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Do Wiggle Effects Depend on Mode Cleaner Length?

by Alex Abramovici

Abstract

It is argued that beam wiggle suppression by the mode cleaner does not depend on mode cleaner length alone, but only on the ratios between the mirror radii and the length. It is shown that the amount of frequency noise generated by a given amount of wiggle (when the mode cleaner is also the frequency reference for the laser) is inversely proportional to mode cleaner length.

1 Introduction

Beam wiggle (i.e. fluctuations in pointing, position and diameter of the beam) was found to be a source of noise in high sensitivity interferometers. It was also found that wiggle can be highly suppressed by passing the beam through a single mode fiber or through an optical resonator. All interferometric gravity wave detector prototypes now have such dewigglers (mode cleaners) before the input to the interferometer.

In our 40 m interferometer, the laser frequency is stabilized with reference to the mode cleaner cavity, which is then followed by a single mode fiber. Since there is no dewiggling device ahead of the mode cleaner (reference) cavity, wiggle can still be turned into frequency noise, which then has to be taken out either by a separate servo, using one of the 40 m arms as a reference, by a phase subtraction arrangement, by a coil subtraction scheme or by some combination of all these.

The present note analyzes the influence of the length of the mode cleaner (reference) cavity on its ability to suppress beam wiggle and on the degree to which it turns wiggle into frequency noise. It is shown that, as far as wiggle suppression is concerned, (for given mirror transmission) only the ratios between the curvatures of the mirrors and the length of the cavity matter, but not the length per se. It is shown, however, that the longer the mode cleaner, the smaller the frequency fluctuations associated with a given amount of input beam wiggle, when the mode cleaner is also the frequency reference for the laser.

Section 2 is a reminder of how wiggle is described in an imaging invariant manner. Sections 3,4 analyze the wiggle/frequency noise relationship and wiggle suppression by a mode cleaner, respectively, as functions of mode cleaner length.

2 Imaging-Invariant Description of Wiggle

Consider the arrangement shown in Fig.1, where the laser beam is passed through a mode cleaner cavity before being injected into the interferometer. The TEM_{00} laser beam is affected by pointing fluctuations of r.m.s. amplitude α, by lateral displacements of the beam axis of r.m.s. amplitude

d and by beam diameter fluctuations of r.m.s. amplitude Δ. α, d and Δ are integrated descriptors of what is called in a general way beam wiggle.

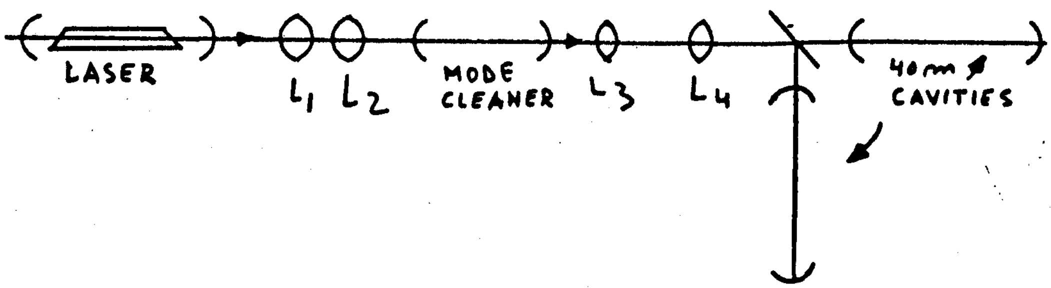

Fig. 1

Beam pointing and beam axis displacement can be described as time varying contamination of the TEM_{00} mode with cartesian higher modes, while beam diameter fluctuations can be described as contamination with cylindrical higher modes:

$$\mathcal{A}(t) = \sum_{n=0}^{\infty} a_n(t)\mathcal{A}_n \tag{1}$$

where $\mathcal{A}_n$ are the mode eigenfunctious, with $\mathcal{A}_0$ describing the TEM_{00} mode. The parameters α, d and Δ can be expressed in terms of the coefficients $a_n(t)$ and viceversa. Since suppressing the higher modes in Eq. (1) will leave a pure TEM_{00} mode, that is a wiggle-free beam, dewiggling is also called mode cleaning.

It is worth noting that while α, d and Δ change when the beam passes through lenses L_1, L_2 (Fig. 1), the coefficients $a_n(t)$ do not, provided the laser and the mode cleaner are mode matched, i.e. when mode $\mathcal{A}_n^{laser}$ is transformed by the lenses into mode $\mathcal{A}_n^{mode\ cleaner}$. A similar comment applies to imaging by lenses L_3, L_4. Thus, if the requirement of mode matching is met, Eq. (1) provides an imaging-invariant description of wiggle. In

what are conditions on cavities to allow $A_n^{laser} \Leftrightarrow A_n^{cavity}$ $\forall n$?

subst perfect conductors for mirrors to define modes?

$\sin\phi \cos(\alpha+\phi)\,d\phi$

$l^2 = 1 + [\ \ ]^2 - 2\frac{T_1\sqrt{R_2}}{1-\sqrt{R_1R_2}}\cos\alpha$

$\frac{dl^2}{d\alpha} = 2$

$= \pi \sin\alpha$

the following it will be assumed, for simplicity, that the amplitude of the beam affected by wiggle has the form:

$$\mathcal{A} = \mathcal{A}_0 + \epsilon(t)\mathcal{A}_N \tag{2}$$

where $\mathcal{A}_N$ is a fixed yet unspecified higher mode and $\epsilon(t)$ is the same at the laser output and at the mode cleaner input, as explained above. N is the sum of mode indices m, n. In the following, wiggle effects will be discussed in terms of Eq. (2), without reference to α, d and Δ.

3 Wiggle/Phase Noise Relationship[1]

In what is now a standard way to stabilize the laser frequency with respect to a reference cavity, the phase of the field which leaks out of the cavity is compared with the phase of the incident field. For an incident TEM_{00} field with unit amplitude, the error signal observed at the output of an appropriate photodiode is:

$$S_0 = C\frac{T_1\sqrt{R_2}\sin\varphi_0}{1+R_1R_2-2\sqrt{R_1R_2}\cos\varphi_0} \qquad C\frac{t_1^2 r_2}{(1+r_1r_2)^2}\frac{4\pi l_N}{c} \tag{3}$$

where T_1 is the transmission of the input mirror, R_1 and R_2 are the intensity reflectivities of the mirrors and φ_0 is the phase which the TEM_{00} field accumulates during a round trip through the cavity. C describes the actual photodetection process and is independent of the cavity parameters. When a higher order mode is added to the beam, $S_0 \rightarrow S = S_0 + S_N$. S_N is calculated by use of an expression similar to Eq. (3), except that the right hand side is multiplied by $|\epsilon(t)|^2$, according to Eq. (2) and $\varphi_0 \rightarrow \varphi_N$: ?

$$S_N = C\,|\,\epsilon(t)\,|^2\,\frac{T_1\sqrt{R_2}\sin\varphi_N}{1+R_1R_2-2\sqrt{R_1R_2}\cos\varphi_N} \tag{4}$$

where[2]:

$$\varphi_N(\nu) = \frac{4\pi}{c}l\nu + 2(N+1)\left[\tan^{-1}(z_2/z_0) - \tan^{-1}(z_1/z_0)\right] \tag{5}$$

[1]See Sheri's thesis. pp. 105-108 for details on Eqs. (3,4)

[2]see e. g. A. Yariv, Introduction to Optical Electronics, Chapter 4

what is numbering system? (N)

where z_1, z_2 are the coordinates of the cavity mirrors with respect to the beam waist, ν is the laser frequency, l is the length of the cavity, c is the speed of light in vacuum and:

for $r_1 = -62\ m = -r_2$
$l = 80\ m$

$$z_0^2 = \frac{l(-r_1 - l)(r_2 - l)(r_2 - r_1 - l)}{(r_2 - r_1 - 2l)^2} \qquad z_0 = 30\ m \qquad (6)$$

$$z_{1,2} = \frac{1}{2}\left(r_{1,2} \pm \sqrt{r_{1,2}^2 - 4z_0^2}\right) \qquad (7)$$

$r_{1,2}$ being the mirror radii.

According to Eq. (5), one can write $\varphi_N(\nu) = \beta l\nu + \psi_0 + \phi_N = \varphi_0(\nu) + \phi_N$, where $\beta = 4\pi/c$, $\psi_0 = 2[\tan^{-1}(z_2/z_0) - \tan^{-1}(z_1/z_0)]$ and $\phi_N = 2N[\tan^{-1}(z_2/z_0) - \tan^{-1}(z_1/z_0)]$. $\phi_1 = 1.87$ rad

If TEM_{00} is the only mode present (no wiggle), the frequency stabilization servo corrects the laser frequency such that $S_0 = 0$, i.e $\varphi_0(\nu_0) = p2\pi$, where p is an integer. The laser frequency is thus locked to the value ν_0. If the laser beam has wiggle, than the servo corrects the laser frequency such that $S_0 + S_N = 0$, which locks the frequency to a value $\nu = \nu_0 + \nu_e$, where ν_e is the frequency error due to wiggle. In order to estimate ν_e, one notes that $\varphi_0(\nu) = \varphi_0(\nu_0) + \beta l\nu_e = p2\pi + \beta l\nu_e$. Furthermore, $\sin\varphi_0(\nu) = \sin(\beta l\nu_e) \sim \beta l\nu_e$ and $\cos\varphi_0(\nu) = \cos(\beta l\nu_e) \sim 1$, since β is a very small number, l is typically a few meters (and can hardly be longer than 4 km) and the frequency error ν_e is assumed to be less than 10 Hz. Similarly, $\sin\varphi_N(\nu) \sim \sin\phi_N$ and $\cos\varphi_N(\nu) \sim \cos\phi_N$. Putting all this together and using also Eqs. (3,4), the frequency fluctuation ν_e is found to be:

$\varphi_0 = 0.9$

β 40m 10Hz = 1.7x10

?

$$\nu_e(t) = \frac{\pi c}{4l\mathcal{F}^2} \frac{|\,\epsilon(t)\,|^2 \sin\phi_N}{1 + R_1R_2 - 2\sqrt{R_1R_2}\cos\phi_N} \qquad (8)$$

where the *finesse* of the cavity is defined as $\mathcal{F} = \pi/(1 - \sqrt{R_1R_2})$.

The mode cleaner currently used in the 40 m prototype has a finesse of about 2,000. It is likely that the mode cleaner used in more advanced versions of the prototype or in LIGO will have similar finesse values. A finesse of 2,000 corresponds to $1 - R_{1,2} = 0.16\%$ and is thus sensitive to small changes in $R_{1,2}$. However, if ϕ_N is not too small[3], the denominator of the second factor in Eq. (8) will not be sensitive to small changes in $R_{1,2}$

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and therefore will have only a weak dependence on finesse. Thus, according to Eq. (8), the frequency fluctuations generated by the interaction of the reference cavity with a wiggling beam are inversely proportional to the length and to the square of the finesse of the cavity.

4 Wiggle Suppression by a Cavity[4]

For an incident field in mode $\mathcal{A}_N$, the fraction of the incident power transmitted by the cavity is:

$$\Theta_N = \frac{T_1 T_2}{(1-\sqrt{R_1 R_2})^2} \frac{1}{1+\frac{4\sqrt{R_1 R_2}}{(1-\sqrt{R_1 R_2})^2}\sin^2\frac{\varphi_N}{2}} \qquad (9)$$

$\sin(\theta_1+\theta_2)$

$= \sin\theta_1\cos\theta_2 + \cos\theta_1 \sin$

where, again, $\varphi_N = \varphi_0 + \phi_N$. If the cavity resonates with the TEM_{00} mode, i.e. $\varphi_0 = p2\pi$, one can see from Eq. (9) that the intensity of mode $\mathcal{A}_N$ is suppressed by a factor:

$$1+\frac{4\sqrt{R_1 R_2}}{(1-\sqrt{R_1 R_2})^2}\sin^2\frac{\phi_N}{2} \qquad (10)$$

at the output of the mode cleaner. From Eqs. (5-7) and the definition of ϕ_N given in Section 3, it results that the wiggle suppression factor given in Eq. (10) depends on the ratios $r_{1,2}/l$, but not just on l.

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[4]see A. Rüdiger et al, Optica Acta **28**, 641 (1981)

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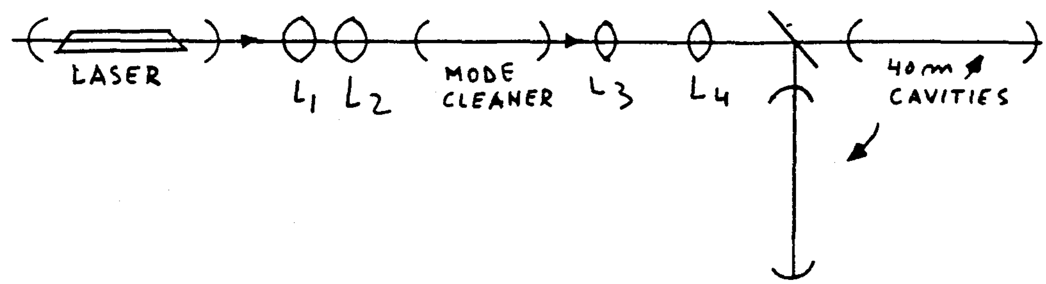

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