Draft Report on Imported Noise

In this report, I will present my best estimate of the imported noise problem. I will first carry out an idealized calculation. Next, I will present numerical estimates for the parameters in the model, supported wherever possible by measurements. Some measurements I have made, and some by others, will be used to point out the systematic uncertainties in the model. Finally, I will summarize the results in terms of guidelines for the LIGO engineering effort.

Trying to calculate the amount of noise imported to a remote site is not as easy as I thought it would be when I embarked on the calculation. I had hoped to model the problem by treating our installation as a set of noise sources, characterized perhaps by their strengths and output impedances. These noise sources are connected to the ground, perhaps through a compliant isolator. The response of the ground can then be calculated, I had hoped, by solving an impedance-divider problem. Finally, using some law for the propagation of waves in the ground, the response at a distant place could be determined. The summation of the effects from all of the installed equipment would give the total "imported noise".

The difficulty with this method is that the objects we are trying to model are less amenable to simplifying idealizations than are the objects that physicists usually choose to work with. Even propagation of waves in the ground is complicated enough that there is no single agreed-upon law for the distance dependence of their amplitude. (In fact, measurements made under seemingly identical conditions often give different results, indicating the existence of uncontrollable variations in conditions, such as the structure of the earth under the sites in question. See, for example, Figures 1, 2, and 5 in the report by Ferahian and Ward.)

In spite of all of these difficulties, it is worth carrying through the calculation to get a feel for the magnitudes involved, to learn some scaling laws, and to see which physical quantities are the crucial ones.

A vibration source can be characterized by a velocity output spectrum (when it is free of any mechanical load), and an output impedance. Surprisingly, there is very little written literature (I wasn't able to find any) characterizing either quantity. I was able to make a library of typical spectra by recording the output of a small piezoelectric accelerometer placed on a variety of pieces of lab equipment. (See Fig. XX) The free vibration spectra can all be crudely described as have white acceleration spectral densities from 100Hz to a cut-off frequency between 500 Hz and 1 kHz, with a level betwee 0.1 and 1 $cm/sec^2\sqrt{Hz}$. Above the cut-off, acceleration spectral density falls approximately as f^{-3}.

Impedance can also be measured, but not with equipment readily available in our labs. For simple cases (in particular, things of simple geometry and infinite extent) it can be calculated. For finite objects, we are likely interested in a frequency range where mechanical resonances are rather closely spaced. In this regime, the typical impedance (geometric mean between peaks and troughs) is given by $\sqrt{kM}$, where k is the DC spring constant at the place in question, and M is a typical mass. Help in estimating these quantities comes from another relation, namely that the lowest resonant frequency ω_0 is given by $\omega_0 = \sqrt{\frac{k}{M}}$. Resonant frequencies for most things in the lab are in the range of 30 Hz to a few hundred Hz. A high impedance source is something massive and stiff. A laser power supply (which is also noisy) might be modelled as something with a mass of 100 kg and a first resonance around 100 Hz. This give a characteristic impedance in the vicinity of $6 \times 10^7 dyne - sec/cm$.

We also need an estimate of the impedance of the floor or ground, since the local motion caused by the equipment is given by

$$v_g = v_e \times (\frac{Z_e}{Z_e + Z_g}).$$

Here Z is the impedance, v the motion spectrum (typically represented as velocity but easily scalable to either position or acceleration), and the subscripts e and g represent equipment and floor or ground, respectively. Measurements of soil impedance are available. Gutowski *et al.* (Noise Control Engineering, vol. 10, no. 3, p. 94, 1978) give graphs of impedance versus frequency for clay and sand, measured over the range 0-100 Hz, using a circular pad with a diameter of 0.4 m. Below 30 Hz, the impedance falls steeply with frequency. At higher frequencies the impedance shows some sharp features, but can be approximately described as a constant with a value of about 10^8 dyn-sec/cm.

I couldn't find similar measurements for floors, but Goyder and White (Journal of Sound and Vibration, vol. 68, no. 1, p.59, 1980) give a method for calculating the impedance of an infinite slab. They give

$$Z_{slab} = 8\sqrt{B\rho h},$$

where

$$B = \frac{h^3 E}{12(1-\nu^2)},$$

and where h is the slab thickness, ρ is the volume density of the slab material, and ν is the Poisson's ratio for the material. Note that this impedance is a real number, representing the "radiation resistance" for launching waves in the slab. Z is proportional to the square of the thickness of the slab, with the constant of proportionality determined by the material properties. Plugging in handbook numbers for concrete (not reinforced), I find $Z_{slab} = 3.75 \times 10^8 dyn - sec/cm$ for h = 6 inches, $5.8 \times 10^9 dyn - sec/cm$ for h = 24 inches.

The floor or the ground is comparable in impedance to the most massive, stiffest equipment found in a lab. Thus, in the simple picture we have been using so far, the ground motion right next to these pieces of equipment should be comparable to the free motion. For smaller, more compliant things, the driven ground motion should be smaller than the free motion by approximately the ratio of the impedances.

If the equipment is resting on a compliant isolator, or even just on feet that are softer than the rest of it, the amount of vibration communicated to the floor may be substantially attenuated. If we can make the approximation that the isolator is much softer than both the equipment and the floor, then the magnitude of induced floor vibration is given by

$$v_g = \frac{Z_i}{Z_g} v_e,$$

where Z_i is the impedance of the isolator. At least at low frequencies, the isolator can be modelled as a simple spring. Attachment to the real, frequency-independent impedance of our model floor gives a net attenuation which is proportional to $\frac{1}{f}$. Because the isolation depends on the ratio of isolator impedance to floor impedance (and not to equipment impedance), there can be substantial isolation below the resonant frequency of the equipment on the isolator. For example, an isolator which makes a 100 kg piece of equipment have a 10 Hz resonance give a transmission of only 0.04 at 1 Hz if placed on a floor with impedance of $1.5\times10^9\, dyn-sec/cm$ (a 12-inch thick concrete floor, according to our model).

The final part of our physical model should be an expression for the amplitude of motion some distance r away from a source. Here there is disagreement among the experts, mainly because physical conditions can vary so much. Surface waves in a lossless medium should have their amplitude fall as $\frac{1}{\sqrt{r}}$, by conservation of energy. The transmission paths of interest to us should be dominated by surface waves, so this would be a good model except for the fact that there are non-negligible losses. It is traditional to model the losses by multiplying the $\frac{1}{\sqrt{r}}$ factor by an exponential with a (frequency-dependent) attenuation length. Gutowski *et al.* claim that empirically it is at least as good to just model the fall-off as $\frac{1}{r}$ instead. Data from blasting, included as Figure 5 in the report of Ferahian and Ward, seem to show $\frac{1}{r} \times exp^{-\alpha r}$. Data in other figures in that report are so ratty that it hardly seems possible to represent them with a simple functional form.

Floors should carry surface waves, perhaps with substantial losses. An extra complicating factor in a floor is the existence of standing waves due to the many impedance discontinuities (edges of building, walls, columns, heavy equipment, etc.)

Last October, I made some measurements to compare with these sorts of calculations. In one set, I compared the acceleration spectrum at one corner of the case of the power supply for our Argon laser with the acceleration spectrum on the concrete slab floor right next to one foot of the power supply. Over a broad band from below 50 Hz to several kHz, turning on the laser power supply substantially increased the vibration of the floor. Below 100 Hz, the magnitude of the transfer function is around -10 dB. It falls rapidly to between -40 dB and -50 dB by 250 Hz, then stays level until around 1 kHz, at which point it falls to about -60 dB. It stays between -50 and -60 dB up to 5 kHz.

If the power supply were as stiff as I had assumed, and if it were in good contact with the floor, then the model given above predicts that the transfer function should be in the range of -10 to -20 dB, independent of frequency except for some sharp features due to resonances. Clearly, that prediction is a poor match to the measurements. We could have a better explanation of the measurements if we assume that the corner of the power supply, or its caster wheels, are behaving like a compliant isolator. This is the sort of effect needed to explain the decline of coupling with frequency. We would then say further that the "isolator" has a constant real part of its impedance which is roughly 10^{-3} times the floor impedance, leading to the constant level of coupling at high frequency.

It is not very satisfying to have such difficulty inventing a model which can match the measured vibration transfer function from noisy equipment to the floor. But perhaps it is unseemly to complain, since the discrepancy has the sense of showing a smaller noise coupling in reality than we had feared.

The level of the noise on the floor is roughly 20 dB higher with the laser on than with it off. If we assume that our site will be about another factor of ten quieter than the MIT lab, we need to have roughly 40 dB of isolation to render this piece of equipment "invisible". I think that this is relatively straightforward to achieve.

We can take a positive lesson from this exercise. That is that it is a relatively straightforward matter to isolate any troublesome vibrating equipment that we install, should the vibration transmitted to the floor be large enough to worry about. The number of items likely to require this sort of attention is small.

The one class of equipment that I still worry about in terms of imported vibration are the power substation and, perhaps, the laser cooling plant. These are massive pieces of equipment which are likely to be inherently noisy. They have the further awkward feature that they will be difficult to isolate, since they are, firstly, physically large and, secondly, likely to be bought as turnkey systems from others. (Is this right?)

We can make a quick-and-quite-dirty estimate of the amount of noise generated by the transformers in the substation. Make the following assumptions: Power being drawn is $\frac{1}{2}$ MW. One part in 10^4 is dissipated as vibration in the ground. The vibration energy is spread equally among the first 20 harmonics of 60 Hz. The impedance of the ground is that given by Gutowski *et al.*, scaled by the radius of the region of force application to an area 10 meters across.

With these assumptions, we find the ground motion amplitude by solving the equation

$$P = Zv^2.$$

With $2.5W$ per harmonic, and an impedance of $2.5 \times 10^6 N - s/m$, we find for the rms velocity at each harmonic about $1mm/sec$, a huge number. If this vibration propagated as $\frac{1}{\sqrt{r}}$, then 1 km away the amplitude is still $10^{-2}\, cm/sec$. Switching to displacement units, we have an amplitude of $2.5 \times 10^{-6}\, cmrms$ at 60 Hz, and $1.3 \times 10^{-7}\, cm$ at 1200 Hz. These peaks would show up strongly above the broadband seismic background.

The assumptions I used above need checking. In particular, the estimate of 10^{-4} of the power transmitted to the ground is just a wild guess.

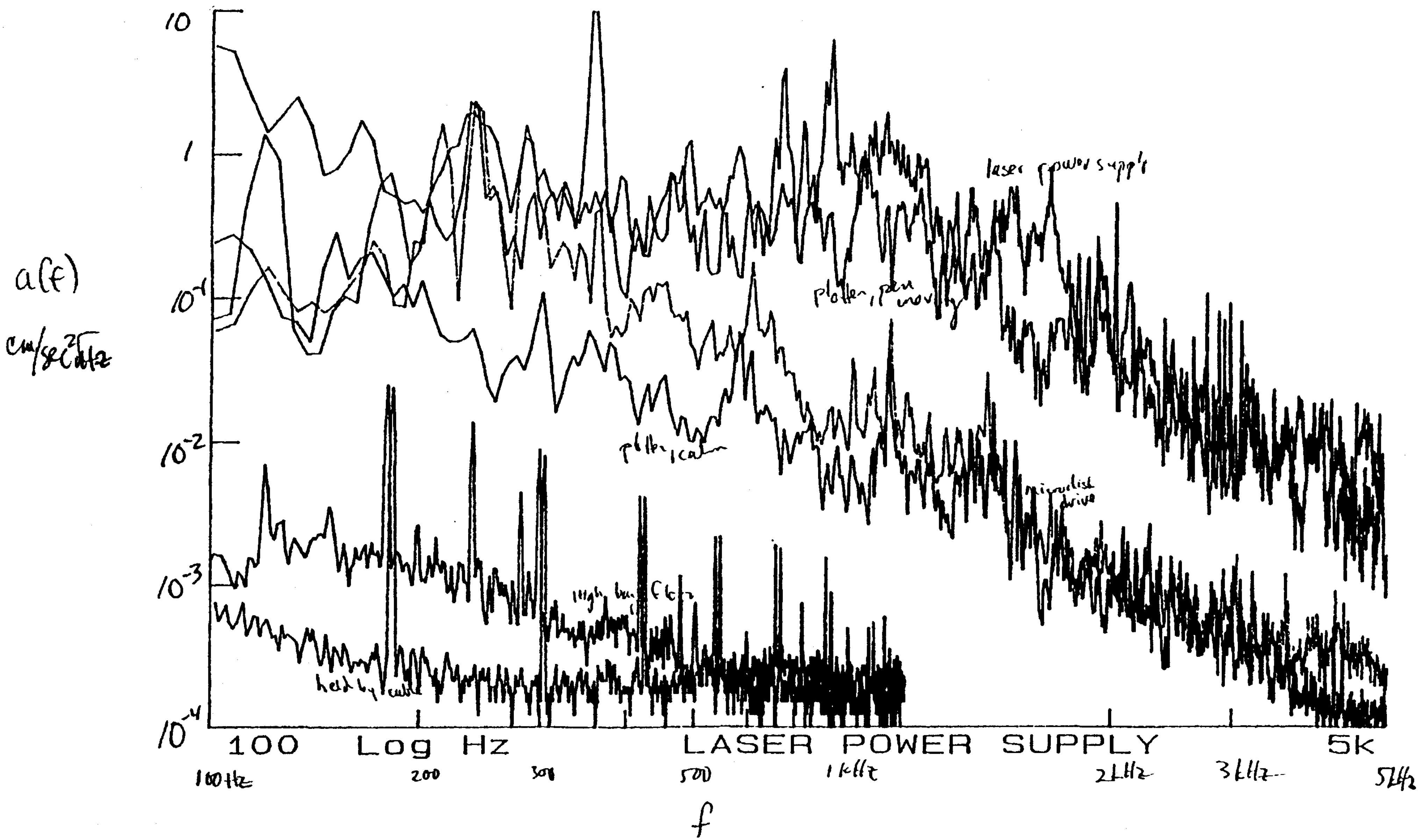
There is one aspect of the calculation above which is almost certainly pessimistic. We ignored the likelihood that there is exponential attenuation (loss) of the wave as it propagates. Figure 5 of Ferahian and Ward, which shows the amplitude of a wave generated by blasting versus the distance of propagation, indicates a steep drop when a wave has to travel farther than 500 feet or so. This number is almost certainly frequency dependent, with the sense of greater losses at higher frequencies. Knopoff (" Attenuation of Elastic Waves in the Earth" in *Physical Acoustics*, vol. IIIB, edited by Mason, 1965) gives graphs of attenuation versus frequency for losses in the propagation of compression waves (not the surface waves we are most concerned about.) For the two kinds of rock tested, attenuation lengths were in the range of 100 to 300 feet at 100 Hz, with attenuation length inversely proportional to frequency.

This points to the one prudent way to reduce vibration from things which are too awkward to isolate. That is to install them as far away as possible. It might make sense, for example, to install the substation halfway along one arm from the corner station to the mid-station, nominally 1 kilometer from the nearest sensitive parts of the system. This is then several to many e-folding lengths away, the next best thing to isolating the source in the first place.

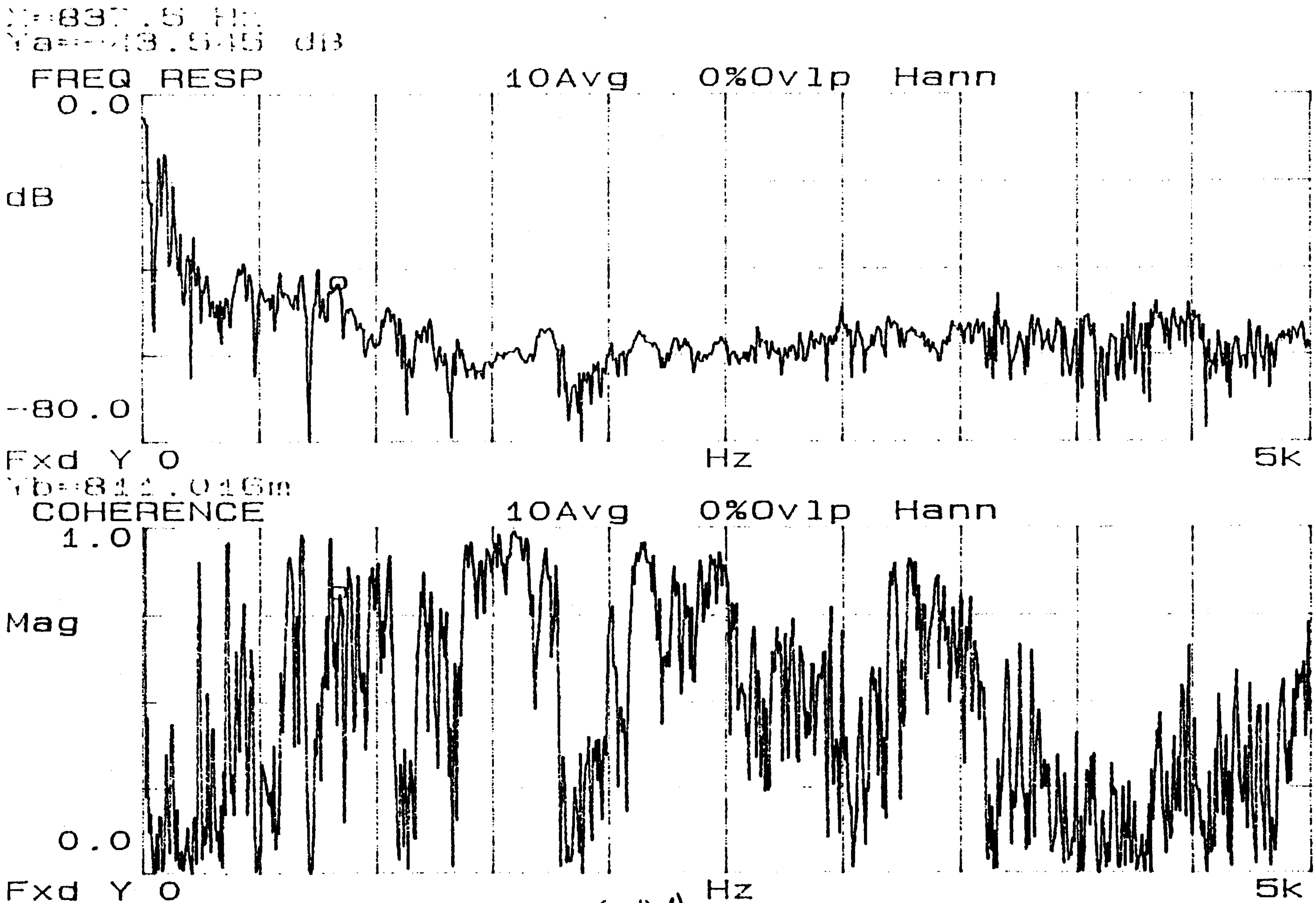
My guess is that it is not a particular burden to place the electrical substation in such a location, since substantial transmission lines would be laid out along the arms in any case. (Is this right?) That argument doesn't hold for the laser cooling system, so it would be more important to know what the amount of noise we could expect from the latter. It may be that the equipment can be isolated there, allowing us to place it closer to where the lasers are.

Peter R. Saulson
January 2, 1989

Acceleration Spectra from Various Pieces of Lab Equipment



Measurements June 3, 1988



$$\text{Xfer Function} = \frac{\text{Accel on (vertical) floor near laser power supply}}{\text{Accel on laser power supply (horiz)}}$$

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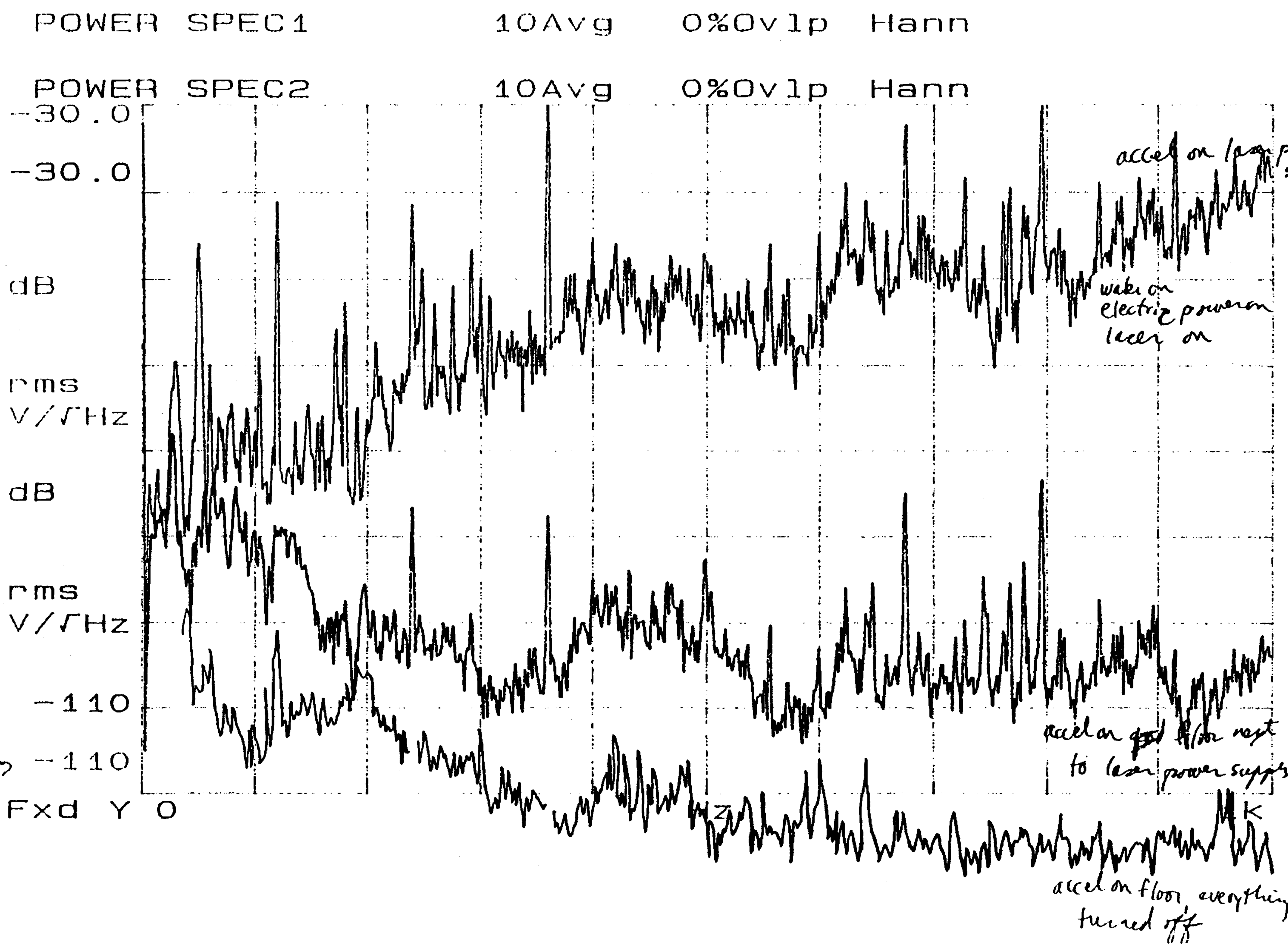
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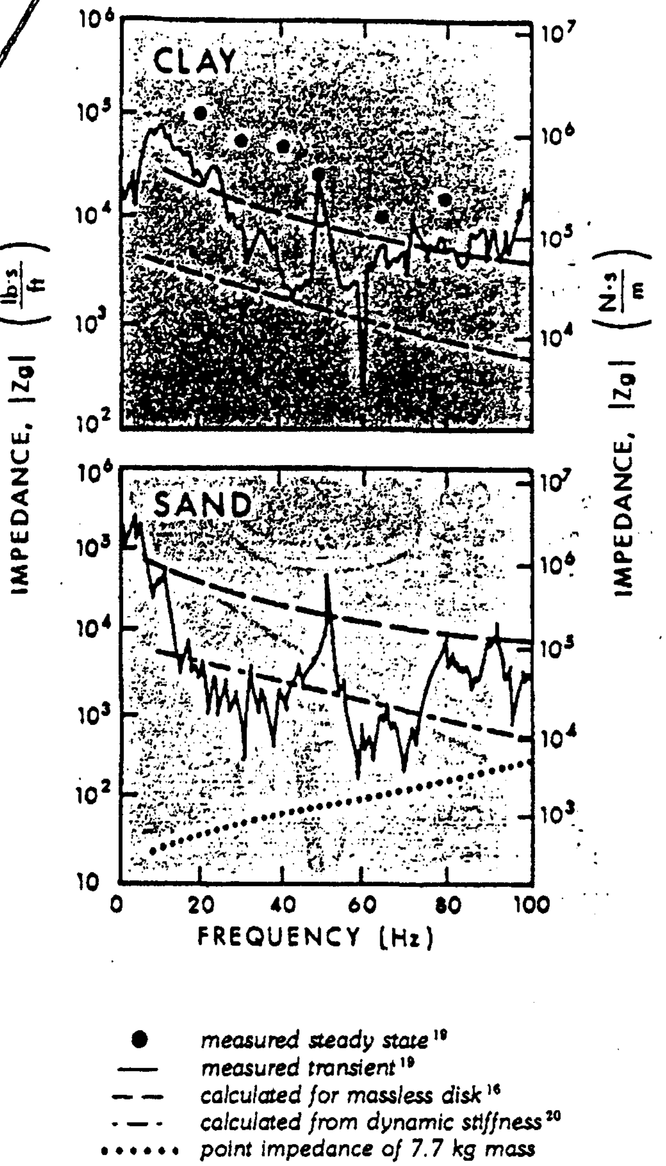

Figure 3 — Measured and estimated vertical soil impedance for a circular area with a 0.4 m diameter

Rayleigh wave is down 10 dB at a distance of λ/10 below the surface.[1] For a Rayleigh wave speed of 150 m/s in clay and a frequency of 50 Hz, λ/10 is 0.3 m.

Of course, when measuring high accelerations from sources such as blasting, the transducer must be firmly attached to the soil. The US Bureau of Mines has suggested stability criteria for three-point surface-mounted transducers.[3]

In addition to these problems, a mass loading effect may occur, especially with high frequencies (for example, groundborne noise from subways). Bycroft and others have developed a theory for disks on the elastic half-space that could be used to estimate this effect.[16–18] An alternate approach is to estimate the respective ground and transducer impedances. If the original ground velocity is V_0 and the ground impedance is Z_g, the new ground velocity V_g will be changed from its original value as a result of the force F between the transducer and the ground. Hence:

$$V_g = V_0 - F/Z_g. \tag{1}$$

The transducer's velocity V_t is a function of the force F and its (assumed point mass) impedance $j\omega m$, so

$$V_t = F/j\omega m. \tag{2}$$

If we assume that the transducer and the ground move together and substitute for F from Eq. 1, we will obtain

$$V_g = V_0 \left[Z_g/(Z_g+j\omega m)\right]. \tag{3}$$

Consequently, if $j\omega m$ is small compared with Z_g, our measured velocity V_g will be very close to the undisturbed velocity V_0.

Estimates of $|Z_g|$ in the vertical direction for a circular area with a 0.4 m diameter are given in Fig. 3. The measured data (both transient and steady state) are provided by White and Mannering.[19] The upper dashed line comes from Bycroft's theory for a massless disk on an elastic half-space[16] and the lower dashed line is calculated from dynamic stiffness values given by Barkan.[20] For both of the calculated curves, nominal soil properties were used. It is worth noting that White and Mannering, who measure their applied force above a disk, do not specifically mention whether they corrected their data for the mass of the disk. A simple $-j\omega m$ correction for the disk's mass would produce a trend in the measured data more in keeping with the calculated results. The general trend of $|Z_g|$ suggested by Bycroft's results is that the soil looks like a spring for these low frequencies. Nevertheless, Fig. 3 provides order-of-magnitude estimates of the absolute value of ground impedance for a circular area with a diameter of 0.4 m.

As an example, consider a 7.7 kg transducer (typical for some moving coil triaxial transducers) with a point impedance of

$$|j\omega m| = 48.5 f, \text{ N s/m},$$

where f is frequency in Hz. This value is plotted as the dotted line on the ground impedance curve for sand. It shows that no serious problems are expected at frequencies below 100 Hz. However, for higher frequencies, softer soil, or heavier transducers, the ground and transducer impedances may cross. This would indicate a resonance (and partial amplification) situation.

Perhaps the most troublesome potential transducer mounting problem is coupling between the ground and the transducer.[21] The vertical response of three different acceleromefrom Ferahian and Ward

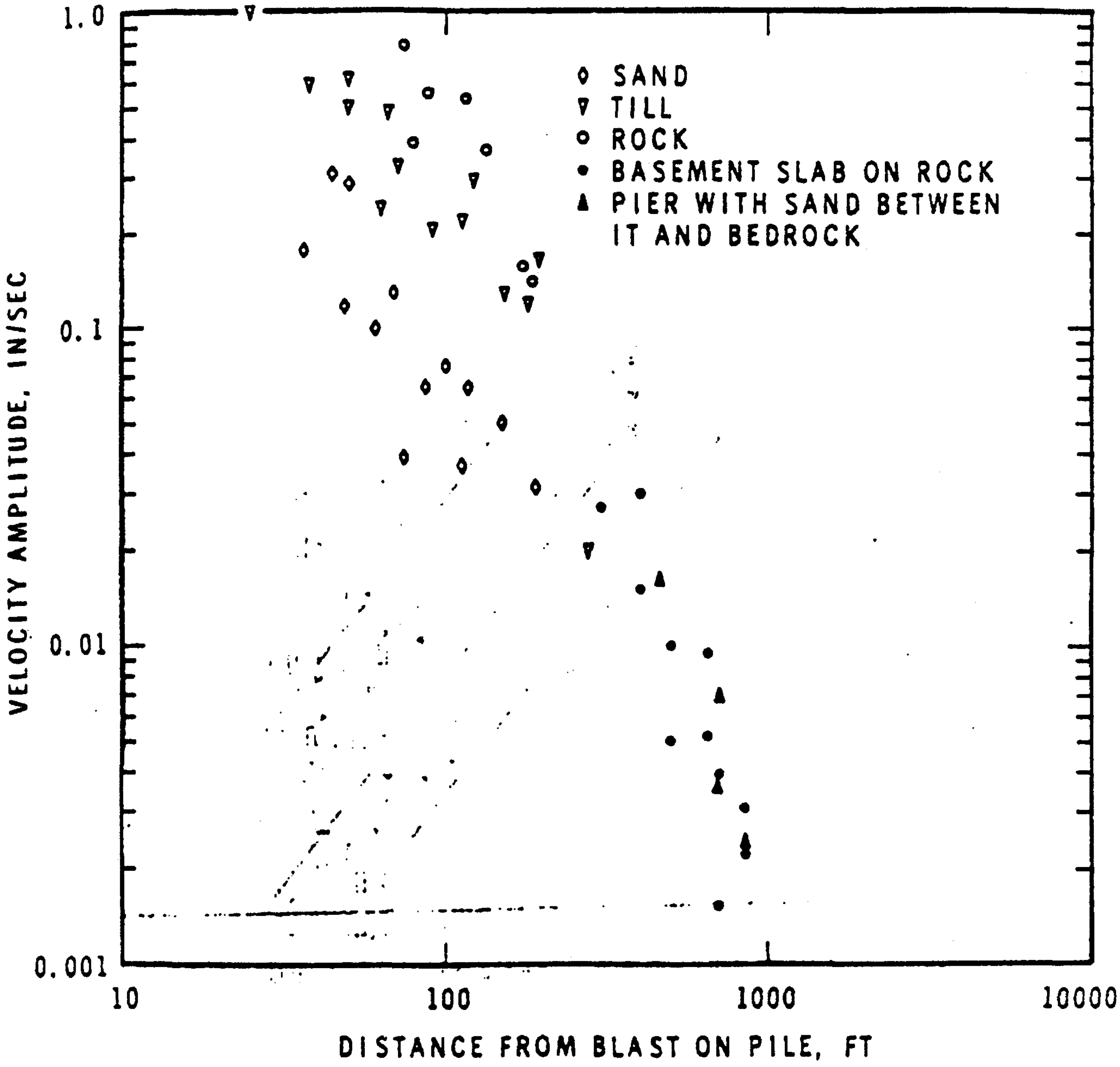

FIGURE 5

VIBRATIONS CAUSED BY BLASTING OF UNIT POUND CHARGE

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in Mason, ed., Physical Acoustics, Vol III B (1965)

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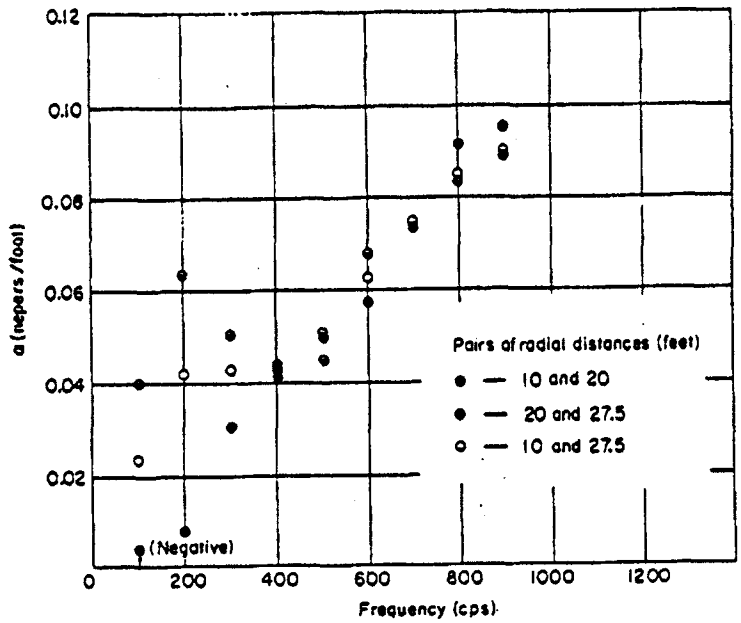

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Fig. 13. Attenuation factor for compression waves in Pottsville sandstone. (After Collins and Lee, 1956.)

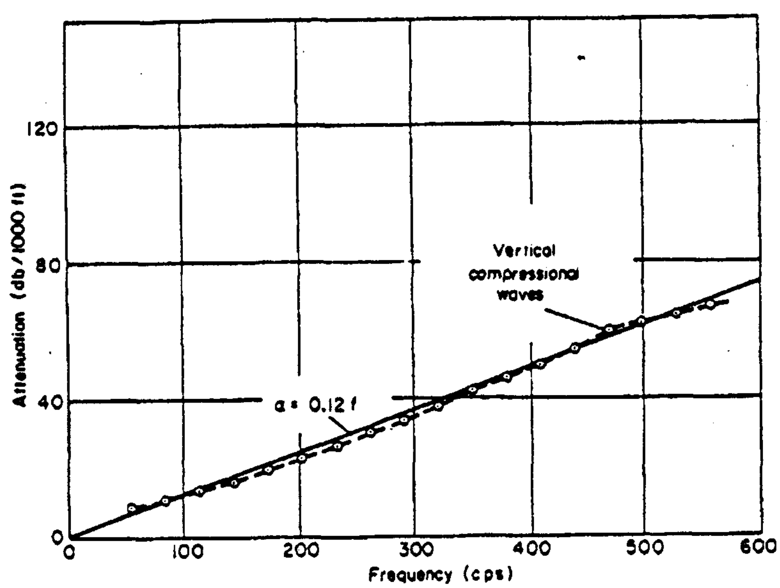

Fig. 14. Attenuation factor for compression waves in Pierre shale. (After McDonal *et al.*, 1958.)