Draft Specification for the Slope of LIGO Arms

Specification

The slopes of the LIGO arms should be level to about 1 part in 10^3. Attempts to make the arms still more level will not help much, as the angle between the vertical at the two ends of the 4 km arms is $\frac{2}{3}mrad$.

The present trial layouts at Edwards and Columbia, which have maximum slope of about 20 feet in 13,000 feet, or 1.5×10^{-3}, are close enough to this target to be acceptable.

Rationale

Suspensions for the test masses in gravitational wave interferometers have always been based on the use of a low frequency pendulum as the innermost stage. A pendulum is typically quite anisotropic in its isolation (vertical motions not nearly so well isolated as horizontal.) It nevertheless has a key advantage – its mechanical Q can be substantially greater than that of an oscillator with a spring made of the same material as the pendulum wire. This is because in a properly designed pendulum by far the largest part of the energy storage is in the gravitational field, which has no mechanical losses. This high Q is valuable since thermal noise (Brownian motion) is inversely proportional to the square root of Q.

Anisotropic isolation is tolerable because the interferometer is sensitive (in first order) only to the degree of freedom of the mass which is parallel to the optic axis. Thus, the natural arrangement which has always been used is to have the optic axes of both arms horizontal.

In laboratory-scale apparatus it is possible to adjust the level of the optic axes. It is a different matter in the case of LIGO with its 4 km long arms. Even at sites chosen especially for flatness, substantial extra expense might have to be incurred to make the elevation of the test masses equal to closer than 20 feet or so. (This is the case both at Edwards and Columbia.) Thus the question arises, "How close to truly level do the arms have to be?"

Surprisingly, this is a question which actually has a well-defined answer. The reason is that there is a characteristic angle to the problem, namely the angle subtended at the center of the earth by an interferometer arm. This angle is

$$\Theta = \frac{L}{R_\oplus} = \frac{4km}{6000km} = \frac{2}{3}mrad.$$

The significance of this angle is that, if the optic axis is precisely horizontal at one end of an arm, then the axis makes an angle Θ with the horizontal at the other end of the arm. A straight line 4 km long can never be level everywhere along its length to better than $\frac{\Theta}{2}$, the value at both ends if the line is level at its middle. Thus one can never find an orientation of the arms which does much better than discount vertical motion by a factor of about 3×10^3.

Here are a few numbers to set the scale of the problem. If the earth were perfectly smooth, then a line 4 km long, set level at the middle and with both ends at the surface of the earth, will be buried about 1 foot below the surface at its midpoint. If we make the line level at one end, and place that end at the surface of the earth, its other end will be about 4 feet above the surface.

The argument given in the previous paragraph says that nothing in the installation of the LIGO can save us from having to face misalignments of the beam with the horizontal at

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about the 10^{-3} level. We can turn the question around and ask, "How large a misalignment can the interferometers tolerate?"

The question comes down to how much anisotropy have we allowed up until now in our suspension designs, and how isotropic could we make them if we paid enough attention to the problem. We need to consider both transmitted seismic noise, and the thermal noise in the suspension.

To study the transmitted seismic noise, I have made numerical estimates based on simple models for suspension behavior. (I have neglected to draw in the resonant peaks themselves, and have not taken account of modal coupling which shifts the frequencies of resonances.) I have made models which are indicative (approximately, at least) of the performance of suspensions built so far, and of some ideas for improving the isotropy of current designs.

Figure 1 shows the vertical and horizontal test mass motion for a suspension design which bears some resemblance to the suspension on the 40-meter interferometer. (The actual performance may be somewhat different. I would welcome better numbers, but I think for the purposes of this exercise it is less important that actual numbers be used than that the issues at stake be pointed out.) The model I took is shown in the upper right hand corner. For horizontal motion I took a 1 Hz pendulum frequency, and assumed the stack gave three resonances near 7 Hz. (I assumed two poles each for 3 layers, or a total of 6 poles at 7 Hz. Is this optimistic?) For vertical motion, I assumed the wire-stretching mode was at 30 Hz, and that the stack modes were at 15 Hz. (These numbers especially need checking.) I multiplied the transfer functions by the same input spectrum for both horizontal and vertical motion, $10^{-7} cm/\sqrt{Hz}$ below 10 Hz, and $10^{-5} cm/\sqrt{Hz} \times (\frac{1Hz}{f})^2$ above 10 Hz. The curve for vertical motion is a factor of 10^5 above the curve for horizontal motion in the 30 Hz to 100 Hz region where the curves cross levels comparable to LIGO shot noise (say $10^{-17} cm/\sqrt{Hz}$ for definiteness). I have also drawn in a curve at a level of 10^{-3} times the vertical motion, to indicate schematically that it would dominate the seismic noise budget in a well-levelled 4 km interferometer.

Figure 2 shows the results for a related model, where I have instead assumed that the vertical resonances in the stack were at the same frequencies as the horizontal ones. Then, in the critical frequency range the vertical motions are 10^3 times larger than horizontal. This means that their effect can be made comparable to or smaller than the horizontal if the interferometer is level to about 1 mrad.

Figure 3 shows the results for a simplified model of a double pendulum, of the type currently being tested at MIT, suspended from one stage of isotropic isolation with a resonant frequency of 5 Hz. Although some thought was given in the design to ways to make the isolation isotropic, this model clearly is not quite isotropic enough, as 10^{-3} times the vertical motion still exceeds the horizontal motion in a crucial frequency band.

Figures 4 and 5 show two modifications of the model of Figure 3. Figure 4 shows the results if we could replace the isotropic 5 Hz spring with an isotropic $\frac{1}{10}$ Hz isolator (a design goal for a magnetic suspension, say.) The levels of both horizontal and vertical motion are reduced, and there is a corresponding narrowing of the gap between vertical and horizontal motions because the crossing point of the "shot noise" comes at a lower frequency where the pendulums are doing less. Figure 5 shows the configuration of Figure

3 with the addition of an extra stage of 3 Hz vertical isolation only. That stage makes the vertical motion of the mass small enough even for a departure of the arms from level of 1%.

The lesson I draw from these numerical examples is that, without some attention to suspension anisotropy, we might find ourselves with a surprisingly large contribution from vertical seismic motion. The more encouraging note is that measures which are not very heroic can tame the problem. There may in fact be other ways to address the problem as well, so I don't think it looks serious.

One other aspect of suspensions which we have to investigate is thermal noise, the very feature which led to the choice of anisotropic suspensions in the first place. For frequencies above the resonance, the thermal noise spectral density is given by

$$x_{thermal}(f) = \frac{1}{f^2}\sqrt{\frac{kTf_0}{2\pi^3 mQ}}$$

(a slightly recast version of equation (5), page V-21 in the Blue Book). If the vertical mode is in the vicinity of 100 Hz instead of 1 Hz, and if its Q is lower by about 10^4, then the vertical thermal noise is 10^3 times larger than in the horizontal direction. Thus as long as the arms are level to 1 part in 10^3 or better, then the noise we expected from the horizontal motion is still the dominant effect.

There are a couple of questions which we should think about more, although I don't see how they would affect the specification we make on the arm slope. One is whether the performance of the anti-seismic interferometer system is limited by these kind of slope effects – in its present version it relies on measuring only a single degree of freedom. Another question to ponder is to what degree we can expect anisotropic motion of the test mass itself without cross-coupling.

Peter R. Saulson
December 10, 1988

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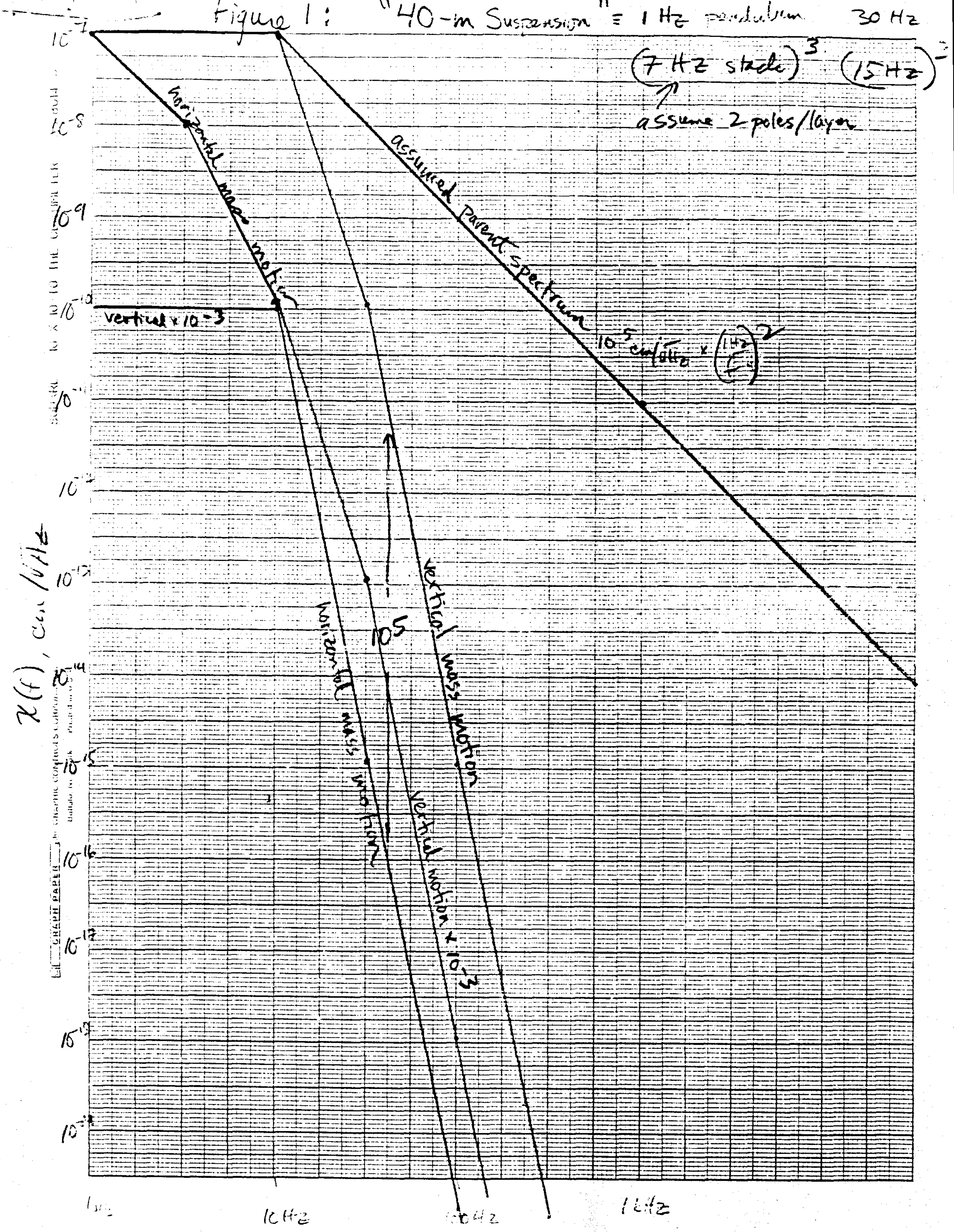


Figure2: "40-m suspension with isotropic stacks" =

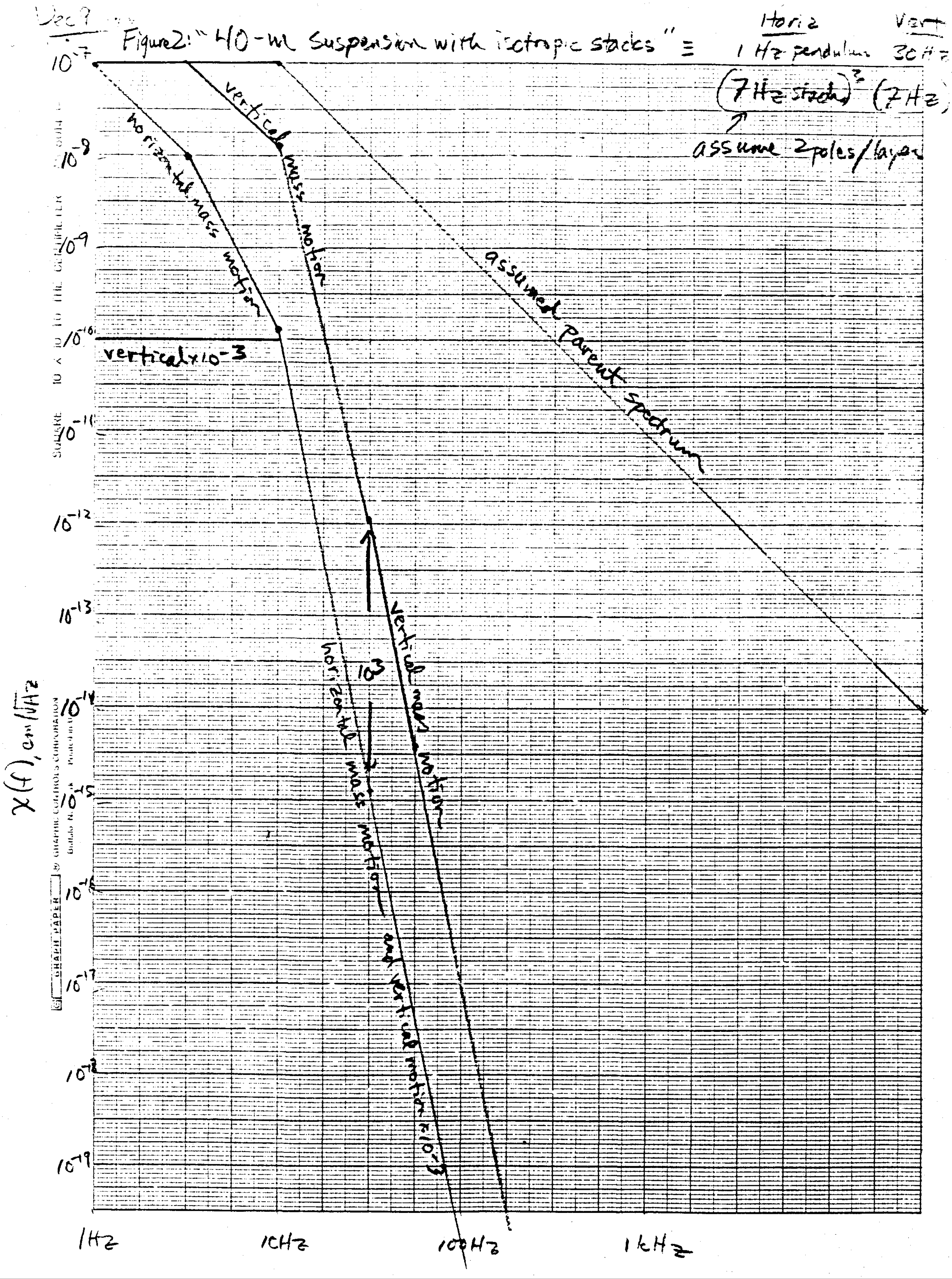
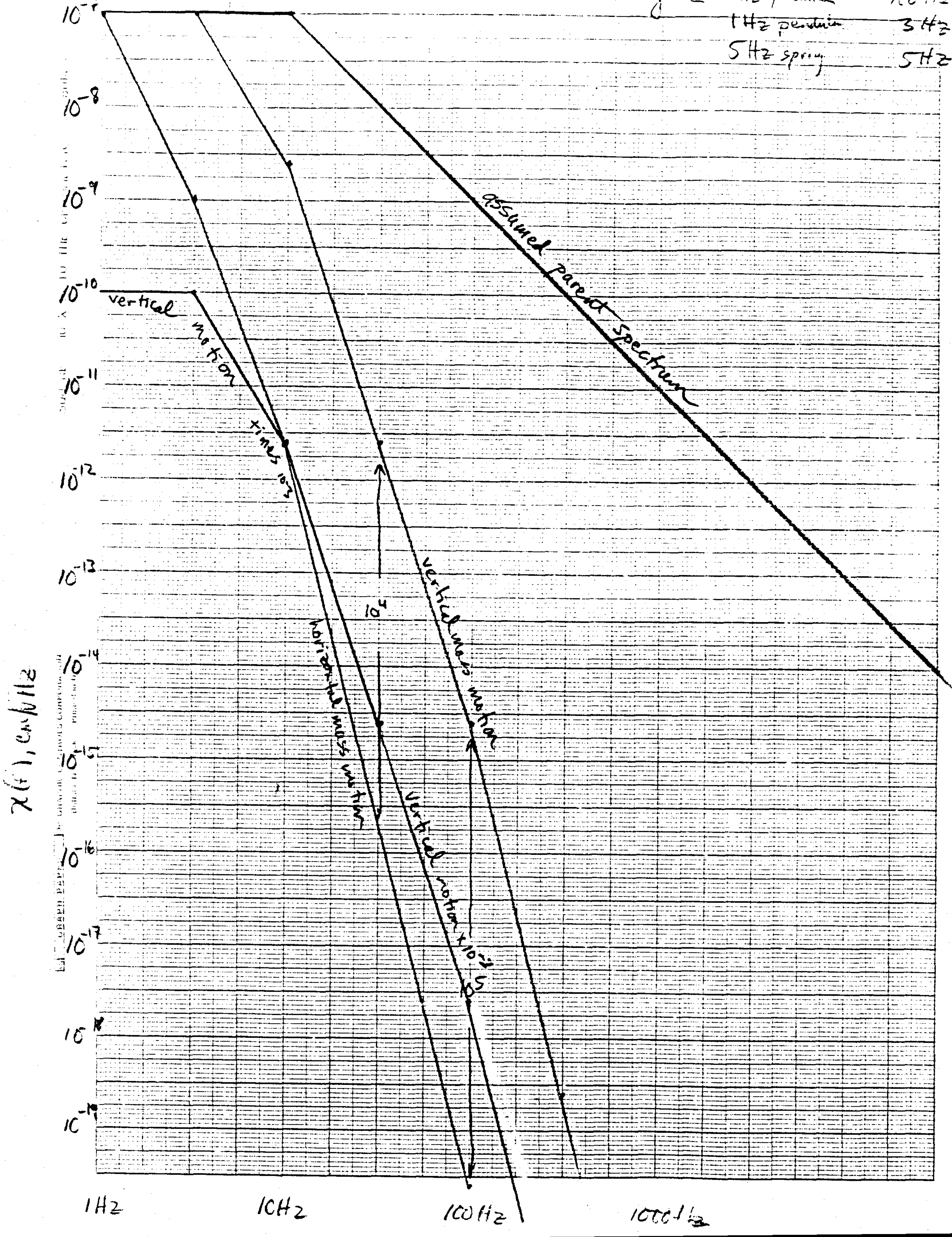
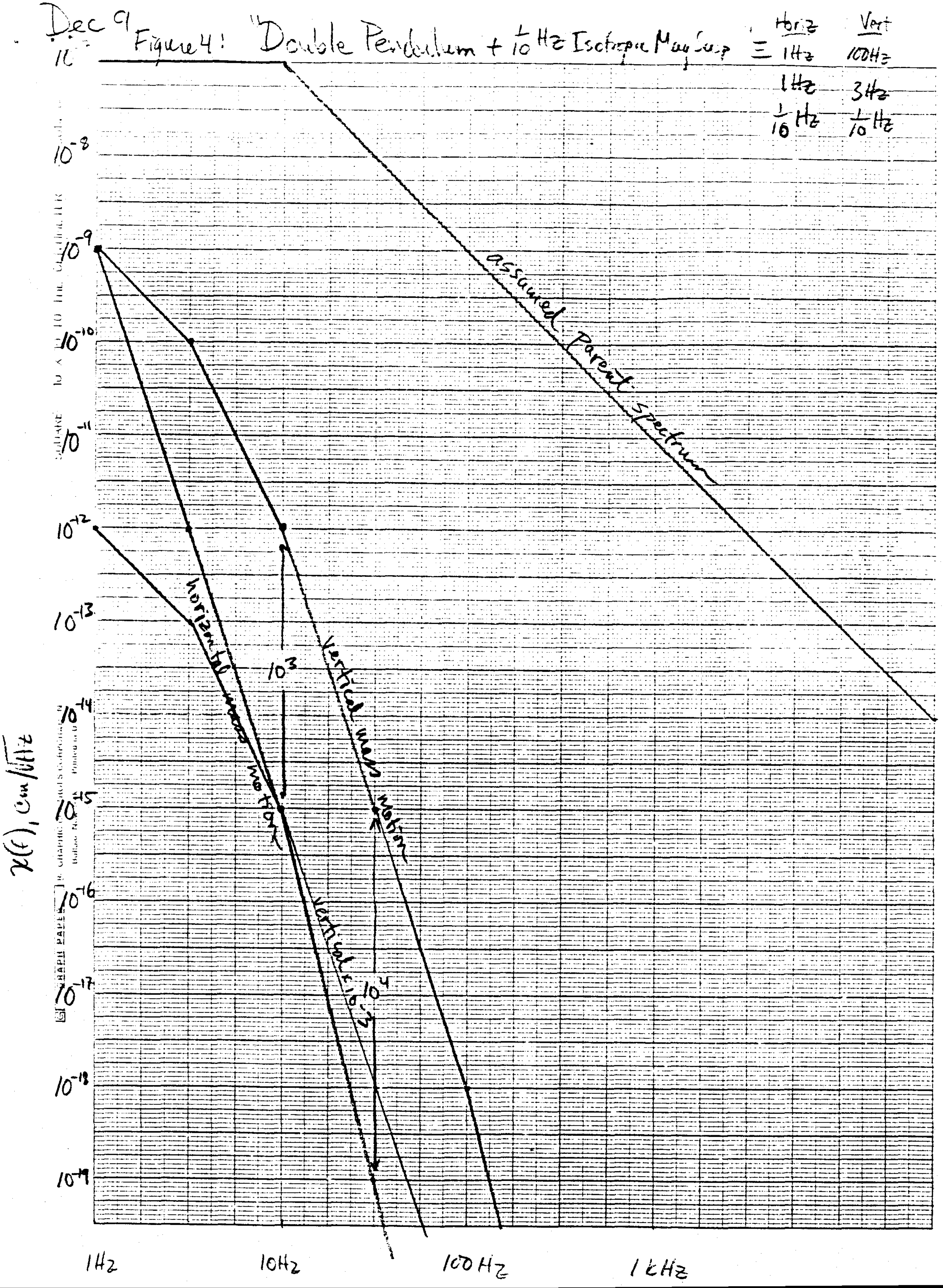


Figure 3: "Double Pendulum + Spring" ≡

Horiz	Vert
1 Hz pendulum	100 Hz
1 Hz pendulum	3 Hz
5 Hz spring	5 Hz

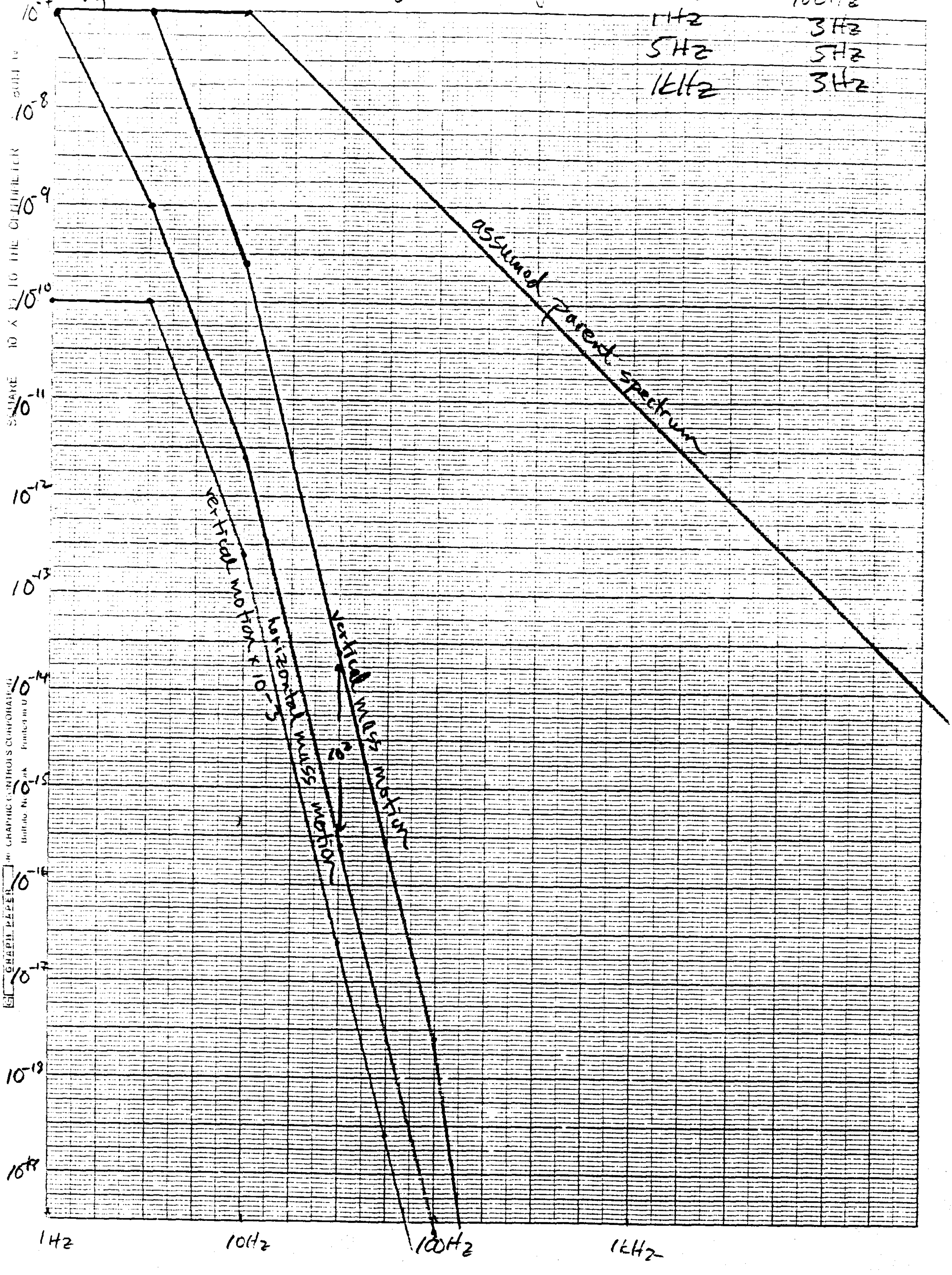


Dec 9 Figure 4: "Double Pendulum + $\frac{1}{10}$ Hz Isotropic Mag Susp" ≡



Dec. 9 Figure 5: Double Pendulum + Spring + Vert Spring" ≡

Horiz	Vert
1 Hz	100 Hz
1 Hz	3 Hz
5 Hz	5 Hz
1 kHz	3 Hz



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December 10, 1988

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Figure 1: "40-m Suspension" ≡ 1 Hz pendulum

Horiz Vert 30 Hz

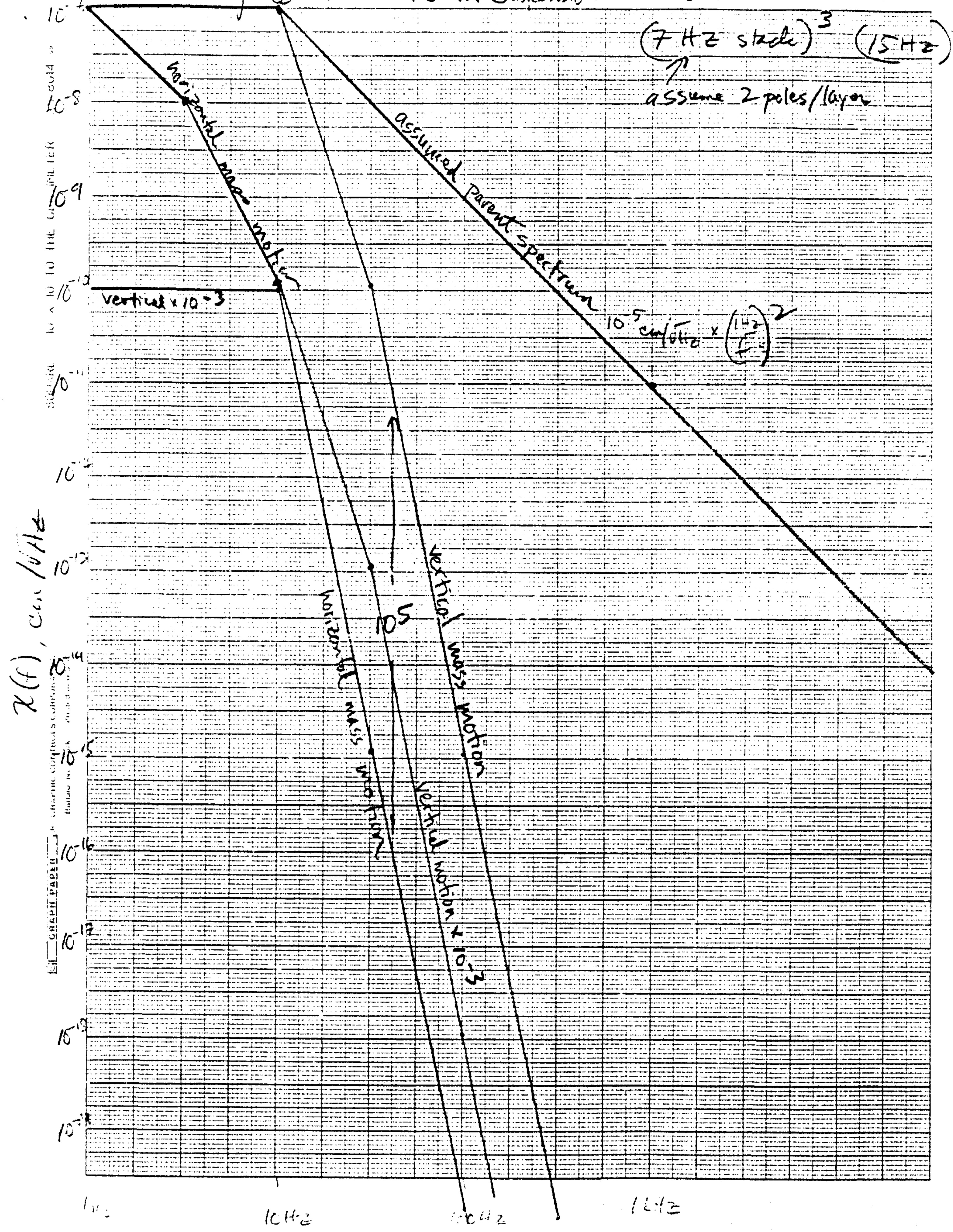
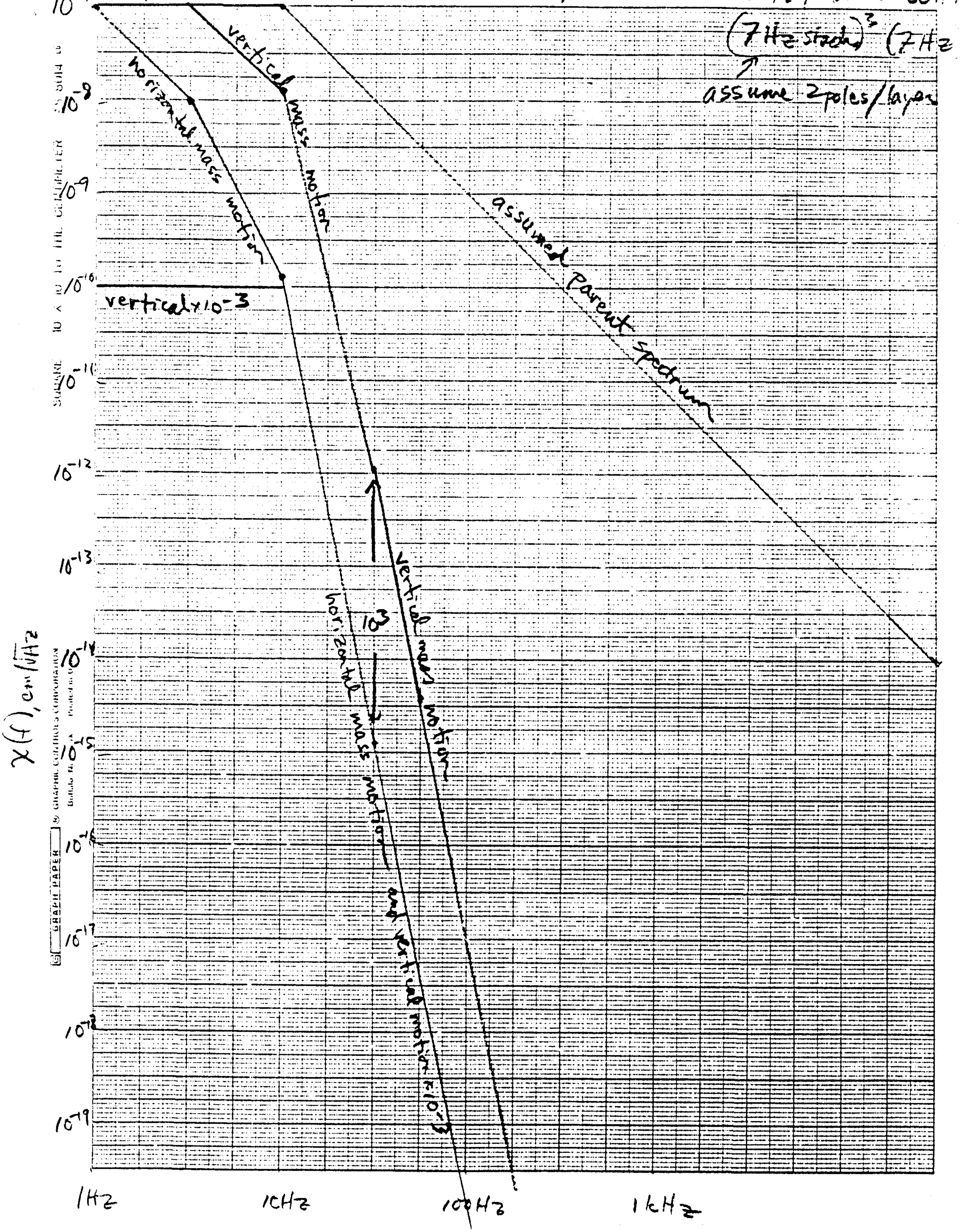


Figure2: "40-m suspension with isotropic stacks" ≡ Horiz 1 Hz pendulum Vert 30 Hz



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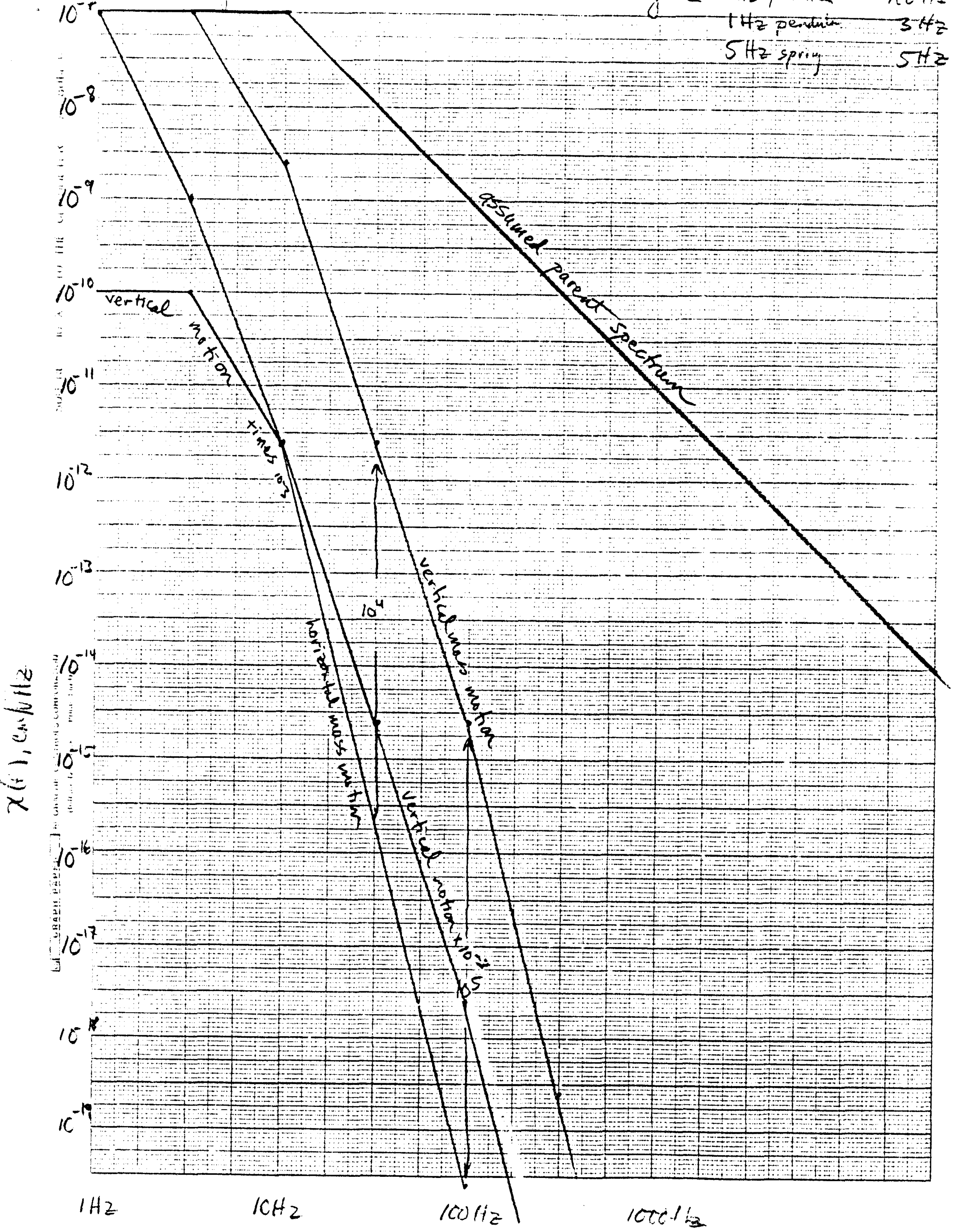

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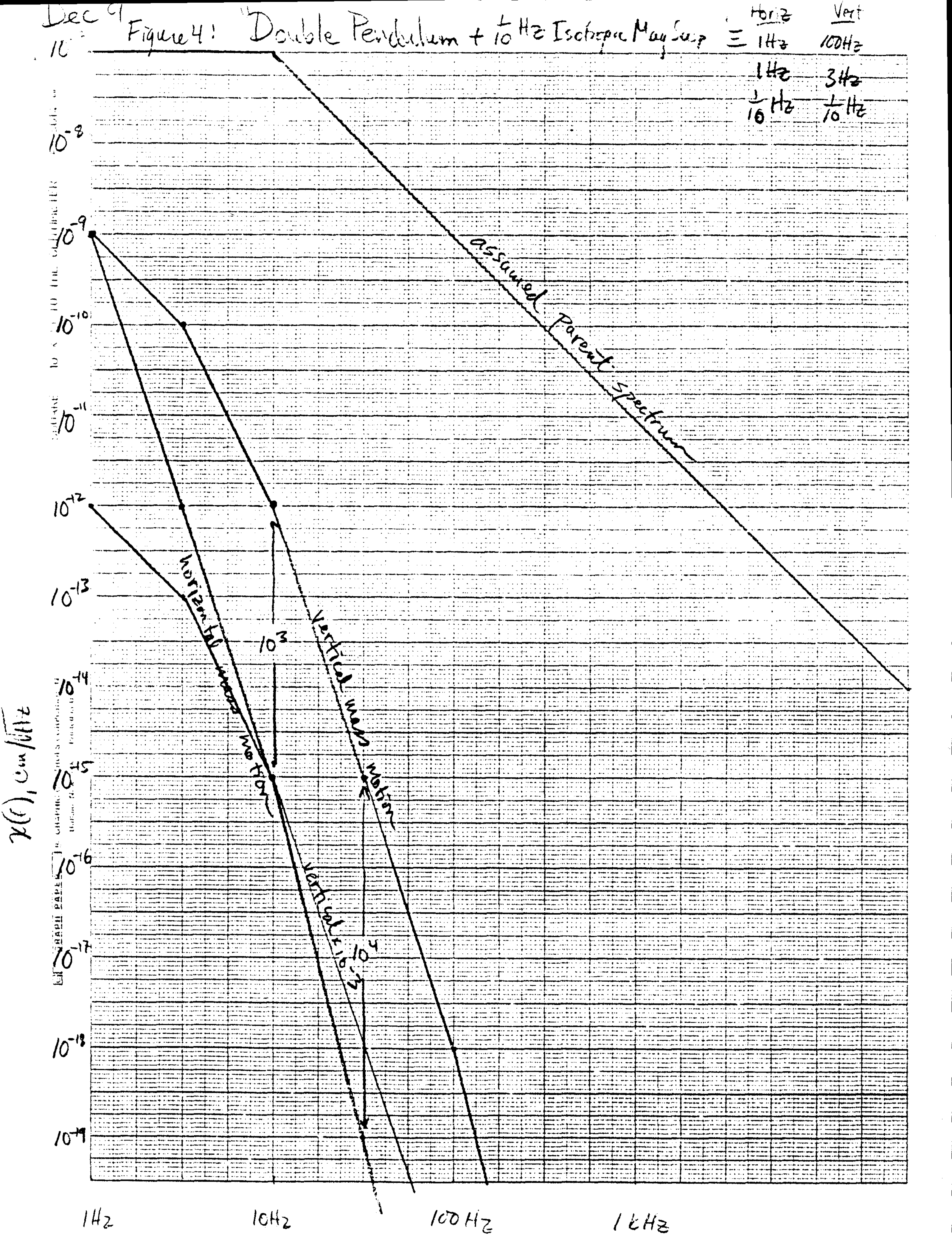
Figure 4: "Double Pendulum + $\frac{1}{10}$ Hz Isotropic Mag Susp"



Dec 9 Figure 5: Double Pend[illegible] + Spring + Vert Spring " ≡

Horiz	Vert
1 Hz	100Hz
1Hz	3Hz
5Hz	5Hz
1kHz	3Hz