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Variable Transmission Mirror for Recycling

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Abstract

It is shown that the maximum possible recycling factor is equal to the inverse of the losses in the interferometer. The formulæ required for the design of a variable transmission mirror (VTM) for recycling are given. It is shown that a VTM which allows to achieve a recycling factor close to the maximum one can be made using standard mirrors.

1 Introduction

In a sample design for a LIGO interferometer, the light from the laser source is divided at the beam splitter, sent to the two Fabry-Perot cavities and recombined at the beam splitter[1]. The system is tuned such that a dark fringe is observed at the port perpendicular to the incident light beam, while all the outcoming power[2] is sent back through the input port. Therefore, the interferometer behaves like a generalized mirror with reflectance R_I and loss L_I such that $R_i + L_I = 1$.

In order to increase the power stored inside the cavities and thus improve interferometer sensitivity, Ron Drever has suggested that one adds a mirror[3] before the beam splitter, thus turning the system into a generalized Fabry-Perot, inside which the optical power is built up[4]. The maximum power level inside this system is reached when

$$\Theta = \Lambda + L_I \tag{1}$$

where Θ and Λ are the transmission and the loss of the recycling mirror, respectively. Since L_I is basicaly given once the best effort to reduce losses in the interferometer has been made, and since one can not tune the usual mirror coating procedure finely enough for Θ to meet the above optimum recycling condition, the capability to tune Θ directly is called for. In Ron's tentative design, this is done by a short cavity with variable mirror spacing. Adjusting the mirror spacing detunes this cavity from resonance to a larger or lesser extent, thus providing a variable transmission mirror (**VTM**).

Section 2 gives an estimate for the interferometer losses L_I under a specific set of assumptions. Section 3 contains a collection of formulae relevant to the characteristics of a VTM. Section 4 gives an estimate for the maximum recycling factor achievable under the assumptions made in Sections 2,3, as well as the corresponding parameters for the VTM. Section 5 is a summary.

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[1] see the tentative design submitted by Ron Drever

[2] which is equal to the input power less the power lost through different mechanisms inside the interferometer

[3] called recycling mirror

[4] R. W. P. Drever, in *Gravitational Radiation*, ed. N. Deruelle and T. Piran (North Holland, Amsterdam, 1983), p. 321

2 Interferometer Losses

2.1 Assumptions

i. Each arm of the interferometer consists of a 4 km Fabry-Perot, with storage time 1 ms;
ii. Mirror loss (absorbtion plus scattering) is 50 ppm[5];
iii. The back surfaces of the coupling mirrors (Fabry-Perot input mirrors) have gyro-grade antireflective coatings so that reflection loss is brought down to 100 ppm[6];
iv. There are no losses due to cavity misalignment or imperfect mode matching or due to wave front distortion;
v. 1% of the light is picked off at the beam splitter for modulation purposes (0.5% loss per arm).

2.2 Losses

The energy storage time τ of a resonator is given by:

$$\tau = \frac{l}{c(1-\sqrt{R_c R_h})} \tag{2}$$

where l is the resonator length, c is the speed of light and R_c R_h are, respectively, the reflection coefficients of the coupling and high reflector mirrors. Eq.(2) and Assumptions i,ii yield $R_c = .97356$ corresponding to a transmission $T_c = .02639$.

On resonance, the cavity reflection coefficient is:

$$\rho_{FP} = \left(\frac{\sqrt{R_c} - (1-L_c)\sqrt{R_h}}{1-\sqrt{R_c R_h}} \right)^2 \tag{3}$$

where L_c is the coupling mirror loss. For R_c as derived above and the assumed losses at each mirror, $\rho_{FP} = 0.985$, i. e. a loss of 1.5% takes place at each 4 km cavity. Adding this up with 0.5% loss due to modulation

[5]such values have been actually measured in our lab, for high reflectors

[6]the laser-gyro industry is currently capable of making such coatings for $\lambda = 632.8$ nm

(Assumption v) and allowing for another loss of 0.5% due to imperfect beam overlap at the beam splitter, one ends up with a loss in the interferometer:

$$L_I = 2.5\% \tag{4}$$

where the reflection losses at the back of the coupling mirror have been neglected since they affect only the next significant digit (Assumption iii) and the losses due to absorbtion in the mirror substrate have also been neglected.

3 The Variable Transmission Mirror

Consider a VTM consisting of two identical, closely spaced mirrors[7], with transmission T and loss L, such that $T \gg L$. Transmission and loss of the VTM are given by:

$$\Theta = \frac{T^2}{(L+T)^2} \times \frac{1}{1+\delta^2} \tag{5}$$

$$\Lambda = \frac{1}{1+\delta^2}\left(\frac{(2-L)L - RL(2+L)}{(L+T)^2} + L\delta^2\right) \tag{6}$$

where δ is the detuning from resonance, defined as $\delta = (\nu - \nu_{res})/(\nu_{1/2} - \nu_{res})$, ν_{res} being the frequency of the resonance and $\nu_{1/2}$ being a half power point. Eqs. (5,6) are valid as long as the detuning is much smaller than the finesse:

$$\delta \ll \frac{\pi}{T} \tag{7}$$

The assumption that $T \gg L$ allows to simplify Eqs. (5,6):

$$\Theta = \frac{1}{1+\delta^2} \tag{8}$$

$$\Lambda = \frac{L}{1+\delta^2}\left(\frac{2}{T} + \delta^2\right) \tag{9}$$

[7] the spacers may be small piezoelectric discs, so that the spacing can be continuously adjusted

4 The Recycling Factor

The recycling factor RF is defined as the ratio between the power in a system with recycling and the power in the same system in the absence of recycling, while the input power generated by the laser source is the same. It has been shown in the Introduction how the recycled interferometer can be regarded as a generalized Fabry-Perot, whereas the interferometer proper is the high reflector with reflectance $R_I = 1 - L_I$ and the recycling mirror with reflectance $R_R = 1 - \Theta - \Lambda$ is the coupler. The recycling factor thus is the ratio between the intensity of the field stored in a generalized Fabry-Perot and the intensity of the incident field, i. e. $RF = \Theta/(1 - \sqrt{(1-\Theta-\Lambda)(1-L_I)}\ )^2$. Since Θ, Λ and R_I are all much smaller than 1:

$$RF = \frac{1}{\Theta} = 1 + \delta^2 \tag{10}$$

where the optimum recycling condition Eq. (1) has been assumed to take place.

For given L_I, the highest recycling factor that can be achieved corresponds to the ideal case $\Lambda = 0$:

$$RF_{max} = \frac{1}{L_I} \tag{11}$$

(see Eqs. (1,10)). In practice, one should require that $\Lambda/\Theta \ll 1$.

It is interesting to note that supermirrors are not required for the VTM. For example, for $T = 3\%$, $RF = 37.3$, which is obtained by first determining $\delta = 6.03$ by use of Eqs. (1,8,9) and then using Eq. (10). One notes that from Eq. (11) one estimates that $RF_{max} = 40$. The above values for δ and T meet the condition Eq. (7).

It is easy to see that for the standard type of VTM mirrors considered above ($T \sim 1\%$, $L \sim 0.1\%$), δ^2 is only weakly dependent on T. Therefore, the recycling factor RF is only marginally affected by the specific choice of T, as long as $T \gg L$ and $T^2 < L_I$[8]. Also, Eq. (7) may put a more stringent upper limit on T. In the example considered here, T can be chosen in the range $1\% - 5\%$.

[8]This condition means that the combined transmission of the two mirrors in the VTM is smaller than L_I, so that Eq. (1) can be valid

5 Summary

It is shown that the maximum achievable recycling factor is the inverse of the losses in the interferometer. Therefore, avoiding losses is critical, e. g. one has to do the best in terms of beam overlap at the beamsplitter and as little light as possible should be diverted for modulation.

In the actual example considered above, most of the losses were ocuring inside the cavities, due to the assumption of a 1 ms storage time. Shorter storage times lead to lower intra-cavity losses and therefore allow for a higher recycling factor. The choice of a particular storage time is a complicated issue depending, among other things, on the particular type of astrophysical gravity wave source one is looking for.

An example is given which shows that one can achieve a recycling factor close to the maximum one corresponding to the losses in the interferometer by using a VTM which consists of mirrors with standard coatings. Also, it is found that the transmission T of the mirrors in the VTM can be chosen within a relatively wide range.

Acknowledgement: A theoretical paper on recycling has been recently published[9]. There is a partial overlap between the present note and that paper. The emphasis in this note is on a quick, practical way to evaluate the recycling factor, as well as on the evaluation of the parameters for a short cavity used as the recycling mirror.

[9] J.-Y Vinet, B Meers, C. N. Man, A. Brillet *Optimization of long-baseline optical interferometers for gravitational-wave detection* Phys. Rev. D **38**, 433 (1988)