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Aspect Ratio and Internal Thermal Noise in Test Masses

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Abstract

The frequency dependence of internal thermal noise in test masses made of fused silica is analyzed. Using existing data on bulk acoustic losses for this material, it is found that the thermal noise spectrum is flat below the lowest internal resonance of the test mass. It is also shown that, for fixed diameter, the level of internal thermal noise is lowest for aspect ratios (thickness/diameter ratios) between 1 and 1.5, and that, inside this range, it is practically constant. Reducing the aspect ratio to 0.3 (the lowest value included in the present analysis) increases the noise level by only ~70% from the minimum value. This noise level is still well below the requirement for the initial LIGO interferometers. Thus, for fixed diameter, the thickness of the test mass can be chosen anywhere in a range exteding at least from 0.3 to 1.5 diameters, so that other test mass design criteria can be met.

1 Introduction

For a test mass with fixed diameter D, the thickness d affects interferometer performance in several ways:

- The mass is proportional to d;
- The thermal noise associated with internal acoustic modes depends on the mass, and also on the aspect ratio $a = d/D$, which determines the frequencies of these modes;
- The thermal noise associated with the test mass as a pendulum depends on the mass;
- The amount of energy absorbed from the laser beam and the resulting thermal lensing and thermal stress induced birefringence increase with d. The wave front distortion due to thermal lensing and stress birefringence degrade the mode matching to the cavities and the beam overlap at the beam splitter, leading to a loss of sensitivity in a nonrecycled interferometer and setting a limit to the recycling factor that can be achieved;
- The effects of optical inhomogeneity and residual birefringence increase with d;
- The amount of light scattered off the beam depends on d.

The present note addresses only the first two issues. Section 2 analyses the frequency dependence of the thermal noise spectrum in a test mass made of fused silica, based on existing data on bulk losses in this material. Section 3 contains an evaluation of thermal noise in a cylindrical test mass of fixed diameter, as a function of its thickness. Section 4 is a summary of the results.

2 Internal Thermal Noise

2.1 Internal Thermal Noise Formulæ

For frequencies below the lowest resonance of the test mass, displacement noise due to thermally driven internal modes is described by[1]:

$$x_{th}^2(\omega) = \frac{8k_BT}{\omega}\sum_n \frac{\phi_n(\omega)}{M\omega_n^2} \tag{1}$$

where M is the mass, ω_n is the resonant frequency of the n-th mode and ϕ_n is the phase lag that describes the losses in the material for that mode, and is related to the quality factor by:

$$\phi_n(\omega) = \frac{1}{Q_n(\omega)} \tag{2}$$

so that Eq. (1) can be rewritten as:

$$x_{th}^2(\omega) = \frac{8k_BT}{\omega}\sum_n \frac{1}{MQ_n(\omega)\omega_n^2} \tag{3}$$

2.2 Frequency Dependence of Internal Thermal Noise

Data on losses in fused silica, at room temperature, is scarce. A summary of Q values, measured below 200 kHz, is shown in Fig.1. The data points at 6.9 kHz where taken with the same sample, which at first had ground surfaces and was subsequently polished, annealed and etched[2]. The points at 28.57 kHz and 35.17 kHz were measured with a single sample having finely ground surfaces[3]. The point at 131.65 kHz was measured with a sample having ground lateral surface[3]. Finally, the point at 37 kHz was measured with an annealed sample[4]. While caution is required when using such a disparate set of data points, it is probably reasonably safe to take the lower $1/f$ line of

[1] P. R. Saulson, Thermal Noise in Mechanical Oscillators, January 1990, unpublished
[2] V. P. Mitrofanov, V .N. Frontov, Vest. Mosk. Univ. **4**, 478 (1974) (in Russian)
[3] A. Čadež, A. Abramovici, J. Phys. E: Sci Instrum., **21**, 453 (1988)
[4] J. W. Marx, J. M. Sivertsen, J. Appl. Phys., **24**, 81 (1953)

Fig. 1 at face value. Indeed, many loss mechanisms lead to a behaviour of the lag angle of the type[5]:

$$\phi(\omega) = \Delta\frac{\omega\tau}{1+\omega^2\tau^2} \quad (4)$$

For $\omega < 1/\tau$, $\phi(\omega) = \Delta\omega\tau$, which, according to Eq. (2), leads to $Q \propto 1/\omega$. This trend is well established down to 7 kHz (Fig. 1), and can therefore be extrapolated towards somewhat lower frequencies. One should be warned, however, that more data are needed at frequencies around 1 kHz. With this in mind, the relation:

$$Q_n(\omega) = \frac{Q_n(\omega_n)\omega_n}{\omega} = \frac{\alpha}{\omega} \quad (5)$$

will be used in what follows, with $\alpha = Q_n(\omega_n)\omega_n = 1.58 \cdot 10^{10}$ rad·s^{-1}. This corresponds to the lower line in Fig. 1. Since the points at 28.57 kHz and 35.17 kHz correspond to two different modes of the same sample, it appears that the magnitude of the losses does not depend on the shape of mode. This is expected to be the case if the loss is caused by a process associated with the bulk of the sample.

Replacing $Q_n(\omega)$ from Eq. (5) into Eq. (3) yields:

$$x_{th}^2(\omega) = \frac{8k_BT}{\alpha M}\sum_n \frac{1}{\omega_n^2} \quad (6)$$

This shows that, if extrapolation of the data from Fig. 1 to lower frequencies is correct, then the spectrum of displacement noise, associated with the internal modes of the test mass, is flat.

[5]see e. g. A. S. Nowick, B. S. Berry, Anelastic Relaxation in Crystalline Solids, Academic Press 1972

3 Dependence of Thermal Noise on Mass Thickness

In order to evaluate the thermal noise according to Eq. (6), one needs to know the frequencies of the relevant acoustic modes. Acoustic modes of cylinders have been studied experimentally by McMahon[6]. Numerical calculations[7] are in very good agreement with the experiment. Fig. 2 shows a variety of mode patterns for a cylinder[6]. Except for the modes in the upper row, which are axially symmetric, each mode shown is characterized by the fact that the average deflection of the surface, along a circle concentric with the cylinder face, vanishes. Thus, if the face is the surface of a Fabry-Perot mirror and if the laser beam is centered on the face, the average change in cavity length due to thermal excitation will vanish, except for the upper row modes. Only those modes will be considered in the present analysis. In the upper row, the modes labeled "1" and "3" can be called membrane modes; when one face of the cylinder bulges, the opposite one recedes. Modes "2" and "4" are compressional modes; opposite faces of the cylinder bulge or recede at the same time.

Eq. (6) has been evaluated for the four lowest lying modes, as a function of test mass thickness, for fixed test mass diameter. Note that the data of Ref's 6,7 refer to aluminum and steel which have Poisson ratio $\nu \sim 0.3$, while for fused silica $\nu = 0.17$. It has been shown, however[7], that the dependence of mode resonant frequencies on Poisson ratio is very weak, so that the relative value of terms in Eq.(6) is affected little by using $\nu = 0.17$ instead of $\nu = 0.3$. There will be a slight error in the absolute value of $x^2_{th}(\omega)$, but the dependence of $x^2_{th}(\omega)$ on the aspect ratio a, or on test mass thickness, when the diameter is kept constant, will be accurately described.

The dependence of thermally driven displacement noise on test mass thickness, for fixed diameter, is shown in Fig. 3. For fused silica and $D = 20$ cm:

$$x_{th,min} \; = \; 7.6 \cdot 10^{-21} \qquad \frac{m}{\sqrt{Hz}} \tag{7}$$

which corresponds to $h(f) = 3.8 \cdot 10^{-24}$ Hz$^{-1/2}$ and is thus approximately

[6]G. W. McMahon, J. Acoustical Soc. America, 36, 85 (1964)

[7]J. R. Hutchinson, J. Appl. Mech., 47, 901 (1980)

5 times lower than the requirement for the Phase A interferometers. It should be pointed out that the quantum noise level for a 10 kg mass, at 150 Hz[8], is $x_q = 9.5 \cdot 10^{-21}$ m·Hz$^{-1/2}$.

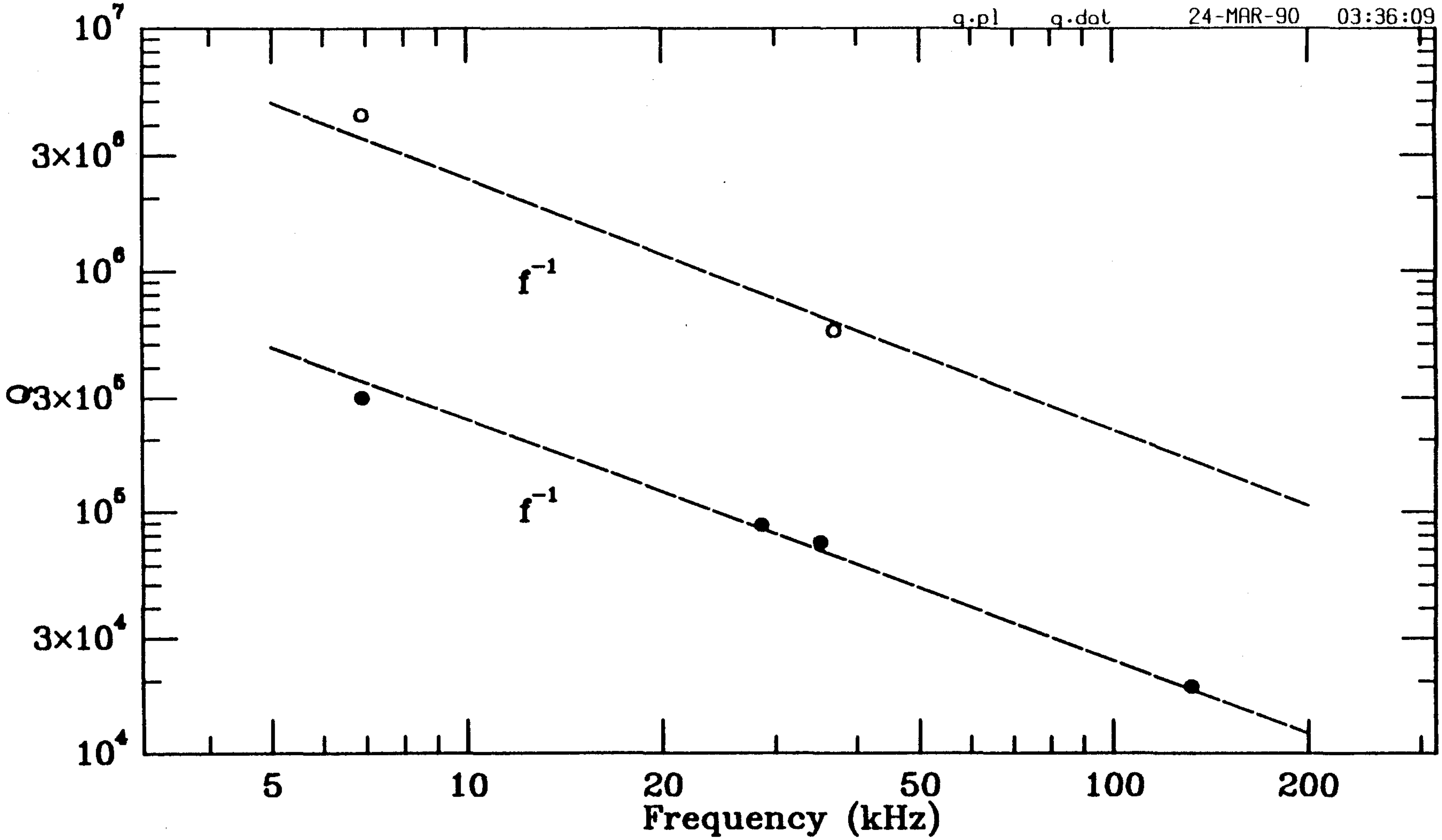
4 Conclusions

The results of this note can be summarized as follows:

1. The available data on frequency dependence of the quality factors of fused silica cylinders seems to indicate that, below the lowest acoustic mode of the test mass, thermally driven displacement noise has flat spectrum, possibly down to ~ 1 kHz and below;

2. For fixed diameter, thermally driven displacement noise depends only weekly on test mass thickness. For $D = 20$ cm, thermal noise in a test mass as thin as 6 cm (see Fig. 3) would still be well below the limit required for the initial LIGO interferometers.

[8]the location of the lowest noise for Phase A interferometers



Measured Q's, for fused silica, at room temperature

Fig. 1

open dots: samples annealed and/or polished, etched
solid dots: samples ground

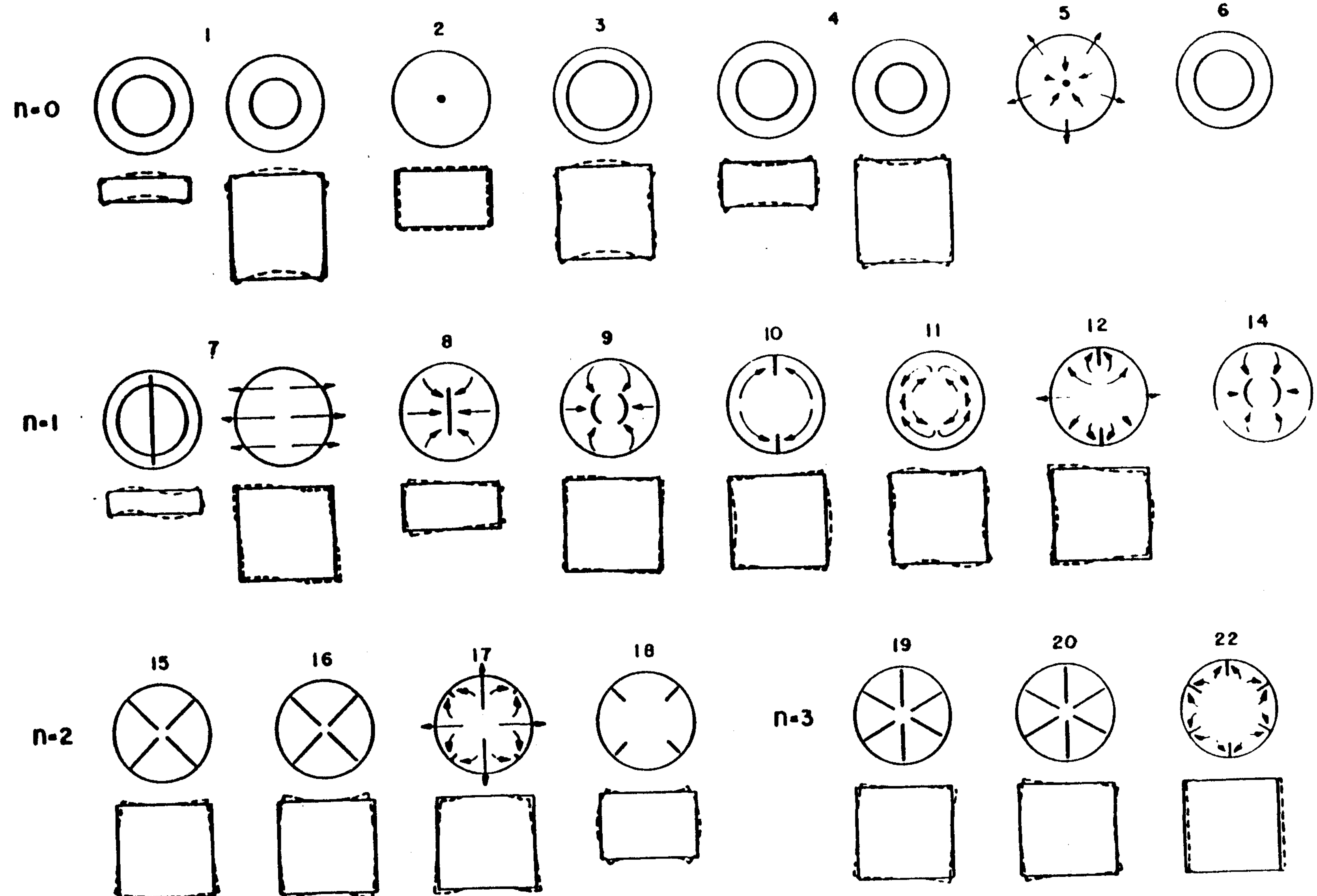


FIG. 2. Mode chart showing sand patterns on the plane surface of the cylinder and approximate form of the vibration at a diametral cross section.

Fig. 2

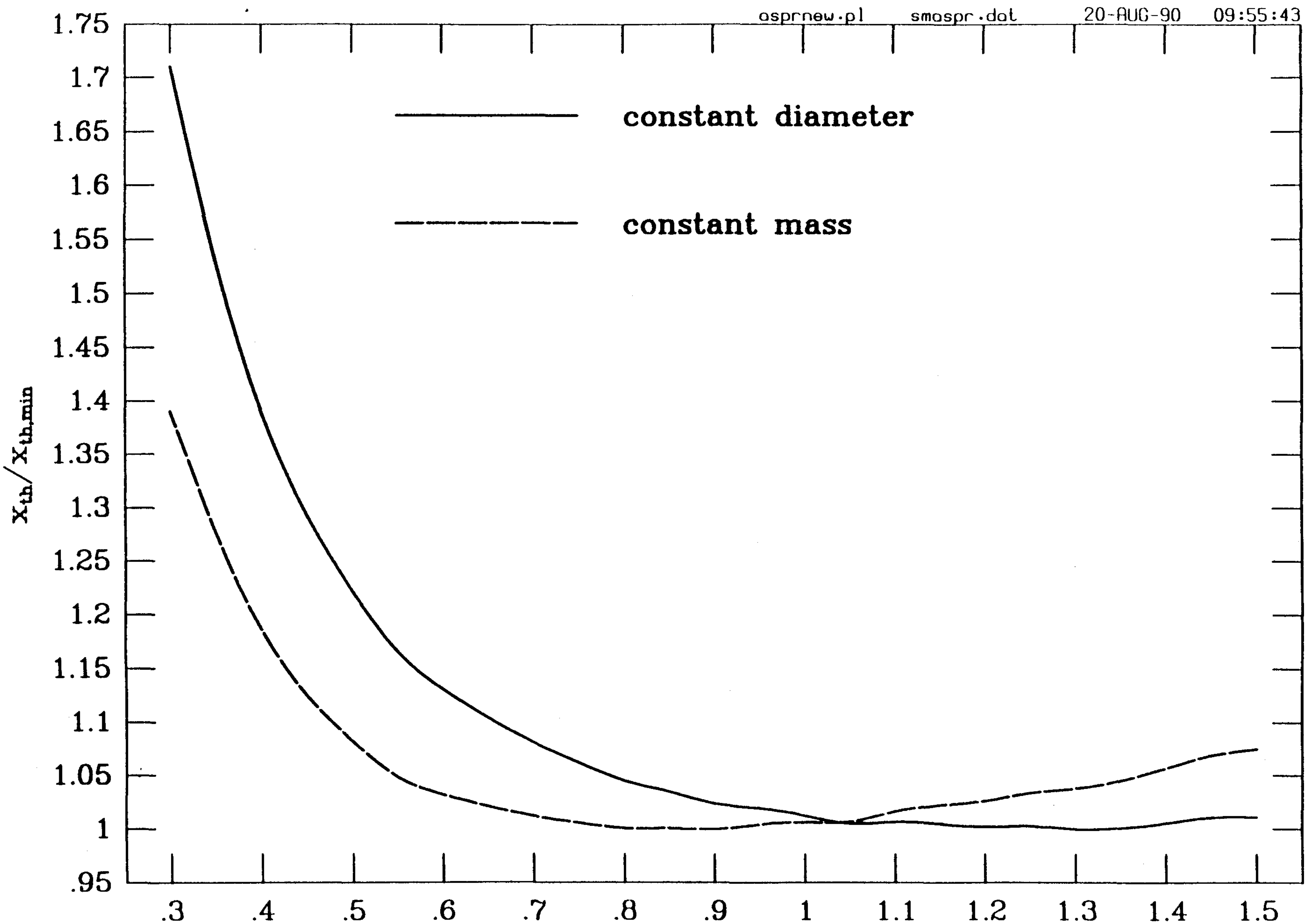


Fig. 3 Test mass thermal noise versus aspect ratio