of η over the beam entering the photodiode

4. Incoming scattered field at position φ on baffle n

$$\mathcal{E}_n(\varphi) = \sqrt{I_0 \left(\frac{dP}{d\Omega}\right)_n} \; \frac{e^{ikr_n}}{r_n}$$

Note: we set the phase at the scattering spot to zero.

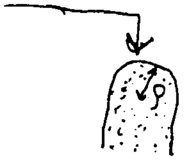
5. Diffracted Light at Receiving Spot.

On the mirror, at the receiving spot, the field $\mathcal{E}_M^{\rm Diff}$ is computed using Fresnel's Form of Huygen's Principle. I include only ~~that~~ the contribution due to the baffles having moved by $x_n(\varphi)$ and thereby having opened up the new area $x_n \frac{D+d}{2}\, d\varphi \simeq x_n \frac{D}{2}\, d\varphi$:

$$\mathcal{E}_M^{\rm Diff} = -\frac{i}{\lambda} \sum_{n=1}^{N} \int_0^{2\pi} \mathcal{E}_n \frac{e^{ikS_n}}{S_n}\, x_n \frac{D}{2}\, d\varphi$$

6. Reflected Light at Receiving Spot.

a. Assume a baffle shape which in the ~~#~~ z-direction (down the tube) has a rounded end with radius of curvature ρ [as suggested by Burka & Saulson]



b. Assume a shape in the azimuthal (φ direction) that is corrugated.

This is needed, I think, to prevent the baffle from acting like a mirror that tends to focus the reflected light back toward the main beam.

c. Then the total area of the reflecting baffle edge is $D\rho$; it ~~can~~ reflects the light approximately equally in all directions, from each & every Fresnel zone (size $\sim \sqrt{\lambda L} \simeq 4$ cm). This means the reflected amplitude at the mirror is

$$\mathcal{E}_M^{\rm Ref} \simeq \sum_{n=1}^{N} \int_0^{2\pi} \mathcal{E}_n \frac{\sqrt{D\rho}}{S_n}\, e^{ikS_n}\, d\varphi$$

[in agreement with Burka & Saulson]

d. I only want the piece of this associated with the displacement of the baffle $x_n(\varphi)$. That displacement causes

$$D \to D + x_n\,, \quad S_n \to S_n - \frac{b x_n}{\sqrt{\ell_n^2 + b^2}} \simeq S_n - \frac{D}{L} x_n\,,$$

$$ikS_n \to ikS_n - i\,\frac{2\pi}{\lambda}\,\frac{D}{L}\, x_n$$

and thus causes a fractional change in the integrand

$$\frac{1}{2}\frac{\delta D}{D} - \frac{\delta s_n}{s_n} + ik\,\delta s_n = \frac{1}{2}\frac{x_n}{D} - \frac{1}{L}\frac{D}{L}x_n - \underbrace{i\frac{2\pi}{\lambda}\frac{D}{L}x_n}_{\text{biggest}}$$

$$\simeq -i\,2\pi\,\frac{D x_n}{\lambda L}$$

e. Thus,

$$\mathcal{E}_M^{Ref} \simeq -\frac{i}{\lambda}\sum_{n=1}^{N}\int_0^{2\pi}\mathcal{E}_n\,\frac{e^{iks_n}}{s_n}\cdot\frac{2\pi D x_n}{L}\cdot\sqrt{D\rho}\;d\varphi$$

f. The ratio of the integrand 6.e for reflected light to that 5. for diffracted light is

$$\frac{\text{Reflect}}{\text{Diffract}} = \frac{\frac{2\pi D x_n}{L}\sqrt{D\rho}}{x_n\frac{D}{2}} = 4\pi\frac{\sqrt{D\rho}}{L} \simeq 10\times\frac{\sqrt{100\,\text{cm}\times10^{-2}\,\text{cm}}}{4\times10^{5}\,\text{cm}}$$

$$\simeq 10^{-5} \ll 1$$

Thus, reflection off the baffles is unimportant compared to diffraction.

7. Diffracted Light at Receiving Spot

a. Return to step 5 and combine with 4 to get

$$\mathcal{E}_M^{Diff} = -\frac{i}{\lambda}\sum_{n=1}^{N}\int_0^{2\pi}\sqrt{I_0\left(\frac{dP}{d\Omega}\right)_n}\;\frac{e^{ik(r_n+s_n)}}{r_n s_n}\,x_n\frac{D}{2}\,d\varphi$$

b. Use $r_n \simeq s_n \simeq L$ in the denominator;

$$\left(\frac{dP}{d\Omega}\right)_n \simeq \beta\frac{(1-R_E)}{\Theta_n^2} \simeq \frac{\beta(1-R_E)}{(D/L)^2}$$

to get

$$\mathcal{E}_M^{Diff} = -i\sqrt{\beta(1-R_E)}\;\frac{\sqrt{I_0}}{\lambda L}\sum_{n=1}^{N}\int_0^{2\pi}e^{ik(r_n+s_n)}\,x_n\,d\varphi$$

c. At the receiving spot $ik(r_n+s_n)$ varies with position $\underset{\sim}{x}$ on the mirror:

$$r_n = \bar{r}_n + \cancel{\frac{(\underset{\sim}{x}\cdot\underset{\sim}{m})b}{\ell_n}}\;,\quad s_n = \bar{s}_n + \frac{(\underset{\sim}{x}\cdot\underset{\sim}{m})\,b}{L-\ell_n}$$

↑ location of scattering center on scattering spot

$$\Rightarrow \quad e^{ik(r_n+s_n)} = e^{ik(\bar{r}_n+\bar{s}_n)}\, e^{ik\left(\frac{b}{L-\ell_n}\right)\underset{\sim}{m}\cdot\underset{\sim}{x}}$$

d. Thus,

$$\mathcal{E}_M^{Diff} = -i\sqrt{\beta(1-R_E)}\ \frac{\sqrt{\overset{QI_L}{I_0}}}{\lambda L}\sum_{n=1}^{N}\int_0^{2\pi} e^{ik(\bar{r}_n+\bar{s}_n)}\, e^{ik\left(\frac{b}{L-\ell_n}\right)\underset{\sim}{m}\cdot\underset{\sim}{x}}\ \chi_n\, d\varphi$$

8. Diffracted Light that Passes Through the Mirror Onto the Beam Splitter

a. This light ~~produces~~ has amplitude $\mathcal{E}_M^{Diff}\frac{1}{\sqrt{Q}}$ since $1/\sqrt{Q}$ is the transmission coefficient through the corner mirror.

b. It superposes on the light from the other arm to produce a power flux at the photodiode

$$\left|\mathcal{E}_M^{Diff}\frac{1}{\sqrt{Q}}\ \underbrace{\sqrt{\frac{I_L}{\lambda L}}\, e^{-\underset{\sim}{x}^2/\lambda L}}_{\text{amplitude from other arm}}\right|$$

c. The result is a "direct" fluctuation in the power into the photodiode

$$\delta I_{pd}^{\mathcal{D}} = \int \frac{|\mathcal{E}_M^{Diff}|}{\sqrt{Q}}\sqrt{\frac{I_L}{\lambda L}}\, e^{-\underset{\sim}{x}^2/\lambda L}\,\eta(x)\, dx\,dy$$

$$\delta I_{pd}^{\mathcal{D}} = \sqrt{\frac{I_L}{Q\lambda L}}\ |\mathcal{E}_M^{Diff}(\underset{\sim}{x}=0)|\ \underbrace{\left|\int \eta(\underset{\sim}{x})\, e^{ik\left(\frac{b}{L-\ell_n}\right)\underset{\sim}{m}\cdot\underset{\sim}{x}}\, e^{-\underset{\sim}{x}^2/\lambda L}\, dx\,dy\right|}$$

This is the factor pointed out by Rai in my Michelson calculation – page 8.
Its value is

$$\simeq \sqrt{\underbrace{S_\eta\left(\frac{kD}{L}\right)}_{\substack{\text{spectral density}\\ \text{of } \eta \text{ at wave}\\ \text{number } kD/L}}\cdot\underbrace{\frac{1}{\sqrt{\lambda L}}}_{\uparrow}} = \sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}}$$

bandwidth determined by size of the beam from other arm

$$\delta I_{pd}^{\mathcal{D}} = \sqrt{\frac{I_L}{Q\lambda L}}\ |\mathcal{E}_M^{Diff}(\underset{\sim}{x}=0)|\cdot\sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}}\ \cdot\lambda L$$

$$\delta I_{pd}^{\mathcal{D}} = \sqrt{\frac{I_L\cdot\lambda L}{Q}}\ \sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\cdot\frac{1}{\sqrt{\lambda L}}}\ \cdot\ |\mathcal{E}_M^{Diff}(\underset{\sim}{x}=0)|$$

9. Light that Scatters Back Into Main Beam

a. Assume there is just one scattering center, located at $\underset{\sim}{x} = 0$ in the receiving spot.

b. Then the amplitude scattered back into the main beam is

$$\mathcal{E}_M^{\mathrm{Diff}}(\underset{\sim}{x}=0) \cdot \sqrt{\underbrace{\left(\frac{dP}{d\Omega}\right)_n}_{\frac{\beta(1-R_E)}{(D/L)^2}} \underbrace{\left(\frac{w_0}{L}\right)^2}_{\uparrow}}$$

$\left.\begin{matrix}\text{solid angle light must}\\ \text{scatter into in order to}\\ \text{re-enter main beam}\end{matrix}\right\} = \left(\frac{\lambda L}{L}\right)^2 = \frac{\lambda}{L}$

$$= \mathcal{E}_M^{\mathrm{Diff}}(\underset{\sim}{x}=0) \cdot \sqrt{\beta(1-R_E)}\ \frac{\sqrt{\lambda L}}{D}$$

c. This builds up in the main beam coherently by a factor $\sqrt{Q}$, then passes out through the mirror with transmission amplitude $\frac{1}{\sqrt{Q}}$ and superposes on the light from the other arm to produce

$$\overset{\leftarrow\text{"scattered"}}{\delta I_{pd}^{S}} = \int \left|\mathcal{E}_M^{\mathrm{Diff}}(\underset{\sim}{x}=0)\right| \sqrt{\beta(1-R_E)}\ \frac{\sqrt{\lambda L}}{D} \cdot \sqrt{\frac{I_L}{\lambda L}}\ e^{-\underset{\sim}{x}^2/\lambda L}\ \eta\ dx\,dy$$

$$\delta I_{pd}^{S} = \sqrt{\beta(1-R_E)}\ \frac{\sqrt{\lambda L}}{D}\sqrt{\frac{I_L}{\lambda L}} \left|\mathcal{E}_M^{\mathrm{Diff}}(\underset{\sim}{x}=0)\right| \cdot \bar{\eta} \cdot \lambda L$$

$$\delta I_{pd}^{S} = \sqrt{\beta(1-R_E)}\ \frac{\sqrt{\lambda L}}{D} \cdot \sqrt{I_L \cdot \lambda L}\ \bar{\eta} \left|\mathcal{E}_M^{\mathrm{Diff}}(\underset{\sim}{x}=0)\right|$$

10. Ratio of Scattered to Direct

a. $$\frac{\delta I_{pd}^{S}}{\delta I_{pd}^{D}} = \sqrt{\beta(1-R_E)}\ \sqrt{\frac{\lambda L}{D^2}}\ \sqrt{Q}\ \frac{\bar{\eta}}{\sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}}}$$

b. $$\frac{\delta I_{pd}^{S}}{\delta I_{pd}^{D}} = \sqrt{\underbrace{\beta(1-R_E)}_{10^{-6}}\ \underbrace{\underbrace{\frac{\lambda L}{D^2}}_{\left(\frac{1}{25}\right)^2}\ \underbrace{Q}_{10^{2}}}_{\sim 20}\ \frac{\bar{\eta}^2}{\underbrace{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}}_{\sim \underbrace{\frac{D}{\sqrt{\lambda L}}}_{25}\underbrace{\frac{1}{(1-\bar{\eta})^2}}_{\simeq 100} \sim 3\times 10^3}}} \sim \sqrt{0.06} \sim \frac{1}{40}$$

11. Conversion into equivalent gravitational-wave amplitude

$$\frac{\delta I_{pd}}{I_L} \simeq \underset{\overset{\uparrow}{\text{phase}}}{\Delta\Phi} \simeq \frac{hLQ}{\lambda} \rightarrow h \simeq \frac{\lambda}{L}\frac{1}{Q}\frac{\delta I_{pd}}{I_L}$$

12. Direct Contribution

$$h^{\mathcal{D}} \simeq \frac{\lambda}{L}\frac{1}{Q}\frac{\delta I_{pd}^{\mathcal{D}}}{I_L} \simeq \frac{\lambda}{L}\frac{1}{Q}\frac{1}{I_L}\cdot\sqrt{\frac{I_L \cdot \lambda L}{Q}}\cdot\sqrt{S_\eta \frac{1}{\sqrt{\lambda L}}}\cdot\underbrace{\left|\mathcal{E}_m^{\text{Diff}}(\underline{x}=0)\right|}$$

$$\hookrightarrow = \sqrt{\beta(1-R_E)}\ \frac{\sqrt{QI_L}}{\lambda L}\sum_{n=1}^{N}\int_0^{2\pi} e^{ik(\bar{r}_n+\bar{s}_n)}\, x_n\, d\varphi$$

$$\boxed{h^{\mathcal{D}} \simeq \sqrt{\beta(1-R_E)}\ \frac{1}{Q}\sqrt{\frac{\lambda}{L}}\ \sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}}\cdot\left|\sum_{n=1}^{N}\int_0^{2\pi} e^{ik(\bar{r}_n+\bar{s}_n)}\frac{x_n}{L}\, d\varphi\right|}$$

due to light that scatters off end mirrors,
diffracts off baffles,
then passes directly through mirrors

This is same as for Michelson, w/ B = (# of bounces in Michelson) changed to Q = (Fabry-Perot Finesse), except for omission of the Michelson's sum over the number of spots on the end mirrors.

13. Scattered contribution

$$\boxed{h^{\mathcal{S}} = \underbrace{\sqrt{\beta(1-R_E)\ \frac{\lambda L}{D^2}\ Q\ \frac{\bar{\eta}^2}{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}}}}_{\sim 1/4\text{, see page 6}}\cdot h^{\mathcal{D}}}$$

due to light that scatters off end mirrors,
diffracts off baffles,
then scatters from corner mirrors
back into main beam

14. Best Guess Estimate of $h^{\mathcal{D}}$ & $h^{\mathcal{S}}$

a. As in my Michelson analysis, baffles probably contribute incoherently so

$$\sum_{n=1}^{N} \rightarrow \frac{N}{\sqrt{N}} \simeq \cancel{\sqrt{\frac{\lambda}{D}}}\sqrt{\frac{D}{d}} \qquad \left[\text{Worst } \overset{\text{case}}{\cancel{\text{guess}}} \text{ would be } \frac{D}{d}\right]$$

b. By the "Cornu-spiral-type" argument in My Michelson calculation

$$\int_0^{2\pi} e^{ik(\bar{r}_n+\bar{s}_n)}\frac{x_n}{L}\,d\varphi \simeq 2\,\frac{\sqrt{\lambda L}}{D}\,\frac{\xi}{L} \qquad \left[\text{Worst } \overset{\text{case}}{\cancel{\text{guess}}} \text{ would be } \frac{\xi}{L}\right]$$

c. In accord with the estimate at the bottom of page 6:

$\sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}}$ involves $\sim \frac{D}{\sqrt{\lambda L}} \sim 25$ wavelengths of a quantity whose rms fluctuations are probably $\sim 1-\bar{\eta}$, so a rough guess is:

$$\sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}} \sim \sqrt{\frac{\sqrt{\lambda L}}{D}}\,(1-\bar{\eta}) \qquad \left[\text{Worst } \overset{\text{case}}{\cancel{\text{guess}}} \text{ would be } 1-\bar{\eta}\right]$$

d. Thus,

$$h^{\mathcal{D}}_{BG} \simeq \sqrt{\beta(1-R_E)}\,\frac{1}{Q}\sqrt{\frac{\lambda}{L}}\cdot\sqrt{\frac{\sqrt{\lambda L}}{D}}\,(1-\bar{\eta})\cdot\sqrt{\frac{D}{d}}\cdot\frac{\sqrt{\lambda L}}{D}\,\frac{\xi}{L}$$

$$h^{\mathcal{D}}_{BG} \simeq \underbrace{\sqrt{\underbrace{\beta(1-R_E)}_{10^{-6}}}}_{10^{-3}}\;\underbrace{\frac{1}{Q}}_{10^{-2}}\;\underbrace{\frac{\lambda}{L}}_{\underbrace{\frac{4\times10^{-5}}{4\times10^{5}}}_{10^{-10}}}\;\underbrace{\sqrt{\underbrace{\frac{\sqrt{\lambda L}}{d}}_{\frac{4\text{cm}}{3\text{cm}}}}}_{1}\;\underbrace{(1-\bar{\eta})}_{\simeq\frac{1}{10}}\;\underbrace{\frac{\xi}{D}}_{\frac{\xi}{100\text{cm}}} \simeq 10^{-18}\,\frac{\xi}{1\text{cm}}$$

Best guess

$$h^{\mathcal{D}}_{WC} \simeq h^{\mathcal{D}}_{BC} \times \underbrace{\underbrace{\sqrt{\frac{D}{d}}}_{10}\cdot\underbrace{\frac{D}{\sqrt{\lambda L}}}_{25}\cdot\underbrace{\sqrt{\frac{D}{\sqrt{\lambda L}}}}_{5}}_{10^3} \simeq 10^3\,h^{\mathcal{D}}_{BG}$$

Worst Case

15. This $h^{\mathcal{D}}_{BG}$ differs from the corresponding Michelson case [my calculation of 5/7/87] by:

① Michelson $\mathcal{B} \to Q$ here

② F.P. is better than Michelson ($h^{\mathcal{D}}_{BG}$ smaller) by a factor $\frac{1}{\sqrt{Q}}$ because it has only one scattering spot rather than $\mathcal{B}$ scattering spots. - This gives a factor $\simeq \frac{1}{10}$.

③ I here have used a more sophisticated estimate of Rai's effort:

$$\left|\int \eta(\underset{\sim}{x})\, e^{ik\left(\frac{b}{L\cdot\theta_n}\right)\underset{\sim}{m}\cdot\underset{\sim}{x}}\, e^{-\underset{\sim}{x}^2/\lambda L}\, dx\,dy\right|$$

$$\simeq \sqrt{S_\eta\left(\frac{2\pi D}{\lambda L}\right)\frac{1}{\sqrt{\lambda L}}} \simeq \underbrace{\sqrt{\frac{\sqrt{\lambda L}}{D}}}_{1/5}\ \underbrace{(1-\bar{\eta})}_{1/10} \simeq \frac{1}{50}\ \text{here}$$

$$\simeq \frac{\sqrt{\lambda L}}{D} \simeq \frac{1}{25}\ \text{in my Michelson Calculation.}$$

~~With the~~

It is easy to verify, in accord with this, that my Michelson best guess formula is $\sqrt{Q} \times \dfrac{\sqrt{\lambda L}/D}{\sqrt{\frac{\sqrt{\lambda L}}{D}}\cdot(1-\bar{\eta})}$ × my Fabry-Perot best guess formula.