New Folder Name Transmission of Optical Power Transients

Transmission of Optical Power Transients by Fabry–Perot Cavities: Laser Power Stabilization and Ringdown Measurement Applications

M. E. Zucker

May 8, 1991

Abstract

The response of a high–finesse optical cavity to a step in input power exhibits different time constants in the respective limits of small and large fractional power changes. For a total step–function shutoff of the input light, as is usually done during a ringdown storage time measurement, the decay time is τ_e, the energy storage time. For small changes in the input power, as might be applied by a corrective power stabilization servo, the effective decay time is actually $2\tau_e$. This is a direct consequence of the response to power, rather than field amplitude, by optical detectors. Some examples and implications for storage time measurement and laser power stabilization servo design are discussed.

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compared to its "storage time" $\tau_e = l/[c(1-\sqrt{R_1R_2})]$, where R_1 and R_2 are the mirror reflectivities, are attenuated by interference with the large stored field inside the cavity. A critical performance factor, the reflectivity product R_1R_2, is generally measured by totally shutting off the light entering the cavity or by offsetting the laser frequency from the resonant frequency of the cavity (so the effective coupling goes to zero) and observing the decay of the stored power with a photodetector at the output. The familiar output response is a pure exponential $P_o(t) = P_o(0)e^{-t/\tau_e}$ [1].

In arranging a power stabilizing servo to control the laser power entering the interferometer, it is advantageous to isolate the reference detector of the loop from spurious geometric and phase fluctuation effects induced by the feedback transducer (generally an acoustooptic or electrooptic modulator) [2, 3, 5]. This can be achieved by placing the feedback transducer before the mode cleaning cavity, and placing the reference beam sampler after the cavity. The transient response of the cavity, however, must then be included in the open loop transfer function of the servo loop. A general Laplace transform analysis is contained in [1]; the following is a simpler treatment.

2 Optical Cavity Step Response

If we take the laser frequency ν to be constant and equal to the resonant frequency of the cavity $\nu_0 = nc/2l$, where n is an integer and l is the mirror separation, we can disregard the optical phase and model the cavity as a simple RC lowpass circuit with $RC = 2\tau_e$. The voltages at the input and output of the network represent the optical electric field amplitudes at the input and output of the cavity, E_i and E_o, respectively. This approximation is appropriate for characteristic timescales $|t_c| \gg 2l/c$, the roundtrip light travel time of the cavity; for very rapid disturbances the cavity field will actually adjust in discrete steps $2l/c$ apart, but the envelope will follow the RC model.

The output field of the cavity thus obeys the equation

$$E_o + 2\tau_e \dot{E}_o = TE_i(t),$$

where $|T|^2$ is the cavity's steady–state power transmission. We are interested in particular solutions for input fields of the form

$$E_i(t) = E_{0i}[1 + \alpha U(t)]$$

where α is some constant (not necessarily small) and $U(t)$ is the unit step function. This field carries input power

$$P_i(t) = |E_{0i}|^2[1 + 2\alpha U(t) + \alpha^2 U^2(t)] = |E_0 i|^2[1 + (\alpha^2 + 2\alpha)U(t)]$$

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The particular solution for the output field is just

$$E_o(t) = \begin{cases} TE_{0i}, & t < 0 \\ TE_{0i}[1 + \alpha(1 - e^{-t/2\tau_e})], & t \geq 0 \end{cases} .$$

Taking the square of this expression, the output power is given by

$$P_o(t \geq 0) = |TE_{0i}|^2[(1 + \alpha)^2 - 2(\alpha^2 + \alpha)e^{-t/2\tau_e} + \alpha^2 e^{-t/\tau_e}]$$

which has, in addition to the asymptotic steady–state value, two exponentially decaying terms with time constants of $2\tau_e$ and τ_e, respectively, whose relative magnitudes depend on the step size α. For $|\alpha| \ll 1$, the output waveform adjusts with time constant $2\tau_e$, while, if α is large, the waveform initially decays with the shorter time constant τ_e before being overtaken by the slower decay at late times.

3 Examples

Substituting $\alpha = -1$ into the above expression we see that for complete removal of the input one obtains the simple output "ringdown" waveform

$$P_o(t) = |TE_{0i}|^2 e^{-t/\tau_e}$$

as expected. However, for $\alpha = -0.01$, a 2% step down in input power, the response is

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which will appear on the detected output as a 2% step with an exponential decay time of $2\tau_e$, not τ_e, since the third term is so small. As a final example, take $\alpha = -0.9$, translating to a step down in input power to 1% of the original value;

$$P_o(t) = |TE_{0i}|^2[0.01 + 0.18e^{-t/2\tau_e} + 0.81e^{-t/\tau_e}].$$

Here, the third term dominates the early evolution, but is overtaken at $t \approx 1.5\tau_e$ by the second term, which decays twice as slowly. The waveform is not exponential at all, except at very late times.

4 Conclusions and Caveats

From the third example above it is clear that to obtain reliable ringdown measurements of the reflectivity product, it is necessary to shut the light off *absolutely* and *entirely*. It can be difficult to achieve perfect extinction with either electrooptic or acoustooptic transducers used in amplitude modulation mode, and the setting can drift with time and temperature. Of course, reducing the light power may induce the laser frequency servo to fall out of lock, but some of the better servos can be quite tenacious, and there is always some possibility that once released the frequency will continue to hang around the cavity resonance. As an alternate method, offsetting the laser frequency and/or alignment with an impulsive voltage to a servo test point (or to an acoustooptic modulator VCO input) by an amount significantly exceeding the acceptance band of the cavity is recommended.

Another alternative, which has the advantage that the cavity can (indeed must) remain in resonance throughout, is to use a very small amplitude squarewave (say, two to five % peak to peak) to modulate the laser input intensity, measure the rise– and fall–times between the asymptotes, and halve the e–folding time to obtain τ_e. Although the individual decays may suffer from poor signal–to–noise ratio, the measurements can be coherently averaged with a digital storage oscilloscope or boxcar averager triggered by the excitation waveform. The averaged measurements may easily meet or exceed the accuracy of a larger "single–shot" ringdown. Also, the perturbation caused by the third $(\alpha^2 e^{-t/\tau_e})$ term will affect the rising and falling transitions oppositely, so averaging the rise and fall times will tend to reduce the residual contribution of that term still further.

For the purpose of determining the proper time constant to allow for in the design of a power stabilizing control loop which includes the cavity, it is clear that the "small perturbation" limit is appropriate, except perhaps for initial and final locking transients. In particular, the effective pole introduced by the cavity storage action is at frequency $f_c = 1/4\pi\tau_e$. The control loop compensation should introduce

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[5] Coherent Associates, Inc., *Model 307 Noise–Eater Operation and Maintenance Manual.*

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M. E. Zucker

31 October, 1990

LIGO TECHNICAL MEMORANDUM

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October 30, 1990

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CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

LIGO PROJECT

Interoffice Memorandum

31 October, 1990

TO: Distribution

FROM: M. E. Zucker

SUBJECT: Cavity Power Transients

Attached is a brief note about the occasionally confusing subject of ringdowns and power transients in high-finesse cavities. There is nothing new, except some emphasis on the potential pitfalls of neglecting the amplitude-sensitive aspects of the transient response. The text may be found in file `~mike/cavity/tcav/transcav.tex` on `ligo`.

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where α is some constant (not necessarily small) and $U(t)$ is the unit step function. This field carries input power

$$P_i(t) = |E_{0i}|^2[1 + 2\alpha U(t) + \alpha^2 U^2(t)] = |E_0 i|^2[1 + (\alpha^2 + 2\alpha)U(t)]$$

since $U^2(t) = U(t)$, and thus also represents a step function of fractional size $\alpha^2 + 2\alpha$ in power (as might arise from a voltage step to an electrooptic modulator).

The particular solution for the output field is just

$$E_o(t) = \begin{cases} TE_{0i}\,, & t < 0 \\ TE_{0i}[1 + \alpha(1 - e^{-t/2\tau_e})]\,, & t \geq 0 \end{cases} .$$

Taking the square of this expression, the output power is given by

$$P_o(t \geq 0) = |TE_{0i}|^2[(1+\alpha)^2 - 2(\alpha^2 + \alpha)e^{-t/2\tau_e} + \alpha^2 e^{-t/\tau_e}]$$

which has, in addition to the asymptotic steady–state value, two exponentially decaying terms with time constants of $2\tau_e$ and τ_e, respectively, whose relative magnitudes depend on the step size α. For $|\alpha| \ll 1$, the output waveform adjusts with time constant $2\tau_e$, while, if α is large, the waveform initially decays with the shorter time constant τ_e before being overtaken by the slower decay at late times.

3 Examples

Substituting $\alpha = -1$ into the above expression we see that for complete removal of the input one obtains the simple output "ringdown" waveform

$$P_o(t) = |TE_{0i}|^2 e^{-t/\tau_e}$$

as expected. However, for $\alpha = -0.01$, a 2% step down in input power, the response is

$$P_o(t) = |TE_{0i}|^2[0.98 + 0.02e^{-t/2\tau_e} + 0.0001e^{-t/\tau_e}]$$

which will appear on the detected output as a 2% step with an exponential decay time of $2\tau_e$, not τ_e, since the third term is so small. As a final example, take $\alpha = -0.9$, translating to a step down in input power to 1% of the original value;

$$P_o(t) = |TE_{0i}|^2[0.01 + 0.18e^{-t/2\tau_e} + 0.81e^{-t/\tau_e}].$$

Here, the third term dominates the early evolution, but is overtaken at $t \approx 1.5\tau_e$ by the second term, which decays twice as slowly. The waveform is not exponential at all, except at very late times.

4 Conclusions and Caveats

From the third example above it is clear that to obtain reliable ringdown measurements of the reflectivity product, it is necessary to shut the light off *absolutely* and *entirely*. It can be difficult to achieve perfect extinction with either electrooptic or acoustooptic transducers used in amplitude modulation mode, and the setting can drift with time and temperature. Of course, reducing the light power may induce the laser frequency servo to fall out of lock, but some of the better servos can be quite tenacious, and there is always some possibility that once released the frequency will continue to hang around the cavity resonance. As an alternate method, offsetting the laser frequency and/or alignment with an impulsive voltage to a servo test point (or to an acoustooptic modulator VCO input) by an amount significantly exceeding the acceptance band of the cavity is recommended.

Another alternative, which has the advantage that the cavity can (indeed must) remain in resonance throughout, is to use a very small amplitude squarewave (say, two to five % peak to peak) to modulate the laser input intensity, measure the rise– and fall–times between the asymptotes, and halve the e–folding time to obtain τ_e. Although the individual decays may suffer from poor signal–to–noise ratio, the measurements can be coherently averaged with a digital storage oscilloscope or boxcar averager triggered by the excitation waveform. The averaged measurements may easily meet or exceed the accuracy of a larger "single–shot" ringdown. Also, the perturbation caused by the third ($\alpha^2 e^{-t/\tau_e}$) term will affect the rising and falling transitions oppositely, so averaging the rise and fall times will tend to reduce the residual contribution of that term still further.

For the purpose of determining the proper time constant to allow for in the design of a power stabilizing control loop which includes the cavity, it is clear that the "small perturbation" limit is appropriate, except perhaps for initial and final locking transients. In particular, the effective pole introduced by the cavity storage action is at frequency $f_c = 1/4\pi\tau_e$. The control loop compensation should introduce

a zero at this frequency if a uniform phase response is required at higher frequencies. The mixture of decay terms points to some interesting possibilities for nonlinear transient behavior, which we hope to explore in a future memo.

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