New Folder Name The Paraxial Wave Equation

THE NUMERICAL SOLUTION OF THE PARAXIAL WAVE EQUATION FOR STABLE RESONATORS: PART I

WRITTEN BY: Yekta Gürsel **September 26, 1990**

I. INTRODUCTION

Many of the problems that need to be solved before the design freeze of the LIGO interferometers can not be analyzed analytically. A large fraction of these are about the behavior of resonant Fabry-Perot cavities which are constructed using state-of-the-art, but nevertheless imperfect optical parts. The laser beams in the interferometers are also distorted by the optical elements in the interferometer input chain before they even reach the cavities. Use of coupled cavities in the recycling interferometer configurations makes the analysis of the effect of the various imperfections on the sensitivity of the interferometer much more difficult.

LIGO project purchased a commercially available program which is capable of performing numerical analysis of complicated optical systems. This program is called GLAD (General Laser Analysis and Design) and it can propagate a two dimensional wavefront through complicated optical systems like Fabry-Perot cavities. Over the course of past two months, I have tested and used the program extensively. I discovered many bugs which were promptly fixed by the author of the program. This memorandum is intended to be the beginning of a "living" document which will be upgraded every few weeks as I analyze more and more complicated problems using GLAD. What follows is an introduction to the theory of numerical simulation of stable cavities, and a few examples of the problems I analyzed using GLAD.

This manuscript contains two major parts: The theoretical analysis and the actual examples of GLAD simulations. The input listings of the simulations in the appendices are included to serve as a tutorial for the reader who wants to learn how to use GLAD. The people who are not interested in using GLAD or in learning the theoretical basis behind the calculations can simply start reading this document at Section VII. In this section results of the simulations are summarized, and some plots are shown. Section VIII summarizes what has been accomplished so far and what is to come in part II of this document.

A complete treatment of the fields inside a resonator requires the use of the vector wave equation which describes the propagation of light waves in vacuum. In this memorandum, we are interested in computing the interference and the diffraction effects inside a resonator. This can be done using the scalar wave equation which takes both the interference and the diffraction terms into account. The polarization effects can not be described using the scalar wave equation. In the remainder of this document, we will ignore the polarization effects. They will be taken into account in the later versions this manuscript.

II. THE SCALAR WAVE EQUATION

The scalar wave equation for the electric field of a light wave propagating in vacuum is given by:

$$\nabla^2 E(\vec{r},t) - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}E(\vec{r},t) = 0, \tag{2.1}$$

where c is the speed of light and

$$\nabla^2 = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right].$$

A mode of a resonator is defined as a field distribution where all parts of the field oscillate in time with the same frequency and phase. Therefore, we assume a solution of the form:

$$E(\vec{r},t) = E_0(\vec{r})\, e^{-i\omega t}, \tag{2.2}$$

where ω is the frequency of the oscillation. Substituting this in Eq. (2.1), we get the Helmholtz equation:

$$\nabla^2 E_0(\vec{r}) + k^2 E_0(\vec{r}) = 0, \tag{2.3}$$

where k is the wave vector defined by $k^2 = \omega^2/c^2$. We try a solution to Eq. (2.3) of the form:

$$E_0(\vec{r}) = E_T(\vec{r})\, e^{ikz} \tag{2.4}$$

Here we assume that the cross section of the beam perpendicular to the direction of propagation z is finite. We further assume that the variations of the function $E_T(\vec{r})$ and $\partial E_T(\vec{r})/\partial z$ within distances of the order of the wavelength in the direction of propagation is negligible. We then obtain the paraxial wave equation for the function $E_T(\vec{r})$:

$$\nabla^2_T E_T(\vec{r}) + 2ik\frac{\partial E_T(\vec{r})}{\partial z} = 0, \tag{2.5}$$

where ∇^2_T is the transverse Laplacian defined by:

$$\nabla^2_T = \left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right].$$

The solution of the paraxial wave equation is given by:

$$E_T(x,y,z) = -\frac{i}{\lambda z}\iint E_T(x',y',0)\, e^{ik[(x-x')^2+(y-y')^2]/2z}\, dx'dy', \tag{2.6}$$

where $E_T(x',y',0)$ is specified on the plane $z=0$ and satisfies Eq. (2.5). The complete solution of the Helmholtz equation [Eq. (2.3)] in the paraxial approximation is then given by:

$$E_0(x,y,z) = E_T(x,y,z)\, e^{ikz} = -\frac{ie^{ikz}}{\lambda z}\iint E_T(x',y',0)\, e^{ik[(x-x')^2+(y-y')^2]/2z}\, dx'dy'. \tag{2.7}$$

III. THE PROPAGATOR

The forward propagator $K_F(x,y;x',y';z,z_0)$ of the paraxial wave equation is defined as:

$$K_F(x,y;x',y';z,z_0)=-\frac{ie^{ik(z-z_0)}}{\lambda(z-z_0)}e^{ik[(x-x')^2+(y-y')^2]/2(z-z_0)}. \tag{3.1}$$

This propagator corresponds to the forward traveling solutions of the paraxial wave equation:

$$E_0(\vec{r})=E_T(\vec{r})e^{ikz}.$$

The electric field $E_0(x,y,z)$ at any $z>z_0$ which is a solution of the paraxial wave equation can then be written in terms of the propagator $K_F(x,y;x',y';z,z_0)$ and the electric field $E_0(x,y,z_0)$ at $z=z_0$ as:

$$E_0(x,y,z)=\iint K_F(x,y;x',y';z,z_0)E_0(x',y',z_0)\,dx'dy'. \tag{3.2}$$

The reverse propagator $K_R(x,y;x',y';z,z_0)$ of the paraxial wave equation is defined as:

$$K_R(x,y;x',y';z,z_0)=-\frac{ie^{-ik(z-z_0)}}{\lambda(z_0-z)}e^{-ik[(x-x')^2+(y-y')^2]/2(z-z_0)}. \tag{3.3}$$

This propagator corresponds to the backward traveling solutions of the paraxial wave equation:

$$E_0(\vec{r})=E_T(\vec{r})e^{-ikz}.$$

The electric field $E_0(x,y,z)$ at any $z<z_0$ which is a solution of the paraxial wave equation can then be written in terms of the propagator $K_R(x,y;x',y';z,z_0)$ and the electric field $E_0(x,y,z_0)$ at $z=z_0$ as:

$$E_0(x,y,z)=\iint K_R(x,y;x',y';z,z_0)E_0(x',y',z_0)\,dx'dy'. \tag{3.4}$$

IV. STABLE RESONATOR

Consider a resonator formed by two mirrors M_0, M_L placed at $z=0$ and $z=L$ respectively. The curvatures of the mirrors R_0 and R_L and the separation of the mirrors L are chosen in such a way to make the cavity stable and make the modes of the cavity as non-degenerate as possible. For the moment, let us assume that there are no scattering and absorptive losses in the mirrors and the mirrors are sufficiently large so that the diffraction losses are negligible.

Let a mode $E_M(x,y,z,t)=E_m(x,y,z)e^{-i\omega t}$ of the cavity bounce back and forth between the mirrors. Consider the field $E_m(x,y,0)$ just after it has reflected from the mirror M_0 at $z=0$: Since $E_m(x,y,z)$ is a mode of the cavity, its shape is unchanged after one round-trip inside the cavity. We first propagate $E_m(x,y,0)$ from $z=0$ to $z=L$:

$$E_m(x,y,L)=\iint K_F(x,y;x',y';L,0)\,E_m(x',y',0)\,dx'dy' . \tag{4.1}$$

Let MO_0 and MO_L be the "mirror operators" for the mirrors at $z=0$ and $z=L$. These operators take the field just in front of the mirror prior to reflection as the "input", and produce the field in front of the mirror just after it is reflected as the "output". Then, the field just after it reflected off the mirror at $z=L$ is given by:

$$E_{mrefL}(x,y,L)=MO_L\,E_m(x,y,L). \tag{4.2}$$

We then propagate this beam backwards to the mirror at $z=0$. The beam just before it reflected off the mirror at $z=0$ is given by:

$$E_m(x'',y'',0)=\iint K_R(x'',y'';x,y;0,L)\,E_{mrefL}(x,y,L)\,dx\,dy . \tag{4.3}$$

and, after reflection off the mirror at $z=0$, the field is given by:

$$E_{mref0}(x'',y'',0)=MO_0\,E_m(x',y',L). \tag{4.4}$$

Since we assumed that $E_m(x,y,z)$ is a mode of the cavity and there are no losses in the cavity, the field $E_{mref0}(x'',y'',0)$ after a round trip should be equal to the field $E_M(x,y,z)$ at the start of the trip. Expressed in the same transverse coordinates, this can be written as [using Eqs. (4.1), (4.2), (4.3) and (4.4)]:

$$E_m(x,y,0)=MO_0\,\Gamma_{RL}\,MO_L\,\Gamma_{FL}\,E_m(x,y,0), \tag{4.5}$$

where Γ_{FL} and Γ_{RL} are operators defined in terms of the propagators K_F and K_R respectively by:

$$\Gamma_{FL}E_m(x,y,0)=\iint K_F(x,y;x',y';L,0)\,E_m(x',y',0)\,dx'dy', \tag{4.6}$$

$$\Gamma_{RL}E_m(x,y,L)=\iint K_F(x,y;x',y';0,L)\,E_m(x',y',L)\,dx'dy'. \tag{4.7}$$

V. COMPUTATION OF THE MODES

One might consider Eq. (4.5) as the defining equation for the modes of the stable cavity and try to solve it to obtain the modes. One way of obtaining the solution is by iteration: We assume an arbitrary initial field for the cavity which is consistent with the boundary conditions at the front of the mirror M_0. We then substitute this field into the right-hand side of Eq. (4.5) to get an "improved" guess for the cavity mode. We repeat the procedure until the field "converges" to one of the modes.

This approach has many pitfalls: First, the realistic cavity has losses. If one puts the losses in, the Eq. (4.5) takes the form:

$$E_m(x,y,0)=\gamma_m\,MO_0\,\Gamma_{RL}\,MO_L\,\Gamma_{FL}\,E_m(x,y,0), \tag{5.1}$$

where γ_m represents the loss of the mode m, and $|\gamma_m| < 1$. If the loss for the mode m we are searching for is large and if the iteration procedure takes too many turns to converge, the converged field amplitude will be near zero increasing the inaccuracy in the results.

Second, if the frequency of the mode is unknown, the wave-number k in the piston term e^{ikz} is also a free parameter [The term "piston" is used for terms that are constant across the wavefront. The piston phase refers to the phase terms which are only functions of z if the wave is traveling in the z-direction]. When this happens, the iteration procedure defined by Eq. (5.1) may never converge.

Third, if the cavity length is long, the numerical inaccuracies may force us to evaluate the operators in Eq. (5.1) in double precision. For example, let us assume that the length of the cavity is $L=40m$. If the wavelength $\lambda=2\pi/k$ of the light is of the order of 1 micron, then there are about 4×10^7 wavelengths along the length of the cavity. To compute the piston phase term will require significantly more than 7 digits of precision which is beyond the range of single precision SUN-Fortran.

Fourth, the modes can only be selected by knowing their frequency exactly or by introducing *ad hoc* mode-shape dependent losses in the cavity. This is not how the modes are selected in practical experiments.

Another approach is to "drop" the piston phase term e^{ikz} in the propagators as the older version of GLAD (3.7) does and iterate the resulting equation to find the mode "shape" but not the exact mode frequency. This is actually an incorrect way of solving the eigenvalue problem. Iteration can only be made to converge to the correct mode shape by numerical "tricks" which amounts to knowing the exact mode shape from the start. I'll try to illustrate this by focusing on Gaussian-Hermite modes of a stable cavity.

A Gaussian-Hermite mode of a stable cavity is given by:

$$E_{mn}(x,y,z)=\frac{Aw_0}{w(z)}H_m\left[\sqrt{2}\frac{x}{w(z)}\right]H_n\left[\sqrt{2}\frac{x}{w(z)}\right]$$

$$\times e^{i[kz-(m+n+1)\tan^{-1}z/z_0]}e^{ik(x^2+y^2)/2R(z)}e^{-(x^2+y^2)/w(z)}, \qquad (5.2)$$

where:

$$w_0=\left[\frac{\lambda L}{\pi}\right]^{1/2}\left[\frac{g_0 g_L(1-g_0 g_L)}{(g_0+g_L-2g_0 g_L)^2}\right]^{1/4}, \qquad (5.3)$$

$$g_0=1-L/R_0\ ;\ g_L=1-L/R_L, \qquad (5.4)$$

$$z_0=\frac{\pi w^2{}_0}{\lambda}, \qquad (5.5)$$

$$w_z=w_0\left[1+\frac{z^2}{z^2{}_0}\right]^{1/2}, \qquad (5.6)$$

$$R(z) = z + \frac{z_0^2}{z}, \tag{5.7}$$

and $H_m(x)$ and $H_n(x)$ are Hermite polynomials of order m and n respectively. The quantity z_0 is called the "Rayleigh" range of the beam, and the quantity w_0 is defined to be the "waist" size of the beam. The diffraction effects start to appear after the beam has traveled over distances which are of the order of the Rayleigh range z_0. The quantity $R(z)$ is called the "radius of curvature" of the beam.

The condition for a beam to be a mode of the cavity is that the mode should not change its shape in a round trip around the cavity. This implies that the curvature of the wavefront matches the curvature of the mirrors at $z=0$ and $z=L$. This can be arranged by choosing the location and the size of the waist of the mode in the cavity. Indeed, the Eqs. (5.3) to (5.7) define the required parameters of the beam in terms of the parameters of the cavity. Furthermore, the overall piston phase of the mode in a round trip around the cavity should change by a multiple of 2π. This results in the resonance condition:

$$\frac{2L}{\lambda} = q + \frac{1}{\pi}(m+n+1)\cos^{-1}(g_0 g_L)^{1/2}, \tag{5.8}$$

where q is an integer labeling the longitudinal mode frequency.

If the piston phase terms which are constant across the wave-front are dropped, then the resonance condition given by Eq. (5.8) is no longer a selector of the modes since it is solely determined by the piston terms in the Hermite-Gaussian modes. In this case, if one starts with an arbitrary waveform which satisfies the boundary conditions on one of the mirrors, there is no guarantee that the iteration will converge to a single mode of the cavity. As a matter of fact, any linear combination of the modes will be a solution to the iteration process. Mode shape dependent losses have to be introduced in order to select between the modes of the cavity. This requires that we know the shape of the mode we want to recover. For example, a simple circular aperture is not sufficient to recover the fundamental mode of the cavity. The opening of the aperture has to be optimal to damp all of the other circularly symmetric modes.

When the iteration is performed numerically, other factors - like the fineness of the transverse grid which defines the wave-front - also play a role. If the grid is too coarse, it will also act as a selector against high-order modes. This happens because the grid can not represent the transverse, spatial, high-frequency oscillations of the wave-front. In this case, the energy in the high-order modes will be transferred to the low order modes because of the "stiffness" of the grid. This is an example of "aliasing".

A way to compute the modes of the cavity is to simulate what happens in reality. We start with an "incoming field" which represents the laser light impinging on a cavity. The frequency of the "incoming field" will not match the resonance frequencies of the modes of the cavity in general. We then compute the field after a round trip in the cavity and add this field coherently to the incoming field. We continue this iteration process for a few iterations. If the frequency of the incoming field is not near the frequency of a mode of the cavity, there will be no significant energy build up in the cavity. We then tune the frequency of the incoming light and "watch" the mode build-up in the cavity as we continue the iteration process. In order to accomplish this, all piston terms have to be computed to sufficient accuracy during the iteration process.

We will now show that it is sufficient to compute only e^{ikz} part of the total piston term $e^{i[kz-(m+n+1)\tan^{-1}z/z_0]}$ in double-precision to perform the procedure described above. Let the wavelength λ of the light be such that the resonance condition Eq. (5.8) is fulfilled for some q, m, n and L. Let us hold λ fixed and change the length of the cavity L by a small amount $\varepsilon = \lambda/100$. We compare the piston phase acquired by a mode of the original cavity as it traverses the length of the cavity to the piston phase acquired by the same beam in the new, slightly longer cavity as it traverses the length of the new cavity. The change in the piston phase due to the kz term in the exponent after a one way trip in the cavity is:

$$\Delta\Phi_{kz} = k(L+\varepsilon) - kL = k\varepsilon = \frac{2\pi}{\lambda}\frac{\lambda}{100} = \frac{\pi}{50} \tag{5.9}$$

The change in the piston phase due to the Guoy phase shift term in the exponent after a one way trip in the cavity is:

$$\Delta\Phi_{Guoy} = (m+n+1)\tan^{-1}\left[\frac{L+\varepsilon}{z_{0\varepsilon}}\right] - (m+n+1)\tan^{-1}\left[\frac{L}{z_0}\right] \tag{5.10}$$

Define:

$$\alpha(L) = \left[\frac{(g_0+g_L-2g_0g_L)^2}{g_0g_L(1-g_0g_L)}\right]^{1/2} . \tag{5.11}$$

Then:

$$\Delta\Phi_{Guoy} = (m+n+1)\tan^{-1}[\alpha(L+\varepsilon)] - (m+n+1)\tan^{-1}[\alpha(L)] \tag{5.12}$$

We assume that the new cavity with length $L+\varepsilon$ is also a stable cavity. Then, ε is very small compared to L, $R_0 - L$, $R_L - L$. Hence, we obtain:

$$\Delta\Phi_{Guoy} = (m+n+1)\frac{1}{1+\alpha^2(L)}\left[L\frac{d\alpha}{dL}\right]\left[\frac{\varepsilon}{L}\right]. \tag{5.13}$$

For relatively small m and n, the right-hand side of Eq. (5.13) is of the order of $\varepsilon/L = \lambda/(100L)$. Hence, we deduce that the Guoy phase shift Φ_{Guoy} varies much less rapidly than Φ_{kz} phase shift as the length of the cavity changes. Note that changing the length of the cavity is equivalent to changing the frequency of the input light. Eq. (5.9) implies that Φ_{kz} must be computed in double precision since realistic cavity lengths will be larger than $10^7\lambda$. The rest of the calculation can proceed in single precision as the Guoy piston term is added to the kz term and it does not vary as rapidly as the kz term as a function of the cavity length.

The e^{ikz} term needs to be computed in double precision only if we want to know the longitudinal mode number q in Eq. (5.9) as well as the transverse mode numbers m and n. In realistic experiments, the longitudinal mode number q in Eq. (5.8) is not a known quantity. The laser frequency is tuned between two longitudinal modes of the cavity until resonance is obtained in the cavity pumped by the laser. If the amount of tuning exceeds the longitudinal mode spacing of the cavity, the laser is made to "hop" one or more longitudinal modes of the cavity to bring the tuning within range again.

The Rayleigh range z_0 of the beam as defined by Eq. (5.5) can be written as:

$$z_0 = \frac{L}{\alpha(L)} \tag{5.14}$$

where $\alpha(L)$ is given by Eq. (5.11). The Guoy phase Φ_{Guoy} acquired after a one way trip in the cavity is completely determined by the Rayleigh range:

$$\Phi_{Guoy} = (m+n+1)\tan^{-1}[\alpha(L)] \tag{5.15}$$

This means that in order to compute the diffractive effects in the cavity to a high degree of accuracy, only the Guoy phase term has to be taken into account in the interference and diffraction calculations. The longitudinal phase term e^{ikz} can be dropped from the calculation. In what follows, I will make this more precise.

Consider the resonance condition given by Eq. (5.8). Let L and λ be given with values not necessarily corresponding to a cavity resonance. Then:

$$\frac{2L}{\lambda} = q' + \frac{\lambda_{frac}}{\lambda} \tag{5.16}$$

where q' is a very large integer and $|\lambda_{frac}| \leq \lambda$. Substituting this into the resonance condition given by Eq. (5.8), we obtain:

$$\frac{\lambda_{frac}}{\lambda} = (q-q') + \frac{1}{\pi}(m+n+1)\cos^{-1}(g_0 g_L)^{1/2}. \tag{5.17}$$

Let us assume that we are looking for the transverse mode determined by a particular choice of the integers m and n. Since $\cos^{-1}(x)$ varies between 0 and π, Eq. (5.17) can be written as:

$$\frac{\lambda_{frac}}{\lambda} = (q-q') + \Pi(L,m,n) + \beta(L,m,n) \tag{5.18}$$

where

$$\Pi(L,m,n) = \text{Integer part of}[\frac{1}{\pi}(m+n+1)\cos^{-1}(g_0 g_L)^{1/2}] \tag{5.19}$$

and

$$\beta(L,m,n) = \text{Fractional part of}[\frac{1}{\pi}(m+n+1)\cos^{-1}(g_0 g_L)^{1/2}]. \tag{5.20}$$

The dropping of the e^{ikz} term from the calculation is equivalent to choosing the arbitrary longitudinal mode number q so that:

$$q = q' - \Pi(L,m,n). \tag{5.21}$$

With this choice, the resonance condition [Eq. (5.18)] becomes:

$$\frac{\lambda_{frac}}{\lambda} = \beta(L, m, n). \tag{5.22}$$

The solution λ_{frac} of this equation in terms of the cavity length L and the transverse mode numbers m and n is the remainder when the cavity length L is divided by the initial wavelength λ. Therefore, the change in the initial wavelength λ required to reach the resonance is:

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_{frac}}{q'\lambda} = \frac{\beta(L, m, n)}{q'} \le \frac{1}{q'}. \tag{5.22}$$

This is equivalent to changing the length L of the cavity by λ_{frac}. As we have shown in Eq. (5.13), the effect of this on the Guoy phase of the cavity is of the order of $1/q'$ which is completely negligible for cavities under consideration.

The discussion above implies that all the transverse modes of the cavity have the same "waist" size $w_0(z)$ and the same "radius of curvature" $R(z)$ to a very high degree of accuracy [See Eqs. (5.6) and (5.7)]. Since we can now drop the e^{ikz} term from the calculation, the computation can be performed in single-precision and the selection of the modes can be accomplished by manipulation of the Guoy phase of the wavefront propagating in the cavity.

The Guoy phase varies between 0 and 2π for the entire set of the transverse modes of the cavity. Since the number of the transverse modes is infinite in general, the cavity eigenmodes will fill the interval $[0, 2\pi]$ densely. Other selection mechanisms, like finite apertures and finite grid sizes have to be employed to damp out the undesired modes. Input beam to the cavity can also be mode-controlled in order not to excite these very high order modes.

Here is a summary of the calculations performed above: Consider a Hermite-Gaussian mode of a given cavity in front of one of the mirrors of the cavity. Compare this mode with a copy of itself which has taken a round trip around the cavity. The copy that has traveled acquires a piston phase which consists of two parts: (a) The Guoy phase which is a function of the "macroscopic" parameters of the cavity. That is to say, it only depends on the "g-factors" of the mirrors of the cavity which are given by Eq. (5.4). (b) The propagation phase $2kL$ which depends very sensitively on the wavelength.

The cavity is said to be resonating in the mode under consideration if and only if the Guoy phase exactly cancels the propagation phase after one round trip in the cavity. As we have shown above, we do not have to compute the propagation phase if we are not interested in determining the longitudinal mode number of the mode in question. As a matter of fact, we can completely drop the e^{ikz} term from the calculation. However, in order to satisfy the resonance condition, a piston phase equal in value to the Guoy phase has to be removed from the mode after a round trip. If we know the mode numbers, then we can compute this Guoy phase and remove it from the mode as it makes round-trips in the cavity. If we do not know the mode numbers, then we "scan" the cavity by changing the amount of phase we remove after each round trip. The resonance will be identified by the decrease of the loss of the mode in question as we change the Guoy phase associated with it. In the following section, I'll describe algorithms which successfully accomplish this even if the mode shape is not known beforehand.

VI. ALGORITHMS FOR DETERMINING THE MODE SHAPE AND THE MODE FREQUENCY

Consider a stable cavity with mirrors M_0, M_L located at $z=0$ and $z=L$. Let $E_{mn\ old}(x,y,0)$ be the amplitude of a mode of the cavity identified by the mode numbers m and n at the location $z=0$ with propagation phase e^{ikz} set to zero. Let $E_{mn\ R\text{-}T}(x,y,0)$ be the amplitude of the same mode after it has taken a round trip in the cavity and after the removal of the e^{2ikL} propagation phase term. Define the "new" mode amplitude $E_{mn\ new}(x,y,0)$ at $z=0$ by:

$$E_{mn\ new}(x,y,0)=\frac{E_{mn\ old}(x,y,0)+E_{mn\ R\text{-}T}(x,y,0)\,e^{i\Phi_{Guoy\ R\text{-}T}(m,n)}}{2} \tag{6.1}$$

where

$$\Phi_{Guoy\ R\text{-}T}(m,n)=2(m+n+1)\tan^{-1}[\alpha(L)] \tag{6.2}$$

with $\alpha(L)$ is as given by Eq. (5.11).

From the form of the Hermite-Gaussian modes given by Eq. (5.2), we see that $E_{mn\ new}(x,y,0)=E_{mn\ old}(x,y,0)$. Therefore, Eq. (6.1) is an iteration equation which is only satisfied by a mode of correct shape and correct frequency. Any mode or field which does not have the correct shape or the correct frequency will give a resulting field with $|E_{mn\ new}(x,y,0)|<|E_{mn\ old}(x,y,0)|$. This means that if the iteration is carried through for a sufficient number of times, the field which does not have either the correct shape or the correct frequency will diminish in amplitude. In what follows, I will prove this assertion.

First assume that the field $E_{mn\ old}(x,y,0)$ has the correct mode shape as given by Eq. (5.2) at $z=0$. This field can be expressed as:

$$E_{mn\ old}(x,y,0)=A(x,y)\,e^{i\theta(x,y)} \tag{6.3}$$

where the real function $A(x,y)$ is the amplitude of the field at the point $(x,y,0)$ and the real function $\theta(x,y)$ is the phase of the field at the point $(x,y,0)$.

Suppose that we do not know the correct resonance frequency of this mode. This means that we have to try a different phase factor in the iteration equation Eq. (6.1) instead of the $\Phi_{Guoy\ R\text{-}T}(m,n)$. Call this phase factor θ'. Using Eq. (6.1), we get:

$$E_{mn\ new}(x,y,0)=E_{mn\ old}(x,y,0)\left[\frac{1+e^{i[\theta'-\Phi_{Guoy\ R\text{-}T}(m,n)]}}{2}\right]. \tag{6.4}$$

Let $\Delta\theta=\theta'-\Phi_{Guoy\ R\text{-}T}(m,n)$. In other words, $\Delta\theta$ is the deviation from the correct Guoy phase. Then Eq. (6.4) can be written as:

$$E_{mn\,new}(x,y,0)=E_{mn\,old}(x,y,0)\,e^{i\Delta\theta/2}\cos(\Delta\theta/2). \tag{6.5}$$

The amplitude of the new field at the point $(x,y,0)$ is given by:

$$|E_{mn\,new}(x,y,0)| = A(x,y)\,|\cos(\Delta\theta/2)| < A(x,y) = |E_{mn\,old}(x,y,0)|. \tag{6.6}$$

After we perform the iteration N times by substituting $E_{mn\,new}(x,y,0)$ in place of $E_{mn\,old}(x,y,0)$ to compute the "new" $E_{mn\,new}(x,y,0)$, the amplitude of the resultant field $E_{mn\,N}(x,y,0)$ is given by:

$$|E_{mn\,N}(x,y,0)| = A(x,y)\,|\cos^N(\Delta\theta/2)| \tag{6.7}$$

Note that the resultant field amplitude goes to zero as the number of iterations N approaches infinity unless $\Delta\theta=0$.

Assume that we are performing the iterations for a fixed, large N. If $\Delta\theta$ is sufficiently small so that $\cos^N(\Delta\theta/2)\simeq 1-(N/2)(\Delta\theta)^2$ holds, the difference between the amplitude $A(x,y)$ of the initial field and the amplitude of the resultant field $E_{mn\,N}(x,y,0)$ is given by:

$$\text{amplitude difference}=A(x,y)\,\frac{N}{2}\,(\Delta\theta)^2 \tag{6.8}$$

which describes a parabola in the variable $\Delta\theta$.

The total energy $I_{mn\,N}$ of the field after N iterations is given by:

$$I_{mn\,N}=I_0\cos^{2N}(\Delta\theta/2) \tag{6.9}$$

where

$$I_0=\int A^2(x,y)\,dx\,dy \tag{6.10}$$

in suitably normalized units. The ratio of the difference between the initial field energy and the field energy after N iterations to the initial field energy for small $\Delta\theta$ is given by:

$$\frac{I_0-I_{mn\,N}}{I_0}=N\,(\Delta\theta)^2. \tag{6.11}$$

Now, suppose that we know the correct resonance frequency of the cavity in a particular mode but we do not know the correct mode shape. In this case we start the iteration with an arbitrary field distribution at $z=0$ which satisfies the relevant boundary conditions there. Call this field $E_{init\,old}(x,y,0)$.

In general, the Hermite-Gaussian modes of a cavity define a set of orthogonal functions. Any field distribution which is defined at $z=0$ and which satisfies the relevant boundary conditions there can be written as a sum of the orthogonal modes of the cavity which are defined at that point. Hence:

$$E_{init\,old}(x,y,0)=\sum_{\substack{i=0\\ j=0}}^{\infty} B_{ij}\,E_{ij\,old}(x,y,0) \tag{6.12}$$

where $E_{ij\,old}(x\,,y\,,0)$ are the transverse modes of the cavity identified by the mode numbers i and j. B_{ij} are constants determined by:

$$B_{ij} = \int E_{init\,old}(x\,,y\,,0)\,E^*{}_{ij\,old}(x\,,y\,,0)\,dx\,dy \tag{6.13}$$

Here, we assume that these modes are normalized in such a way that

$$\int E_{ij\,old}(x\,,y\,,0)\,E^*{}_{kl\,old}(x\,,y\,,0)\,dx\,dy = \delta_{ik}\,\delta_{jl}\,, \tag{6.14}$$

where δ_{ik} is the Kronecker-delta function which has the value 1 when $i=k$ and the value 0 when i is different from j. "*" denotes the complex conjugation operation.

Suppose we know the frequency of the mode labeled by the mode numbers m and n. We define the iteration equation which will give the required mode shape after a sufficient number of iterations by:

$$E_{init\,new}(x\,,y\,,0) = \frac{E_{init\,old}(x\,,y\,,0) + E_{init\,R-T}(x\,,y\,,0)\,e^{i\Phi_{Guoy\,R-T}(m\,,n)}}{2} \tag{6.15}$$

where

$$\Phi_{Guoy\,R-T}(m\,,n) = 2\,(m+n+1)\tan^{-1}[\alpha(L)] \tag{6.16}$$

with $\alpha(L)$ is as given by Eq. (5.11). $E_{init\,R-T}(x\,,y\,,0)$ is the field $E_{init\,old}(x\,,y\,,0)$ after one round-trip in the cavity.

As we have shown in the previous case, the mode $E_{mn\,old}(x\,,y\,,0)$ in the expansion for the initial field $E_{init\,old}(x\,,y\,,0)$ will not be altered by the iteration equation since its Guoy phase is indeed $\Phi_{Guoy\,R-T}(m\,,n)$. All of the other modes in the expansion will suffer a degradation of their amplitude after an iteration. The amplitude of the mode labeled by the mode numbers i and j in the expansion for $E_{init\,old}(x\,,y\,,0)$ after N iterations is given by:

$$A_{ij\,N}(x\,,y\,,0) = |B_{ij}\,E_{ij\,old}(x\,,y\,,0)|\;|\cos^N(\Delta\theta_{ij}/2)| \tag{6.17}$$

where $\Delta\theta_{ij} = 2\,[(m-i)+(n-j)]\tan^{-1}[\alpha(L)]$. We see from Eq. (6.17) that all of the modes other than the mode labeled by the mode numbers m and n will lose amplitude as the number of iterations N gets larger. In the end, we will be left only with the correct mode.

It is possible that the mode we are looking for does not enter into the expansion of the initial waveform by an unfortuitous choice of the starting waveform. In general, this is very unlikely. One always has an inkling of the approximate mode shape one expects from the cavity in question. Specifying that approximate mode shape as the starting waveform will solve this problem.

Now, assume that we know neither the frequency nor the shape of the mode we are looking for. We again choose an appropriate starting waveform $E_{init\,old}(x\,,y\,,0)$ which satisfies the boundary conditions in front of one of the mirrors of the cavity at $z=0$. We expand this waveform in terms of the orthogonal modes of the cavity defined at that point:

$$E_{init\,old}(x,y,0) = \sum_{\substack{i=0\\j=0}}^{\infty} C_{ij}\, E_{ij\,old}(x,y,0) \qquad (6.18)$$

where $E_{ij\,old}(x,y,0)$ are the transverse modes of the cavity identified by the mode numbers i and j. C_{ij} are constants determined by:

$$C_{ij} = \int E_{init\,old}(x,y,0)\, E^{*}_{ij\,old}(x,y,0)\, dx\, dy \qquad (6.19)$$

These modes are normalized in such a way that

$$\int E_{ij\,old}(x,y,0)\, E^{*}_{kl\,old}(x,y,0)\, dx\, dy = \delta_{ik}\,\delta_{jl}\,, \qquad (6.20)$$

where δ_{ik} is the Kronecker-delta function.

We introduce the iteration equation:

$$E_{init\,new}(x,y,0) = \frac{E_{init\,old}(x,y,0) + E_{init\,R-T}(x,y,0)\, e^{i\Delta\Phi}}{2} \qquad (6.21)$$

where $\Delta\Phi$ is an arbitrary trial phase angle and $E_{init\,R-T}(x,y,0)$ is the field $E_{init\,old}(x,y,0)$ after one round-trip in the cavity. The amplitude of the mode labeled by the mode numbers i and j in the expansion for $E_{init\,old}(x,y,0)$ after N iterations is given by:

$$A_{ij\,N}(x,y,0) = |C_{ij}\, E_{ij\,old}(x,y,0)|\ |\cos^{N}(\Delta\theta_{ij}/2)| \qquad (6.22)$$

where $\Delta\theta_{ij} = \Delta\Phi - 2(i+j+1)\tan^{-1}[\alpha(L)]$ and $\alpha(L)$ is given by Eq. (5.11).

We see from Eq. (6.22) that all of the modes will lose amplitude. As N gets larger, the energy in the iterated field goes to zero unless for some integers m and n, $\Delta\theta_{mn} = 0$. This implies that

$$\Delta\Phi = 2(m+n+1)\tan^{-1}[\alpha(L)]\,. \qquad (6.23)$$

When this happens, the iterated field will converge to the mode labeled by m and n of the cavity and the Guoy phase of the mode will be given by Eq. (6.23).

In summary: In order to find the shape and the frequency of a mode of the cavity, we use the iteration equation (6.21) with an arbitrary initial field and an arbitrary phase. We iterate this equation a number of times and plot the energy left in the iterated field as a function of the phase we put in. When the phase we substitute is equal to the Guoy phase of the mode, the energy left in the iterated field will be the largest. The iterated field will be equal to the mode of the cavity and the frequency of the mode can be extracted from the Guoy phase thus determined.

It is informative to estimate the number of iterations N which is needed to extract the desired mode from an arbitrary input waveform. Assume that the trial phase $\Delta\Phi$ in Eq. (6.21) is 1 degree larger than the actual Guoy phase for the mode labeled by a particular pair of integers m and n. In the units we are using, the spacing between two longitudinal modes of the cavity is 360 degrees. Hence, this separation corresponds to a frequency difference of $1/360$ of the frequency difference between two longitudinal modes. The ratio of the intensity left in the m,n component of the starting waveform after N iterations to the intensity of the m, n component of the starting waveform before the iterations is given by:

$$\text{Intensity Ratio} = \cos^{2N}(\Delta\theta / 2) \tag{6.24}$$

where $\Delta\theta = 1$ degree. We see that for $N = 100$ iterations, the intensity ratio is 0.9924 which corresponds to a loss of 7.6×10^{-3} for the mode with the wrong frequency. This loss is easily detectable with the default 7 digit accuracy of single-precision FORTRAN. As a matter of fact, one percent of this loss is easily discernible with $N = 100$ iterations. This implies a resolution of $1/3600$ of the frequency difference between two longitudinal modes with just 100 iterations.

The argument above was presented for the Hermite-Gaussian modes of an ideal cavity. When the mirrors are not perfect, the modes of the cavity will deviate from the modes of the ideal cavity. However, imperfect cavities which deviate slightly from ideal, stable cavities do have an orthogonal or bi-orthogonal set of modes in general. The analysis described above can be carried out for not-quite-perfect cavities. I will present such an analysis in a later manuscript.

VII. EXAMPLES OF GLAD SIMULATIONS

In the following appendices I will give listings of GLAD input commands which implement some of the algorithms described above. For the sake of brevity, I will not explain the action of each and every GLAD command given in these files. For a detailed explanation of GLAD commands, please see GLAD Version 3.8 Command Reference Manual.

In these appendices, a cavity closely resembling the cavities of the 40 meter interferometer at Caltech is simulated. The input mirror of the cavity is flat and the output mirror has a radius of curvature of 61 meters. The mirrors are assumed to be free of any losses. The size of the mirrors are restricted by an aperture of 1 cm in radius placed in front of the mirrors. The mirrors are separated by 40 meters. The cavity lies along the z direction, and x and y coordinates are used to describe the transverse field. With these parameters, the magnitude of the Guoy phase for the lowest mode of the cavity (TEM_{00}) is 108.14816805 degrees.

In the simulations, the iterations are performed until either a convergence criterion is satisfied or an upper bound for iterations is reached. The convergence criterion used in the iterations is that the iterations stop when the magnitude of the difference between 1 and the ratio of the total energy in the mode before the round trip to the total energy in the mode after the round trip is smaller than a prescribed amount.

The "differences" between two waveforms in the cavity are computed by an orthogonalization procedure. Let Ψ_1 and Ψ_2 be two waveforms in the cavity. The correlation α between Ψ_1 and Ψ_2 is a complex number defined by:

$$\alpha = \frac{\int \Psi^*{}_2 \Psi_1 \, dx \, dy}{\left[\int |\Psi_2|^2 dx \, dy \int |\Psi_1|^2 dx \, dy \right]^{1/2}} \tag{7.1}$$

Let $\Psi_{\parallel}$ be the component of Ψ_1 "along" Ψ_2 and let $\Psi_{\perp}$ be the component of Ψ_1 "orthogonal" to Ψ_2.

$$\Psi_1 = \Psi_{\parallel} + \Psi_{\perp} \tag{7.2}$$

$$\int \Psi^*_{\parallel} \Psi_{\perp} \, dx \, dy = 0, \tag{7.3}$$

$$\int \Psi^*_{\perp} \Psi_2 \, dx \, dy = 0. \tag{7.4}$$

Then:

$$\Psi_{\parallel} = \frac{\int \Psi^*_2 \Psi_1 \, dx \, dy}{\int |\Psi_2|^2 \, dx \, dy} \Psi_2 \tag{7.5}$$

with

$$\Psi_{\perp} = \Psi_1 - \Psi_{\parallel}. \tag{7.6}$$

In the Appendix Ia, the listing for the GLAD simulation of the lowest mode of the cavity described above with the correct input light frequency and the correct mode shape is shown. The dimensions of the array which describes the transverse field is 256 by 256. The distance between the array elements are 0.01 centimeters. Appendix Ib shows the output produced by the GLAD program for the input file of Appendix I(a). The mode satisfies the convergence criterion (one part in a million difference between 1 and the ratio of energies before and after the iteration) in one iteration. The actual error is 2.4×10^{-7}. Figure Ia shows the orthogonal component of the converged mode to the ideal mode of the cavity in an isometric plot at the flat mirror. Figure Ib shows the section of the same plot at $y = 0$ as a function of x. The peak intensity of the converged mode is 1. These figures show the effects of the apertures and the amount of numerical errors.

In the Appendix IIa, a similar simulation to the one given in the Appendix Ia is performed with the exception that the transverse grid size is now 64 by 64. The distance between the array elements is scaled up to 0.04 centimeters so that the array covers the same area as it did in the previous simulation. Appendix IIb shows the corresponding GLAD output. The mode again converges in one iteration, with one surprising exception: The energy loss (Energy before/Energy after)-1 = 1.79×10^{-7} is less! One would expect that the finer grid results in a more accurate result. It indeed does: The finer grid can represent variations of the field caused by the apertures and other numerical errors better, and that manifests itself as a larger loss. Figures IIa and IIb show the orthogonal component of the converged mode to the ideal mode at the flat mirror in an isometric plot and in a cross-section plot at $y = 0$ respectively. The peak intensity of the converged mode is 1.

In the Appendix IIIa, a similar simulation to the one given in the Appendix IIa is performed. The convergence criterion is disabled and the initially perfect mode is subjected to 100 iterations to track the errors. In Appendix IIIb, only the last page of GLAD output is shown. The energy loss as defined above is about 1.0×10^{-6} after 100 iterations. The Figures IIIa and IIIb again show the orthogonal component of the converged mode to the ideal mode at the flat mirror in an isometric plot and in a cross-section plot at $y = 0$ respectively. The peak intensity of the converged mode is 0.998.

In the Appendix IVa, a simulation to find the mode shape of the lowest mode of the cavity when the frequency of the mode is known is performed. The initial waveform is assumed to be flat (constant amplitude) at the flat mirror. The convergence criterion is that the energy loss as defined above is equal to 1 part in 10^4. The procedure converges is 21 iterations with an energy loss of 8.5×10^{-5} as shown in Appendix IVb. Figures IVa and IVb show the orthogonal component of the converged mode to the ideal mode at the flat mirror in an isometric plot and in a cross-section plot at $y = 0$ respectively. The peak intensity of the converged mode is 4.03.

Appendix Va shows the GLAD output for an identical simulation to the one given in Appendix IVa. The convergence criterion is disabled and the iterations are performed 100 times. The final energy loss is about 1.5×10^{-6}. Figures Va and Vb show the orthogonal component of the converged mode to the ideal mode at the flat mirror in an isometric plot and in a cross-section plot at $y = 0$ respectively. The peak intensity of the converged mode is 4.00.

Appendix VIa shows the GLAD output for an identical simulation to the one given in Appendix IVa. The convergence criterion is disabled and the iterations are performed 25 times. The final energy loss is about 1.8×10^{-5}. Figures VIa and VIb show the orthogonal component of the converged mode to the ideal mode at the flat mirror in an isometric plot and in a cross-section plot at $y = 0$ respectively. The peak intensity of the converged mode is 4.00.

Appendix VIIa gives the input listing of a simulation in which neither the cavity mode shape nor its frequency are known. The Guoy phase is scanned around the correct Guoy phase of the cavity. In this simulation the scanning step is 2 degrees. The initial waveform has a constant amplitude at the flat mirror. The iteration process is repeated for 25 times at each value of the Guoy phase and the energy left in the waveform after 25 iterations is printed and plotted as a function of the trial Guoy phase. The voluminous GLAD output is not shown. Appendix VIIb tabulates the energy left in the waveform after the iterations as a function of the Guoy phase. The second column is the Guoy phase in degrees, the third column is the energy left in the waveform. Figure VII shows a plot of this table. Note that the maximum occurs at the correct Guoy phase and the shape of the curve near the maximum is a parabola.

Appendix VIIIa gives the input listing for an identical simulation to the one shown in Appendix VIIa with a finer resolution scan. Appendix VIIIb tabulates the energy left in the mode after the iterations as a function of the initial Guoy phase. Figure VIII shows a plot of this table.

Appendix IX gives the input listing for a simulation in which the input waveform is a Gaussian that matches the lowest mode of the cavity, but its phase is tilted around the peak of the waveform in the y direction in such a way to produce a 0.68 wavelength tilt at 1 cm from the center. y cross-section of the phase of the input wave at $x = 0$ is shown in Figure IXa. The iterations are performed at the correct Guoy phase for the lowest mode of the cavity, and the orthogonal component of the input waveform to the resulting mode of the cavity is computed and displayed in an isometric plot as shown in Figure IXb. Figure IXc shows the y cross-section of the isometric plot at $x = 0$. Note that the π phase shift usually associated with the beams that come out of the cavity is not put in because it makes no difference in computing the intensity of the orthogonal component. This orthogonal component is the a representation of the "reflected" wave from the cavity. The plots are produced at a lower resolution than the calculation. The peak intensity of the input waveform is 1.

The input listing for a simulation with distorted mirrors are given in Appendix Xa. The distortion in the mirrors are simulated by adding a random phase error to each wavefront every time it bounces off a mirror. The correlation length of the random phase front is 0.5 cm in the x direction and 0.6 cm in the y direction for the flat mirror. For the curved mirror, the correlation lengths are 0.65 cm in the x direction and 0.7 cm in the y direction. The random phase fronts are chosen once in the beginning of the program for each mirror. The initial field is assumed to be a flat waveform with constant amplitude at the flat mirror. The waveform is subjected to 25 iterations in the cavity at each value of the scanning Guoy phase and the resulting energy is printed and plotted as a function of the Guoy phase. Appendix Xb tabulates the energy as a function of the Guoy phase and Figure X shows a plot of this table. Note that the Guoy phase is near 104 degrees which is 4 degrees away from the phase of the lowest mode of an ideal cavity with the parameters.

Appendix XIa shows an identical simulation to the one shown in Appendix Xa with a high resolution scan to determine the Guoy phase better. Appendix XIb tabulates the results of the iterations, and Figure XI shows a plot of the resulting table. We determine that the Guoy phase is approximately 104.750 degrees.

Appendix XIIa shows a listing of the GLAD input which checks whether the mode shape and the frequency determined by the previous two simulations corresponds to a "healed" mode. The mode shape is computed at the frequency determined above and the round trip loss of this mode is computed. This loss is compared to the loss of the lowest mode of an ideal cavity with the same parameters at the same frequency. As Appendix XIIb shows the round-trip loss of the healed mode is 1.3×10^{-5} while the loss of the ideal mode in the distorted cavity is 8.1×10^{-5}. Figure XIIa and figure XIIb show the orthogonal component of the healed mode after a round-trip to the healed mode before the round-trip in the cavity in an isometric plot and in a x cross-section plot respectively at the flat mirror. Note that this orthogonal component is consistent with the aperture and the numerical errors indicating that the mode is indeed healed. However, the shape of the mode is not Gaussian as shown in Figures XIIc and XIId. These figures show the orthogonal component of the healed mode to the ideal lowest mode of the cavity in an isometric and in a x cross-section plot respectively. The peak intensity of the healed mode is 3.97 at the flat mirror.

VIII. CONCLUSIONS

It is shown that the program GLAD can successfully be used in simulating cavities which are relevant to the work done in the LIGO project. The feasibility of single-precision simulation is demonstrated. Algorithms to find the mode shapes and resonant frequencies of the cavities are developed. Theoretical limits on the rate of convergence of these algorithms are established and these limits are verified in the numerical experiments. It is shown that randomly distorted mirrors result in healed modes if the correlation length of the distortion is comparable to the size of the mode in the cavity.

In part II of this document, I will give a theoretical analysis of the slightly distorted cavities as well as the results of a complete simulation of distorted cavities with varying correlation lengths. The immediate extension of the results given in this manuscript to much longer cavities will be presented. Most of the work for the problems mentioned above is already complete. The next difficult stage of the simulation is the development and the analysis of the algorithms for coupled cavities used in recycling interferometers. Some advances in this front will also be presented in the next installment of this document.

```
C  The simulation of 40 meter cavity with correct input light frequency
C  and the correct mode shape.  This is a comparison between what goes
C  in and what comes out.  This one tries the paraxial coordinate system with
C  the energy ratios as the convergence criterion.  Array dimensions are
C  256 by 256.
C
C  Define the iteration macro:
C
macro/def cavity/overwrite
C
   register/add/int 1 1                # Increment pass counter
C
   prop 4000.0                         # Propagate 40 m. forward
C
   clap/cir/con 1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph/con 1 -6100.0            # Mirror of 61 m. radius
C
   prop 4000.0                         # Propagate 40 m. backwards
C
   clap/cir/con 1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph/con 1 1.0e15             # Flat mirror
C
   phase/piston 1 108.14816805         # Add the correct Guoy phase for
                                       # TEM00 mode (mode frequency
                                       # selection).
C
   add/coh/con 1 2                     # Add beam 2 to beam 1 coherently
                                       # (interference with the incoming beam)
C
   mult/scalar 1 .25000000             # Normalize it by multiplying
                                       # the intensity by .25000000.
C
   copy/con 1 2                        # Copy the resultant beam to
                                       # the accumulator beam
C
   register/set/param  1 1 energy/real # Put the energy of the Beam 1 in
                                       # real register number 1.
C
   register/div/real   1 %r1 %r2       # Calculate the ratio of the current
                                       # and the previous energies.
C
   register/sub/real 1 1.0             # Subtract 1.0 from it.
C
C  If its is negative, change its sign:
C
   if %r1 < 0.0 register/mul/real 1 %r1 -1.0
C
   if %r1 <= 0.000001 macro/exit       # Exit from macro on convergence
C
   register/set/param 2 1 energy/real  # Otherwise, put the current energy
                                       # in real register number 2.
C
macro/end                              # End of macro definition.
C
nbeam 2                                # Define two beams; beam 2 is the
                                       # accumulating beam.
C
array/set 1 256 256 0                  # Set the array dimensions for beam 1
array/set 2 256 256 0                  # Set the array dimensions for beam 2
C
color 1 0.5145                         # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                         # Set the mean wavelength for beam 2 (microns)
```

```
C
units/set 1 0.01 0.01                  # The distance between the array elements
                                       # for beam 1 (cm)
units/set 2 0.01 0.01                  # The distance between the array elements
                                       # for beam 2 (cm)

C
units/field 1 1.27 1.27                # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27                # Set the field radius for beam 2 (cm)
C
gaussian/cir/con 1 1.0 .217865054304   # Make the initial beam the
                                       # correct mode (beam 1)
gaussian/cir/con 2 1.0 .217865054304   # Make the initial beam the
                                       # correct mode (beam 2)
C
zbound/set 1 .217865054304             # Set the waist size for beam 1 (cm)
                                       # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304             # Set the waist size for beam 2 (cm)
                                       # (ideal, defines the Rayleigh range)
C
register/set/param 2 1 energy/real     # Put the energy of the beam 1
                                       # in real register number 2.
C
C  Plot the initial mode shape (beam 1):
C
title
initial mode shape
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  initial_mode.6
plot/isometric first=1 last=1
C
C  Ready to start. Initialize variables:
C
beams/all/off                          # Turn all beams off (no propagation)
C
beams/on 1                             # Beam 1 propagates, beam 2 accumulates
C
status/p                               # Print the status
C
register/set/int 1 0                   # Macro pass integer register
C
C  Start running the macro:
C
mac/run cavity/50                      # Run the cavity macro for 50
                                       # round trips (maximum)
C
C  Either 50 trips have taken place or the mode is converged:
C
C
C  Plot the converged mode shape at the flat mirror:
C
title
converged mode shape at flat
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat.6
plot/isometric first=1 last=1
C
copy/con 1 2                           # Copy converged beam to beam 2.
C
C
C  Propagate beam forward for 40 m to the other mirror:
C
prop 4000.0                            # Propagate 40 m.
```

```
C
clap/cir/con 1 1.0                     # 1.0 cm. radius aperture
C
mirror/sph 1 -6100.0                   # Mirror of 61 m. radius
C
C  Plot the mode shape at the other mirror:
C
title
converged mode shape at curved
set/density 64 64
set/window/abs -1.28 1.28 -1.28 1.28
plot/disk final_curved.6
plot/isometric first=1 last=1
C
C  Propagate the beam 40 m backwards to the flat mirror:
C
prop 4000.0
C
clap/cir/con      1 1.0                # 1.0 cm. radius aperture
C
mirror/sph    1 1.0e15                 # Flat mirror
C
phase/piston 1 108.14816805            # Add the correct Guoy phase for
                                       # TEM00 mode (mode frequency
                                       # selection).
C
add/coh/con   1 2                      # Add beam 2 to beam 1 coherently
                                       # (interference with the incoming beam)
C
mult/scalar   1 .25000000              # Normalize it by multiplying
                                       # the intensity by .25000000.
C
gaussian/cir/con 2 1.0 .217865054304   # Make beam 2 the correct mode
C
mult/mode/orthogonal  1 2              # Compute the orthogonal component to the
                                       # perfect mode in the resulting mode.
C
C  Plot the residual intensity:
C
title
Difference between input and converged beams
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat_diff.6
plot/isometric first=1 last=1
C
C  Plot the x-slice of the residual intensity:
title
x-slice of the difference
plot/disk final_fl_df_x.6
plot/xslice/intensity 1 0.0 -0.5 0.5
C
C  This is it:
C
end
```

```
end of macro commands

Expanding number of beams.

Maximum beam number =   2

Generic name (  ): beam

 Beam Array Dimensions and Disk Files
                                                          File Names
    Beam No.      Nlinxs       Nlinys   Nlinxs*Nlinys    P 1          P 2
        1          256          256       65536           (  )s01
Generic name (  ): beam

 Beam Array Dimensions and Disk Files
                                                          File Names
    Beam No.      Nlinxs       Nlinys   Nlinxs*Nlinys    P 1          P 2
        2          256          256       65536           (  )s02
 Beam No.  Vac. wavel. (mic.)     Eff. wavel. (mic),       Index
    1                0.514500                0.514500    1.000000
 Beam No.  Vac. wavel. (mic.)     Eff. wavel. (mic),       Index
    2                0.514500                0.514500    1.000000
 beam no.  unitsx  unitsy     fieldx  fieldy
    1     1.000E-02 1.000E-02  1.27      1.27
          CHG UNIT  CHG UNIT

 beam no.  unitsx  unitsy     fieldx  fieldy
    2     1.000E-02 1.000E-02  1.27      1.27
          CHG UNIT  CHG UNIT

 beam no.  unitsx  unitsy     fieldx  fieldy
    1     1.000E-02 1.000E-02  1.27      1.27
          CHG UNIT  CHG UNIT

 beam no.  unitsx  unitsy     fieldx  fieldy
    2     1.000E-02 1.000E-02  1.27      1.27
          CHG UNIT  CHG UNIT

              circle   gaussian fluence distribution
 beam no.  I (w/cm2) R0 x       R0 y       Sgxp  Sgyp    Dec x      Dec y
      1    1.00      0.218     0.218        1.0   1.0      0.         0.

 Beam No.  Unitsx   Unitsy     Fieldx  Fieldy
    1     1.000E-02 1.000E-02  1.27      1.27
              circle   gaussian fluence distribution
 beam no.  I (w/cm2) R0 x       R0 y       Sgxp  Sgyp    Dec x      Dec y
      2    1.00      0.218     0.218        1.0   1.0      0.         0.

 Beam No.  Unitsx   Unitsy     Fieldx  Fieldy
    2     1.000E-02 1.000E-02  1.27      1.27

              Gaussian parameters
no   waistx        waisty         zwastx        zwasty

 1    2.17865E-01   2.17865E-01   0.00000E+00   0.00000E+00

                 constant unit boundaries
      x direction                       y  direction
no left            right           left            right
 1 -2.89828E+03    2.89828E+03     -2.89828E+03    2.89828E+03


              Gaussian parameters
no   waistx        waisty         zwastx        zwasty
```

```
  2    2.17865E-01    2.17865E-01    0.00000E+00    0.00000E+00

                    constant unit boundaries
       x direction                              y  direction
 no left             right            left              right
  2 -2.89828E+03    2.89828E+03     -2.89828E+03     2.89828E+03

  energy     = 7.455740E-02 real register no.  2 =  7.455740E-02
PLOT  1, Fri Sep 21 16:46:14 1990

           coord. beam center (cm)  beam radius (cm)     field size (cm)
 beam no.      x          y          x          y          x          y
     1        0.         0.        0.218      0.218      1.27       1.27
     2        0.         0.        0.218      0.218      1.27       1.27

 beam no.   zreff    direction    zwaistx      zwaisty      iplanx iplany
    1        0.      positive       0.           0.           1      1
    2        0.      positive       0.           0.           1      1

 integer register no.  1 set to          0

  integer register no.  1 changed to         1

    chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
  no.    Xg         Yg        Zg         Xb        Yb         Xb         Yb
  1      0.         0.     4.000E+03 0.371      0.371      10.2       10.2
              circular aperture
               Beam Half Widths       Coord. of Apt. Center
 Beam No.   X          Y            X          Y      Units X    Units Y
  1     0.371      0.371          0.          0.      8.039E-02  8.039E-02

  mirror  with curvature radii = -6100.     in x  -6100.     in y

 beam n. Tilt x  Tilt y       Radx      Rady       Unitsx     Unitsy
   1      0.        0.       6100.      6100.      8.0391E-02 8.0391E-02

    chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
  no.    Xg         Yg        Zg         Xb        Yb         Xb         Yb
  1      0.         0.     8.000E+03 0.218      0.218      1.27       1.27
              circular aperture
               Beam Half Widths       Coord. of Apt. Center
 Beam No.   X          Y            X          Y      Units X    Units Y
  1     0.218      0.218          0.          0.      1.000E-02  1.000E-02

  mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

 beam n. Tilt x  Tilt y       Radx      Rady       Unitsx     Unitsy
   1      0.        0.     -5.0000E+14-5.0000E+14 1.0000E-02 1.0000E-02

 accumulating into beam no.   1 unitsx      unitsy
                              1.000E-02   1.000E-02
 beam no.  2 accumulated
  beam no.  Fac
     1    0.25000

  energy     = 7.455742E-02 real register no.  1 =  7.455742E-02
  Real register no.  1 changed to   1.00000

  real register no.  1 changed to  2.384186E-07
```

```
PLOT   2, Fri Sep 21 16:48:23 1990
     chief  Ray Propagation of    4000.      (cm).

 Beam Beam Center Coordinates (cm)    Beam Radius (cm)    Field Radius (cm)
  no.     Xg         Yg          Zg         Xb         Yb         Xb         Yb
   1      0.         0.     1.200E+04 0.371      0.371       10.2       10.2
               circular aperture
                Beam Half Widths      Coord. of Apt. Center
 Beam No.    X          Y          X          Y     Units X     Units Y
   1    0.371      0.371        0.         0.      8.039E-02  8.039E-02

   mirror  with curvature radii = -6100.     in x  -6100.      in y

 beam n. Tilt x  Tilt y        Radx       Rady        Unitsx      Unitsy
    1       0.          0.     6100.      6100.      8.0391E-02 8.0391E-02

PLOT   3, Fri Sep 21 16:49:34 1990
     chief  Ray Propagation of    4000.      (cm).

 Beam Beam Center Coordinates (cm)    Beam Radius (cm)    Field Radius (cm)
  no.     Xg         Yg          Zg         Xb         Yb         Xb         Yb
   1      0.         0.     1.600E+04 0.218      0.218       1.27       1.27
               circular aperture
                Beam Half Widths      Coord. of Apt. Center
 Beam No.    X          Y          X          Y     Units X     Units Y
   1    0.218      0.218        0.         0.      1.000E-02  1.000E-02

   mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

 beam n. Tilt x  Tilt y        Radx       Rady        Unitsx      Unitsy
    1       0.          0.   -5.0000E+14-5.0000E+14 1.0000E-02 1.0000E-02

 accumulating into beam no.   1 unitsx      unitsy
                              1.000E-02   1.000E-02
 beam no.   2 accumulated
  beam no.   Fac
     1     0.25000

                circle   gaussian fluence distribution
  beam no.   I (w/cm2) R0 x       R0 y        Sgxp  Sgyp   Dec x       Dec y
       2     1.00      0.218      0.218       1.0   1.0      0.          0.

  Beam No.  Unitsx   Unitsy      Fieldx  Fieldy
    2     1.000E-02 1.000E-02  1.27      1.27
   Kbeam    Mbeam
       1       2
 correlation, real-imaginary                 1.000001E+00        4.020324E-07

 correlation, magnitude-angle(deg) =         1.000001E+00      0.0000

PLOT   4, Fri Sep 21 16:51:46 1990
PLOT   5, Fri Sep 21 16:51:48 1990
Fri Sep 21 16:51:48 1990
```

DIFFERENCE BETWEEN INPUT AND CONVERGED BEAMS

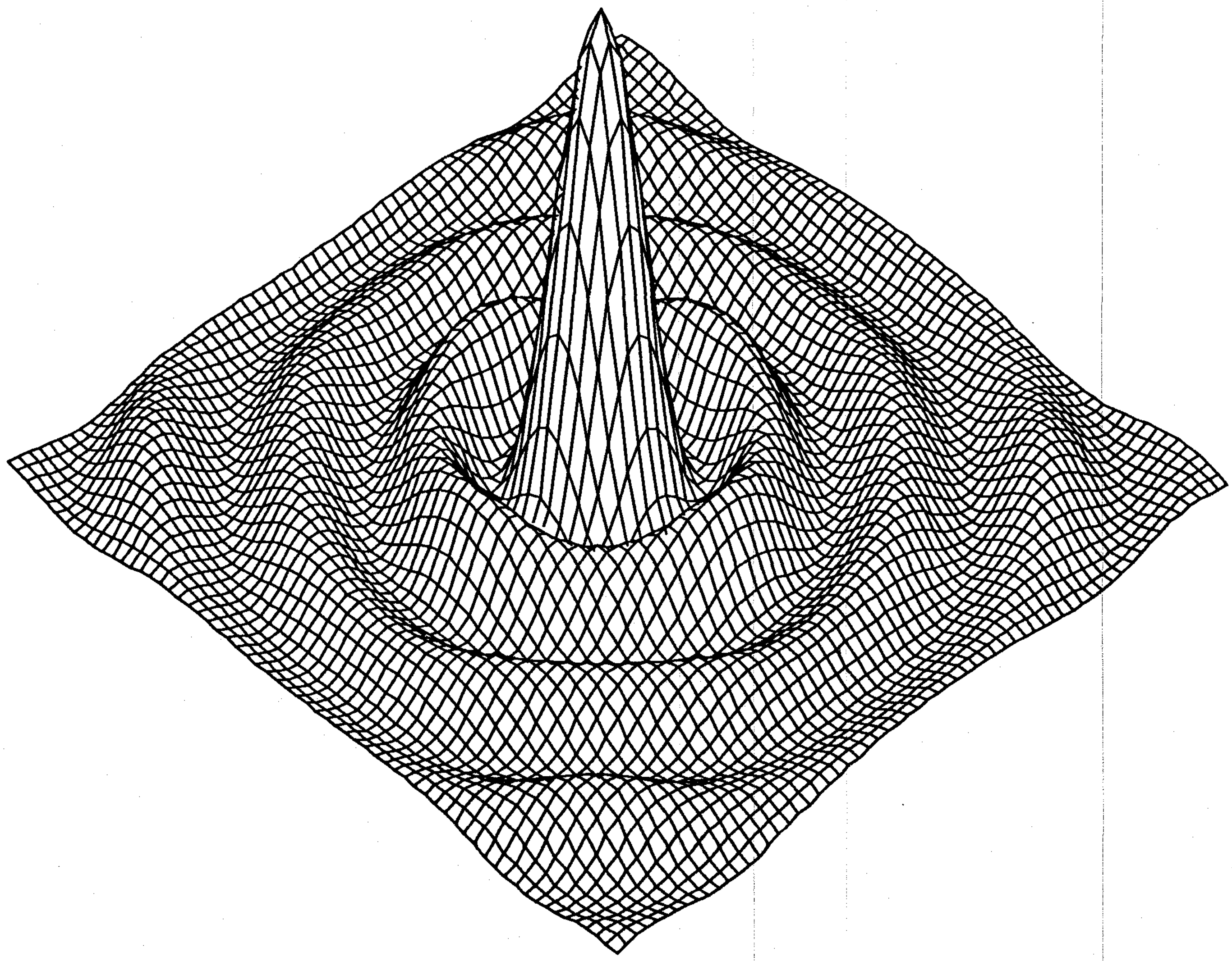
PLOT LIMITS
(X AND Y IN CM)

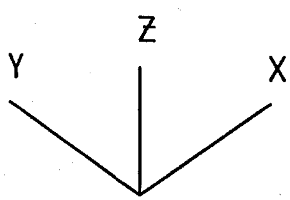
	MIN	MAX
X	-0.300	0.320
Y	-0.320	0.300
Z	1.30E-10	2.60E-07

INTENSITY

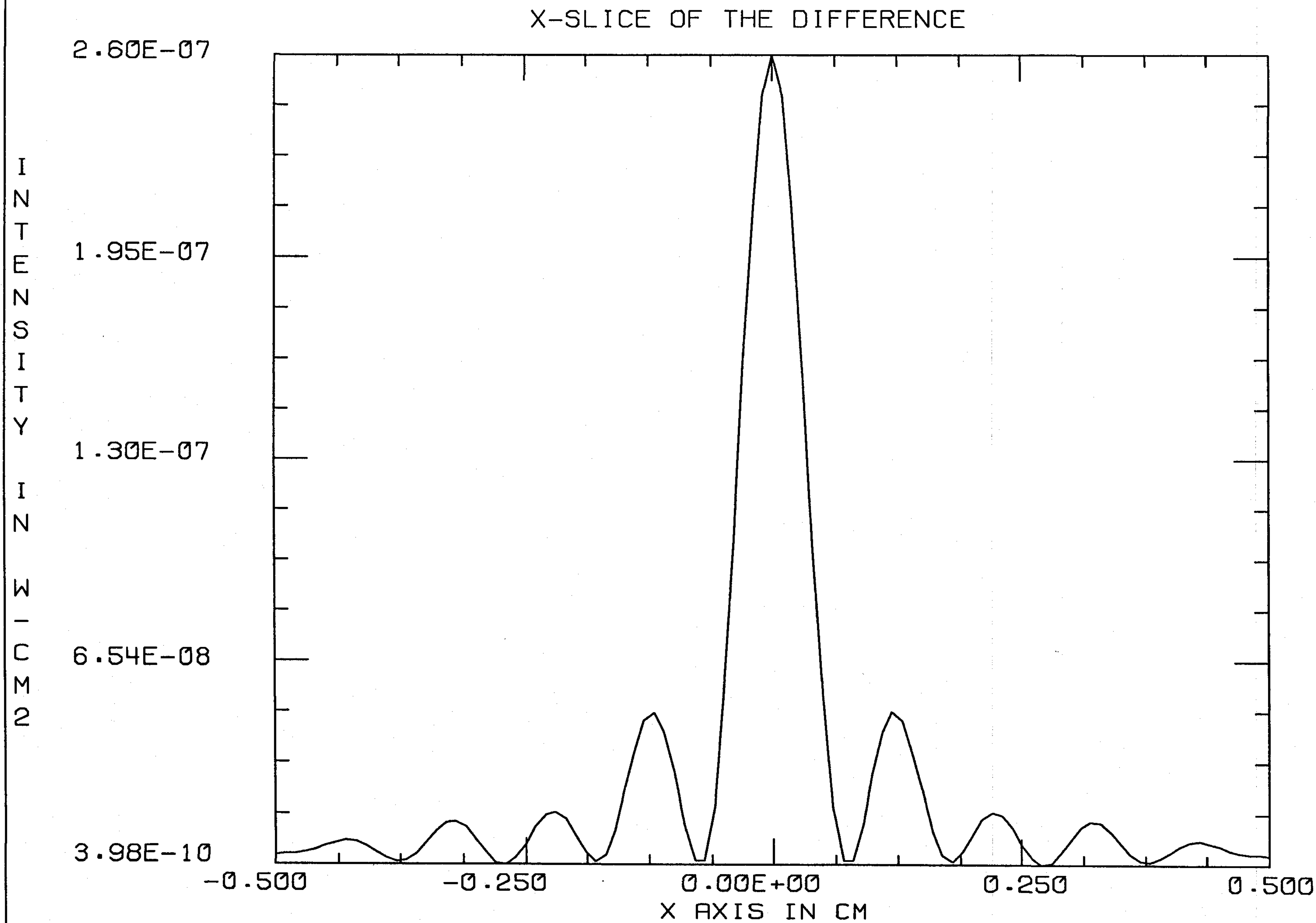
BEAM NO. 1

WAVELEN = 0.515 MIC





PLOT 4, FRI SEP 21 16:51:46 1990



WAVELENGTH 0.515 MIC
BEAM NUMBER- 1
Y COOR 0. CM
PLOT 5, FRI SEP 21 16:51:48 1990

```
C  The simulation of 40 meter cavity with correct input light frequency
C  and the correct mode shape.  This is a comparison between what goes
C  in and what comes out.  This one tries the paraxial coordinate system with
C  the energy ratios as the convergence criterion. Array dimensions are
C  64 by 64.
C
C  Define the iteration macro:
C
macro/def cavity/overwrite
C
   register/add/int 1 1                    # Increment pass counter
C
   prop 4000.0                             # Propagate 40 m. forward
C
   clap/cir/con 1 1.0                      # 1.0 cm. radius aperture
C
   mirror/sph/con 1 -6100.0                # Mirror of 61 m. radius
C
   prop 4000.0                             # Propagate 40 m. backwards
C
   clap/cir/con 1 1.0                      # 1.0 cm. radius aperture
C
   mirror/sph/con 1 1.0e15                 # Flat mirror
C
   phase/piston 1 108.14816805             # Add the correct Guoy phase for
                                           # TEM00 mode (mode frequency
                                           # selection).
C
   add/coh/con 1 2                         # Add beam 2 to beam 1 coherently
                                           # (interference with the incoming beam)
C
   mult/scalar 1 .25000000                 # Normalize it by multiplying
                                           # the intensity by .25000000.
C
   copy/con 1 2                            # Copy the resultant beam to
                                           # the accumulator beam
C
   register/set/param  1 1 energy/real     # Put the energy of the Beam 1 in
                                           # real register number 1.
C
   register/div/real   1 %r1 %r2           # Calculate the ratio of the current
                                           # and the previous energies.
C
   register/sub/real 1 1.0                 # Subtract 1.0 from it.
C
C  If its is negative, change its sign:
C
   if %r1 < 0.0 register/mul/real 1 %r1 -1.0
C
   if %r1 <= 0.000001 macro/exit           # Exit from macro on convergence
C
   register/set/param 2 1 energy/real      # Otherwise, put the current energy
                                           # in real register number 2.
C
macro/end                                  # End of macro definition.
C
nbeam 2                                    # Define two beams; beam 2 is the
                                           # accumulating beam.
C
array/set 1 64 64 0                        # Set the array dimensions for beam 1
array/set 2 64 64 0                        # Set the array dimensions for beam 2
C
color 1 0.5145                             # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                             # Set the mean wavelength for beam 2 (microns)
```

```
C
units/set 1 0.01 0.01                   # The distance between the array elements
                                        # for beam 1 (cm)
units/set 2 0.01 0.01                   # The distance between the array elements
                                        # for beam 2 (cm)
C
units/field 1 1.27 1.27                 # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27                 # Set the field radius for beam 2 (cm)
C
gaussian/cir/con 1 1.0 .217865054304    # Make the initial beam the
                                        # correct mode (beam 1)
gaussian/cir/con 2 1.0 .217865054304    # Make the initial beam the
                                        # correct mode (beam 2)
C
zbound/set 1 .217865054304              # Set the waist size for beam 1 (cm)
                                        # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304              # Set the waist size for beam 2 (cm)
                                        # (ideal, defines the Rayleigh range)
C
register/set/param 2 1 energy/real      # Put the energy of the beam 1
                                        # in real register number 2.
C
C  Plot the initial mode shape (beam 1):
C
title
initial mode shape
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  initial_mode.7
plot/isometric first=1 last=1
C
C  Ready to start. Initialize variables:
C
beams/all/off                           # Turn all beams off (no propagation)
C
beams/on 1                              # Beam 1 propagates, beam 2 accumulates
C
status/p                                # Print the status
C
register/set/int 1 0                    # Macro pass integer register
C
C  Start running the macro:
C
mac/run cavity/50                       # Run the cavity macro for 50
                                        # round trips (maximum)
C
C  Either 50 trips have taken place or the mode is converged:
C
C
C  Plot the converged mode shape at the flat mirror:
C
title
converged mode shape at flat
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat.7
plot/isometric first=1 last=1
C
copy/con 1 2                            # Copy converged beam to beam 2.
C
C
C  Propagate beam forward for 40 m to the other mirror:
C
prop 4000.0                             # Propagate 40 m.
```

```
C
clap/cir/con 1 1.0                      # 1.0 cm. radius aperture
C
mirror/sph 1 -6100.0                    # Mirror of 61 m. radius
C
C  Plot the mode shape at the other mirror:
C
title
converged mode shape at curved
set/density 64 64
set/window/abs -1.28 1.28 -1.28 1.28
plot/disk final_curved.7
plot/isometric first=1 last=1
C
C  Propagate the beam 40 m backwards to the flat mirror:
C
prop 4000.0
C
clap/cir/con      1 1.0                 # 1.0 cm. radius aperture
C
mirror/sph    1 1.0e15                  # Flat mirror
C
phase/piston 1 108.14816805             # Add the correct Guoy phase for
                                        # TEM00 mode (mode frequency
                                        # selection).
C
add/coh/con   1 2                       # Add beam 2 to beam 1 coherently
                                        # (interference with the incoming beam)
C
mult/scalar   1 .25000000               # Normalize it by multiplying
                                        # the intensity by .25000000.
C
gaussian/cir/con 2 1.0 .217865054304    # Make beam 2 the correct mode
C
mult/mode/orthogonal  1 2               # Compute the orthogonal component to the
                                        # perfect mode in the resulting mode.
C
C  Plot the residual intensity:
C
title
Difference between input and converged beams
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat_diff.7
plot/isometric first=1 last=1
C
C  Plot the x-slice of the residual intensity:
title
x-slice of the difference
plot/disk final_fl_df_x.7
plot/xslice/intensity 1 0.0 -0.5 0.5
C
C  This is it:
C
end
```

```
end of macro commands

Expanding number of beams.

Maximum beam number =    2

Generic name (  ): beam

 Beam Array Dimensions and Disk Files
                                                              File Names
    Beam No.      Nlinxs       Nlinys   Nlinxs*Nlinys      P 1          P 2
       1            64           64         4096            (  )s01
Generic name (  ): beam

 Beam Array Dimensions and Disk Files
                                                              File Names
    Beam No.      Nlinxs       Nlinys   Nlinxs*Nlinys      P 1          P 2
       2            64           64         4096            (  )s02
 Beam No.  Vac. wavel. (mic.)     Eff. wavel. (mic),        Index
   1                 0.514500                0.514500    1.000000
 Beam No.  Vac. wavel. (mic.)     Eff. wavel. (mic),        Index
   2                 0.514500                0.514500    1.000000
 beam no.  unitsx  unitsy     fieldx  fieldy
   1      1.000E-02 1.000E-02 0.310     0.310
          CHG UNIT  CHG UNIT

 beam no.  unitsx  unitsy     fieldx  fieldy
   2      1.000E-02 1.000E-02 0.310     0.310
          CHG UNIT  CHG UNIT

 beam no.  unitsx  unitsy     fieldx  fieldy
   1      4.097E-02 4.097E-02  1.27      1.27
          CHG UNIT  CHG UNIT

 beam no.  unitsx  unitsy     fieldx  fieldy
   2      4.097E-02 4.097E-02  1.27      1.27
          CHG UNIT  CHG UNIT

              circle   gaussian fluence distribution
 beam no.  I (w/cm2) R0 x       R0 y      Sgxp  Sgyp    Dec x      Dec y
     1     1.00      0.218     0.218       1.0   1.0      0.          0.

 Beam No.  Unitsx   Unitsy     Fieldx  Fieldy
   1      4.097E-02 4.097E-02  1.27      1.27
              circle   gaussian fluence distribution
 beam no.  I (w/cm2) R0 x       R0 y      Sgxp  Sgyp    Dec x      Dec y
     2     1.00      0.218     0.218       1.0   1.0      0.          0.

 Beam No.  Unitsx   Unitsy     Fieldx  Fieldy
   2      4.097E-02 4.097E-02  1.27      1.27

              Gaussian parameters
no   waistx         waisty         zwastx         zwasty

 1    2.17865E-01   2.17865E-01   0.00000E+00   0.00000E+00

                 constant unit boundaries
     x direction                         y  direction
no left              right           left            right
 1 -2.89828E+03    2.89828E+03     -2.89828E+03    2.89828E+03


              Gaussian parameters
no   waistx         waisty         zwastx         zwasty
```

```
  2    2.17865E-01    2.17865E-01    0.00000E+00    0.00000E+00

                    constant unit boundaries
       x direction                           y  direction
 no left               right              left           right
  2 -2.89828E+03    2.89828E+03       -2.89828E+03    2.89828E+03

  energy     = 7.455814E-02 real register no.  2 =  7.455814E-02
PLOT  1, Fri Sep 21 17:02:06 1990

            coord. beam center (cm)  beam radius (cm)     field size (cm)
 beam no.      x           y          x          y          x          y
     1         0.          0.       0.218      0.218      1.27       1.27
     2         0.          0.       0.218      0.218      1.27       1.27

 beam no.    zreff    direction     zwaistx       zwaisty       iplanx iplany
    1         0.      positive        0.            0.           1      1
    2         0.      positive        0.            0.           1      1

 integer register no.  1 set to          0

  integer register no.  1 changed to         1

    chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)  Beam Radius (cm)   Field Radius (cm)
  no.   Xg        Yg        Zg         Xb        Yb         Xb         Yb
  1     0.        0.     4.000E+03 0.371     0.371       2.43       2.43
                circular aperture
                 Beam Half Widths     Coord. of Apt. Center
 Beam No.  X           Y           X          Y     Units X    Units Y
  1    0.371       0.371          0.         0.     7.849E-02  7.849E-02

   mirror  with curvature radii = -6100.     in x  -6100.     in y

 beam n. Tilt x  Tilt y       Radx      Rady       Unitsx      Unitsy
   1       0.         0.      6100.     6100.      7.8492E-02 7.8492E-02

    chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)  Beam Radius (cm)   Field Radius (cm)
  no.   Xg        Yg        Zg         Xb        Yb         Xb         Yb
  1     0.        0.     8.000E+03 0.218     0.218       1.27       1.27
                circular aperture
                 Beam Half Widths     Coord. of Apt. Center
 Beam No.  X           Y           X          Y     Units X    Units Y
  1    0.218       0.218          0.         0.     4.097E-02  4.097E-02

  mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

 beam n. Tilt x  Tilt y       Radx      Rady       Unitsx      Unitsy
   1       0.         0.    -5.0000E+14-5.0000E+14 4.0968E-02 4.0968E-02

 accumulating into beam no.    1 unitsx      unitsy
                               4.097E-02    4.097E-02
 beam no.  2 accumulated
  beam no.  Fac
     1     0.25000

  energy     = 7.455812E-02 real register no.  1 =  7.455812E-02
  Real register no.  1 changed to   1.00000

  real register no.  1 changed to -1.788139E-07
```

```
 real register no.  1 changed to  1.788139E-07

PLOT  2, Fri Sep 21 17:02:20 1990
     chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
  no.    Xg        Yg        Zg       Xb       Yb        Xb       Yb
   1     0.        0.     1.200E+04 0.371    0.371     2.43     2.43
                circular aperture
                 Beam Half Widths      Coord. of Apt. Center
 Beam No.   X          Y          X          Y      Units X    Units Y
   1    0.371      0.371        0.         0.      7.849E-02  7.849E-02

  mirror  with curvature radii = -6100.    in x  -6100.     in y

 beam n. Tilt x  Tilt y      Radx      Rady       Unitsx      Unitsy
    1     0.        0.      6100.     6100.     7.8492E-02 7.8492E-02

PLOT  3, Fri Sep 21 17:02:31 1990
     chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
  no.    Xg        Yg        Zg       Xb       Yb        Xb       Yb
   1     0.        0.     1.600E+04 0.218    0.218     1.27     1.27
                circular aperture
                 Beam Half Widths      Coord. of Apt. Center
 Beam No.   X          Y          X          Y      Units X    Units Y
   1    0.218      0.218        0.         0.      4.097E-02  4.097E-02

  mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

 beam n. Tilt x  Tilt y      Radx      Rady       Unitsx      Unitsy
    1     0.        0.    -5.0000E+14-5.0000E+14 4.0968E-02 4.0968E-02

 accumulating into beam no.   1 unitsx      unitsy
                              4.097E-02   4.097E-02
 beam no.  2 accumulated
  beam no.  Fac
     1    0.25000

                circle   gaussian fluence distribution
  beam no.  I (w/cm2) R0 x      R0 y       Sgxp  Sgyp   Dec x      Dec y
      2     1.00     0.218     0.218       1.0   1.0      0.         0.

  Beam No.  Unitsx   Unitsy    Fieldx  Fieldy
     2     4.097E-02 4.097E-02  1.27     1.27
   Kbeam    Mbeam
      1        2
 correlation, real-imaginary                1.000000E+00       -1.660381E-07

 correlation, magnitude-angle(deg) =        1.000000E+00      0.0000

PLOT  4, Fri Sep 21 17:02:43 1990
PLOT  5, Fri Sep 21 17:02:45 1990
Fri Sep 21 17:02:45 1990
```

DIFFERENCE BETWEEN INPUT AND CONVERGED BEAMS

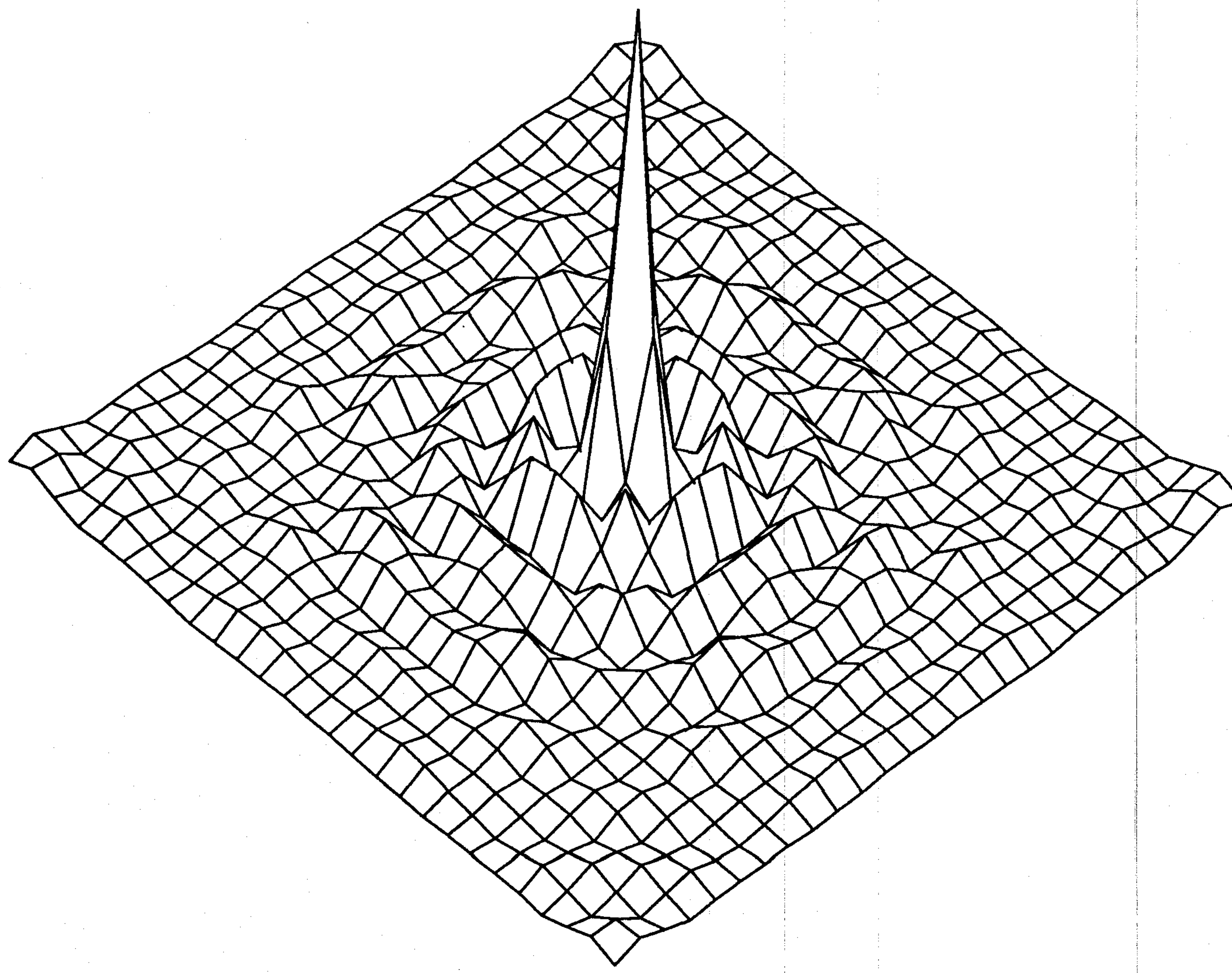
PLOT LIMITS
(X AND Y IN CM)

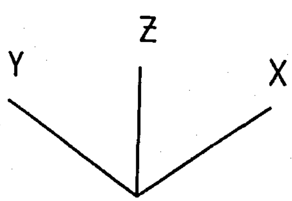
	MIN	MAX
X	-0.533	0.533
Y	-0.533	0.533
Z	1.48E-11	2.94E-07

INTENSITY

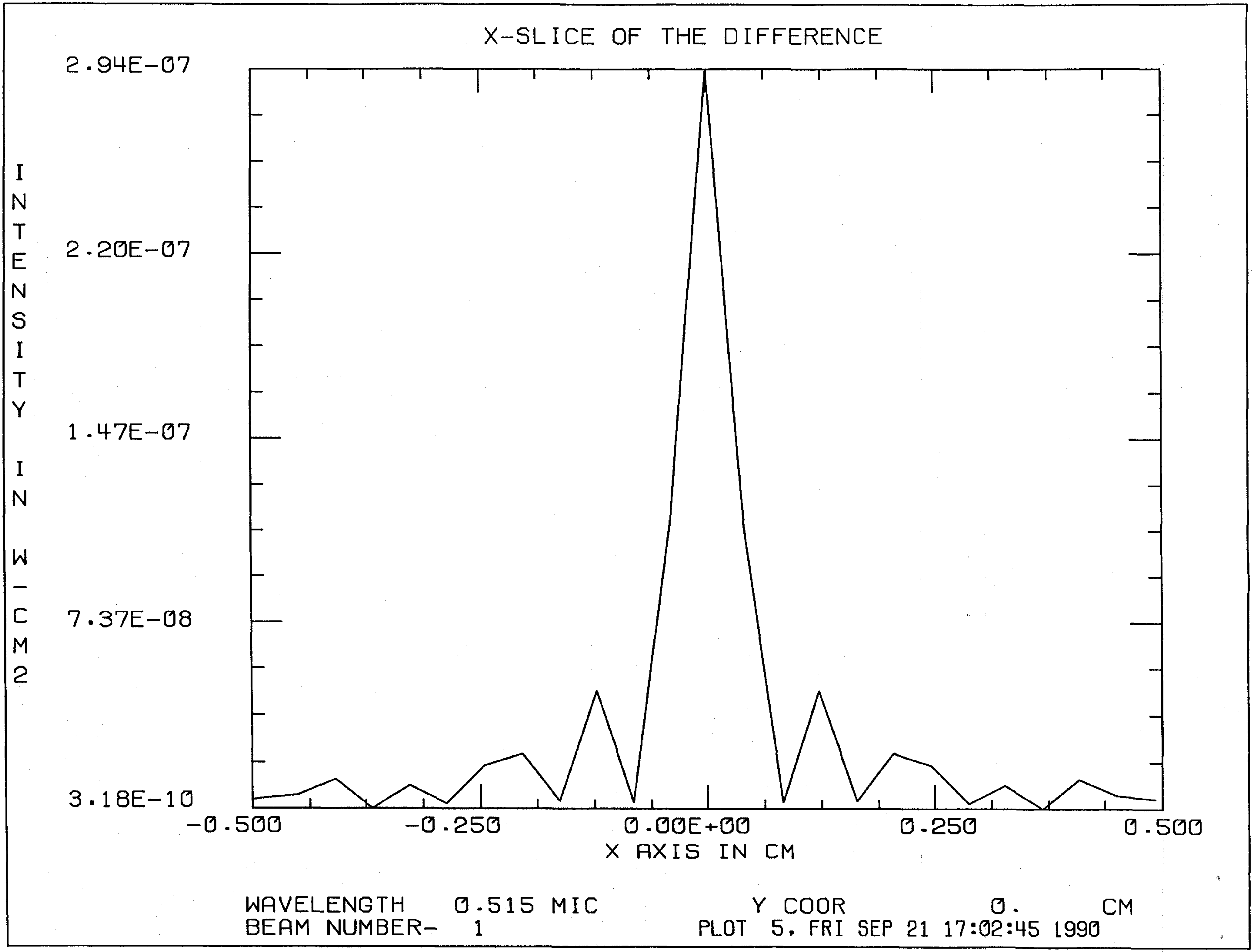
BEAM NO. 1

WAVELEN = 0.515 MIC





PLOT 4, FRI SEP 21 17:02:43 1990


X-SLICE OF THE DIFFERENCE
2.94E-07
2.20E-07
1.47E-07
7.37E-08
3.18E-10
INTENSITY IN W-CM2
-0.500
-0.250
0.00E+00
0.250
0.500
X AXIS IN CM
WAVELENGTH 0.515 MIC
Y COOR 0. CM
BEAM NUMBER- 1
PLOT 5, FRI SEP 21 17:02:45 1990

```
C  The simulation of 40 meter cavity with correct input light frequency
C  and the correct mode shape.  This is a comparison between what goes
C  in and what comes out.  This one tries the paraxial coordinate system with
C  the energy ratios as the convergence criterion. Array dimensions are
C  64 by 64.  This does not exit on convergence, to track the
C  numerical errors.
C
C  Define the iteration macro:
C
macro/def cavity/overwrite
C
   register/add/int 1 1                     # Increment pass counter
C
   prop 4000.0                              # Propagate 40 m. forward
C
   clap/cir/con 1 1.0                       # 1.0 cm. radius aperture
C
   mirror/sph/con 1 -6100.0                 # Mirror of 61 m. radius
C
   prop 4000.0                              # Propagate 40 m. backwards
C
   clap/cir/con 1 1.0                       # 1.0 cm. radius aperture
C
   mirror/sph/con 1 1.0e15                  # Flat mirror
C
   phase/piston 1 108.14816805              # Add the correct Guoy phase for
                                            # TEM00 mode (mode frequency
                                            # selection).
C
   add/coh/con 1 2                          # Add beam 2 to beam 1 coherently
                                            # (interference with the incoming beam)
C
   mult/scalar 1 .25000000                  # Normalize it by multiplying
                                            # the intensity by .25000000.
C
   copy/con 1 2                             # Copy the resultant beam to
                                            # the accumulator beam
C
   register/set/param  1 1 energy/real      # Put the energy of the Beam 1 in
                                            # real register number 1.
C
   register/div/real   1 %r1 %r2            # Calculate the ratio of the current
                                            # and the previous energies.
C
   register/sub/real 1 1.0                  # Subtract 1.0 from it.
C
C  If its is negative, change its sign:
C
   if %r1 < 0.0 register/mul/real 1 %r1 -1.0
C
   register/set/param 2 1 energy/real       # Put the current energy
                                            # in real register number 2.
C
macro/end                                   # End of macro definition.
C
nbeam 2                                     # Define two beams; beam 2 is the
                                            # accumulating beam.
C
array/set 1 64 64 0                         # Set the array dimensions for beam 1
array/set 2 64 64 0                         # Set the array dimensions for beam 2
C
color 1 0.5145                              # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                              # Set the mean wavelength for beam 2 (microns)
C
```

```
units/set 1 0.01 0.01                  # The distance between the array elements
                                       # for beam 1 (cm)
units/set 2 0.01 0.01                  # The distance between the array elements
                                       # for beam 2 (cm)
C
units/field 1 1.27 1.27                # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27                # Set the field radius for beam 2 (cm)
C
gaussian/cir/con 1 1.0 .217865054304   # Make the initial beam the
                                       # correct mode (beam 1)
gaussian/cir/con 2 1.0 .217865054304   # Make the initial beam the
                                       # correct mode (beam 2)
C
zbound/set 1 .217865054304             # Set the waist size for beam 1 (cm)
                                       # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304             # Set the waist size for beam 2 (cm)
                                       # (ideal, defines the Rayleigh range)
C
register/set/param 2 1 energy/real     # Put the energy of the beam 1
                                       # in real register number 2.
C
C  Plot the initial mode shape (beam 1):
C
title
initial mode shape
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  initial_mode.8
plot/isometric first=1 last=1
C
C  Ready to start. Initialize variables:
C
beams/all/off                          # Turn all beams off (no propagation)
C
beams/on 1                             # Beam 1 propagates, beam 2 accumulates
C
status/p                               # Print the status
C
register/set/int 1 0                   # Macro pass integer register
C
C  Start running the macro:
C
mac/run cavity/100                     # Run the cavity macro for 100
                                       # round trips (maximum)
C
C  100 trips have taken place.
C
C
C  Plot the converged mode shape at the flat mirror:
C
title
converged mode shape at flat
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat.8
plot/isometric first=1 last=1
C
copy/con 1 2                           # Copy converged beam to beam 2.
C
C
C  Propagate beam forward for 40 m to the other mirror:
C
prop 4000.0                            # Propagate 40 m.
C
```

```
clap/cir/con 1 1.0                      # 1.0 cm. radius aperture
C
mirror/sph 1 -6100.0                    # Mirror of 61 m. radius
C
C  Plot the mode shape at the other mirror:
C
title
converged mode shape at curved
set/density 64 64
set/window/abs -1.28 1.28 -1.28 1.28
plot/disk final_curved.8
plot/isometric first=1 last=1
C
C  Propagate the beam 40 m backwards to the flat mirror:
C
prop 4000.0
C
clap/cir/con      1 1.0                 # 1.0 cm. radius aperture
C
mirror/sph    1 1.0e15                  # Flat mirror
C
phase/piston 1 108.14816805             # Add the correct Guoy phase for
                                        # TEM00 mode (mode frequency
                                        # selection).
C
add/coh/con   1 2                       # Add beam 2 to beam 1 coherently
                                        # (interference with the incoming beam)
C
mult/scalar   1 .25000000               # Normalize it by multiplying
                                        # the intensity by .25000000.
C
gaussian/cir/con 2 1.0 .217865054304    # Make beam 2 the correct mode
C
mult/mode/orthogonal  1 2               # Compute the orthogonal component to the
                                        # perfect mode in the resulting mode.
C
C  Plot the residual intensity:
C
title
Difference between input and converged beams
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat_diff.8
plot/isometric first=1 last=1
C
C  Plot the x-slice of the residual intensity:
title
x-slice of the difference
plot/disk final_fl_df_x.8
plot/xslice/intensity 1 0.0 -0.5 0.5
C
C  This is it:
C
end
```

```
 energy    = 7.454822E-02 real register no.  1 =  7.454822E-02
 Real register no.  1 changed to  0.999999

 real register no.  1 changed to -1.013279E-06

 real register no.  1 changed to  1.013279E-06

 energy    = 7.454822E-02 real register no.  2 =  7.454822E-02
PLOT  2, Fri Sep 21 18:22:52 1990
     chief  Ray Propagation of    4000.     (cm).

Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg         Yg         Zg         Xb         Yb        Xb         Yb
  1     0.         0.      8.040E+05 0.371      0.371      2.43       2.43
                circular aperture
                 Beam Half Widths     Coord. of Apt. Center
Beam No.   X          Y          X          Y      Units X    Units Y
  1    0.371      0.371        0.         0.       7.849E-02  7.849E-02

  mirror  with curvature radii = -6100.     in x  -6100.     in y

beam n. Tilt x  Tilt y       Radx       Rady       Unitsx     Unitsy
   1      0.         0.      6100.      6100.      7.8492E-02 7.8492E-02

PLOT  3, Fri Sep 21 18:23:06 1990
     chief  Ray Propagation of    4000.     (cm).

Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg         Yg         Zg         Xb         Yb        Xb         Yb
  1     0.         0.      8.080E+05 0.218      0.218      1.27       1.27
                circular aperture
                 Beam Half Widths     Coord. of Apt. Center
Beam No.   X          Y          X          Y      Units X    Units Y
  1    0.218      0.218        0.         0.       4.097E-02  4.097E-02

  mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

beam n. Tilt x  Tilt y       Radx       Rady       Unitsx     Unitsy
   1      0.         0.    -5.0000E+14-5.0000E+14 4.0968E-02 4.0968E-02

accumulating into beam no.   1 unitsx      unitsy
                             4.097E-02   4.097E-02

beam no.   2 accumulated
 beam no.   Fac
     1    0.25000

                circle   gaussian fluence distribution
 beam no.  I (w/cm2) R0 x       R0 y       Sgxp  Sgyp   Dec x      Dec y
      2    1.00      0.218      0.218       1.0   1.0     0.         0.

 Beam No.  Unitsx    Unitsy     Fieldx   Fieldy
    2     4.097E-02 4.097E-02  1.27      1.27
  Kbeam   Mbeam
     1       2
correlation, real-imaginary                9.999990E-01       -1.150573E-05

correlation, magnitude-angle(deg) =        9.999990E-01     -0.0007

PLOT  4, Fri Sep 21 18:23:19 1990
PLOT  5, Fri Sep 21 18:23:21 1990
Fri Sep 21 18:23:21 1990
```

DIFFERENCE BETWEEN INPUT AND CONVERGED BEAMS

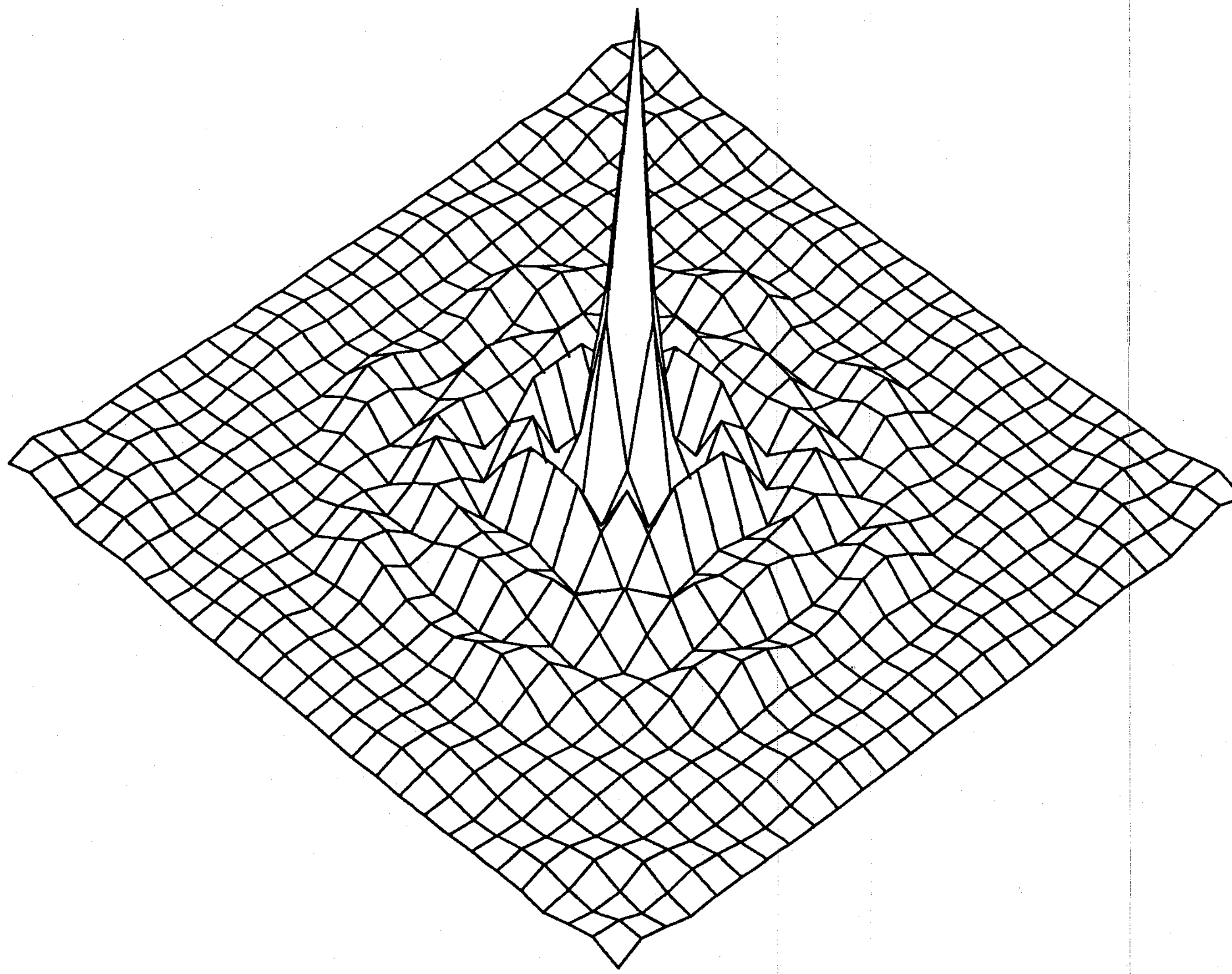
PLOT LIMITS
(X AND Y IN CM)

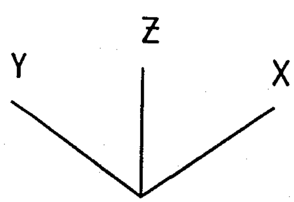
	MIN	MAX
X	-0.533	0.533
Y	-0.533	0.533
Z	9.14E-10	9.28E-07

INTENSITY

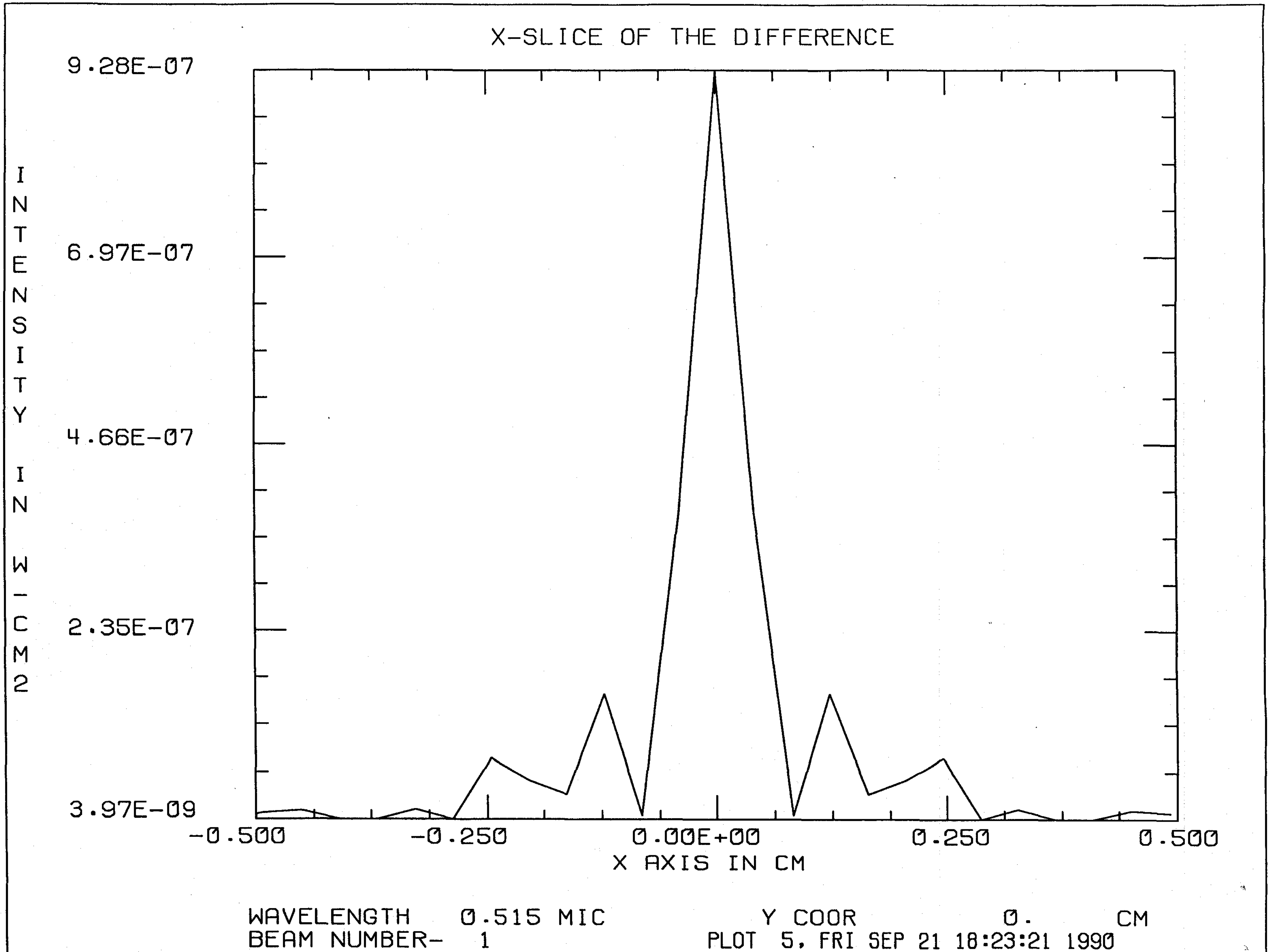
BEAM NO. 1

WAVELEN = 0.515 MIC





PLOT 4, FRI SEP 21 18:23:19 1990



X-SLICE OF THE DIFFERENCE
9.28E-07
6.97E-07
4.66E-07
2.35E-07
3.97E-09
INTENSITY IN W-CM2
-0.500
-0.250
0.00E+00
0.250
0.500
X AXIS IN CM
WAVELENGTH 0.515 MIC
Y COOR 0. CM
BEAM NUMBER- 1
PLOT 5, FRI SEP 21 18:23:21 1990

Appendix_IV_a


```
C  The simulation of 40 meter cavity with correct input light frequency
C  and flat (incorrect) mode  shape.  This is a comparison between the ideal
C  mode and what comes out.  This one tries the paraxial coordinate system with
C  the energy ratios as the convergence criterion (1e-4).  The units are auto
C  adjusting and the array size is 64 by 64.
C
C  Define the iteration macro:
C
macro/def cavity/overwrite
C
   register/add/int 1 1                   # Increment pass counter
C
   prop 4000.0                            # Propagate 40 m. forward
C
   clap/cir          1 1.0                # 1.0 cm. radius aperture
C
   mirror/sph        1 -6100.0            # Mirror of 61 m. radius
C
   prop 4000.0                            # Propagate 40 m. backwards
C
   clap/cir          1 1.0                # 1.0 cm. radius aperture
C
   mirror/sph        1 1.0e15             # Flat mirror
C
   phase/piston 1 108.14816805            # Add the correct Guoy phase for
                                          # TEM00 mode (mode frequency
                                          # selection).
C
   add/coh/con       1 2                  # Add beam 2 to beam 1 coherently
                                          # (interference with the incoming beam)
C
   mult/scalar       1 .25000000          # Normalize it by multiplying
                                          # the intensity by .25000000.
C
   copy/con          1 2                  # Copy the resultant beam to
                                          # the accumulator beam
C
   register/set/param  1 1 energy/real    # Put the energy of the Beam 1 in
                                          # real register number 1.
C
   register/div/real   1 %r1 %r2          # Calculate the ratio of the current
                                          # and the previous energies.
C
   register/sub/real 1 1.0                # Subtract 1.0 from it.
C
C  If it is negative, change its sign:
C
   if %r1 < 0.0 register/mul/real 1 %r1 -1.0
C
   if %r1 <= 0.0001 macro/exit            # Exit from macro on convergence
C
   register/set/param 2 1 energy/real     # Otherwise, put the current energy
                                          # in real register number 2.
C
macro/end                                 # End of macro definition.
C
nbeam 2                                   # Define two beams; beam 2 is the
                                          # accumulating beam.
C
array/set 1 64  64  0                     # Set the array dimensions for beam 1
array/set 2 64  64  0                     # Set the array dimensions for beam 2
C
color 1 0.5145                            # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                            # Set the mean wavelength for beam 2 (microns)
```

```
C
units/set 1 0.04 0.04                   # The distance between the array elements
                                        # for beam 1 (cm)
units/set 2 0.04 0.04                   # The distance between the array elements
                                        # for beam 2 (cm)
C
units/field 1 1.27 1.27                 # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27                 # Set the field radius for beam 2 (cm)
C
clear 1 1                               # Make initial beam a plane
                                        # wave (beam 1)
clear 2 1                               # Make initial beam a plane
                                        # wave (beam 2)
C
zbound/set 1 .217865054304              # Set the waist size for beam 1 (cm)
                                        # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304              # Set the waist size for beam 2 (cm)
                                        # (ideal, defines the Rayleigh range)
C
register/set/param 2 1 energy/real      # Put the energy of the beam 1
                                        # in real register number 2.
C
C  Plot the initial mode shape (beam 1):
C
title
initial mode shape (9)
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  initial_mode.9
plot/isometric first=1 last=1
C
C  Ready to start. Initialize variables:
C
beams/all/off                           # Turn all beams off (no propagation)
C
beams/on 1                              # Beam 1 propagates, beam 2 accumulates
C
status/p                                # Print the status
C
register/set/int 1 0                    # Macro pass integer register
C
C  Start running the macro:
C
mac/run cavity/50                       # Run the cavity macro for 50
                                        # round trips (maximum)
C
C  Either 50 trips have taken place or the mode is converged:
C
C
C  Plot the converged mode shape at the flat mirror:
C
title
converged mode shape at flat (9)
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat.9
plot/isometric first=1 last=1
C
C
C  Propagate beam forward for 40 m to the other mirror:
C
prop 4000.0                             # Propagate 40 m.
C
clap/cir    1 1.0                       # 1.0 cm. radius aperture
```

```
C
mirror/sph 1 -6100.0                          # Mirror of 61 m. radius
C
C  Plot the mode shape at the other mirror:
C
title
converged mode shape at curved (9)
set/density 64 64
set/window/abs -1.28 1.28 -1.28 1.28
plot/disk final_curved.9
plot/isometric first=1 last=1
C
C  Propagate the beam 40 m backwards to the flat mirror:
C
prop 4000.0
C
clap/cir      1 1.0                          # 1.0 cm. radius aperture
C
mirror/sph    1 1.0e15                       # Flat mirror
C
phase/piston 1 108.14816805                  # Add the correct Guoy phase for
                                             # TEM00 mode (mode frequency
                                             # selection).
C
add/coh/con   1 2                            # Add beam 2 to beam 1 coherently
                                             # (interference with the incoming beam)
C
mult/scalar   1 .25000000                    # Normalize it by multiplying
                                             # the intensity by .25000000.
C
gaussian/cir/con 2 1.0 .217865054304         # Make beam 2 the correct mode
C
mult/mode/orthogonal  1 2                    # Compute the orthogonal component to the
                                             # perfect mode in the resulting mode.
C
C  Plot the intensity of the orthogonal component:
C
title
Diff. between perfect and converged beams (9)
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat_diff.9
plot/isometric first=1 last=1
C
C  Plot the x-slice of the orthogonal intensity:
title
x-slice of the difference (9)
plot/disk final_fl_df_x.9
plot/xslice/intensity 1 0.0 -0.5 0.5
C
C  This is it:
C
end
```

```
 integer register no.  1 changed to       21

    chief  Ray Propagation of    4000.     (cm).

Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg         Yg         Zg        Xb        Yb        Xb        Yb
 1      0.         0.     1.640E+05 0.371     0.371      2.43      2.43
             circular aperture
              Beam Half Widths      Coord. of Apt. Center
Beam No.   X           Y           X          Y      Units X    Units Y
 1     0.371       0.371         0.         0.      7.849E-02  7.849E-02

  mirror  with curvature radii = -6100.    in x  -6100.    in y

beam n. Tilt x  Tilt y       Radx       Rady       Unitsx     Unitsy
  1       0.         0.     6100.      6100.     7.8492E-02 7.8492E-02

    chief  Ray Propagation of    4000.     (cm).

Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg         Yg         Zg        Xb        Yb        Xb        Yb
 1      0.         0.     1.680E+05 0.218     0.218      1.27      1.27
             circular aperture
              Beam Half Widths      Coord. of Apt. Center
Beam No.   X           Y           X          Y      Units X    Units Y
 1     0.218       0.218         0.         0.      4.097E-02  4.097E-02

  mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

beam n. Tilt x  Tilt y       Radx       Rady       Unitsx     Unitsy
  1       0.         0.    -5.0000E+14-5.0000E+14 4.0968E-02 4.0968E-02

accumulating into beam no.    1 unitsx      unitsy
                              4.097E-02   4.097E-02
beam no.  2 accumulated
 beam no.  Fac
    1     0.25000

 energy    = 0.299062     real register no.  1 =  0.299062
 Real register no.  1 changed to  0.999915

 real register no.  1 changed to -8.517504E-05

 real register no.  1 changed to  8.517504E-05

PLOT  2, Fri Sep 21 18:44:01 1990
    chief  Ray Propagation of    4000.     (cm).

Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg         Yg         Zg        Xb        Yb        Xb        Yb
 1      0.         0.     1.720E+05 0.371     0.371      2.43      2.43
             circular aperture
              Beam Half Widths      Coord. of Apt. Center
Beam No.   X           Y           X          Y      Units X    Units Y
 1     0.371       0.371         0.         0.      7.849E-02  7.849E-02

  mirror  with curvature radii = -6100.    in x  -6100.    in y

beam n. Tilt x  Tilt y       Radx       Rady       Unitsx     Unitsy
  1       0.         0.     6100.      6100.     7.8492E-02 7.8492E-02

PLOT  3, Fri Sep 21 18:44:10 1990
    chief  Ray Propagation of    4000.     (cm).
```

Appendix_IV_b


```
Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg        Yg        Zg        Xb         Yb        Xb        Yb
  1     0.        0.    1.760E+05  0.218      0.218      1.27      1.27
               circular aperture
                 Beam Half Widths      Coord. of Apt. Center
Beam No.   X          Y          X          Y     Units X      Units Y
  1      0.218      0.218        0.         0.    4.097E-02   4.097E-02

  mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

beam n. Tilt x  Tilt y      Radx       Rady       Unitsx      Unitsy
  1       0.       0.    -5.0000E+14-5.0000E+14 4.0968E-02 4.0968E-02

accumulating into beam no.   1 unitsx      unitsy
                             4.097E-02   4.097E-02
beam no.   2 accumulated
 beam no.   Fac
    1      0.25000

              circle    gaussian fluence distribution
beam no.  I (w/cm2) R0 x       R0 y       Sgxp  Sgyp    Dec x      Dec y
    2     1.00      0.218      0.218       1.0   1.0      0.         0.

Beam No.  Unitsx    Unitsy     Fieldx   Fieldy
  2      4.097E-02 4.097E-02  1.27      1.27
 Kbeam   Mbeam
   1       2
correlation, real-imaginary                  9.999498E-01     -6.226447E-04

correlation, magnitude-angle(deg) =          9.999499E-01     -0.0357

PLOT  4, Fri Sep 21 18:44:21 1990
PLOT  5, Fri Sep 21 18:44:22 1990
Fri Sep 21 18:44:22 1990
```

DIFF. BETWEEN PERFECT AND CONVERGED BEAMS (9)

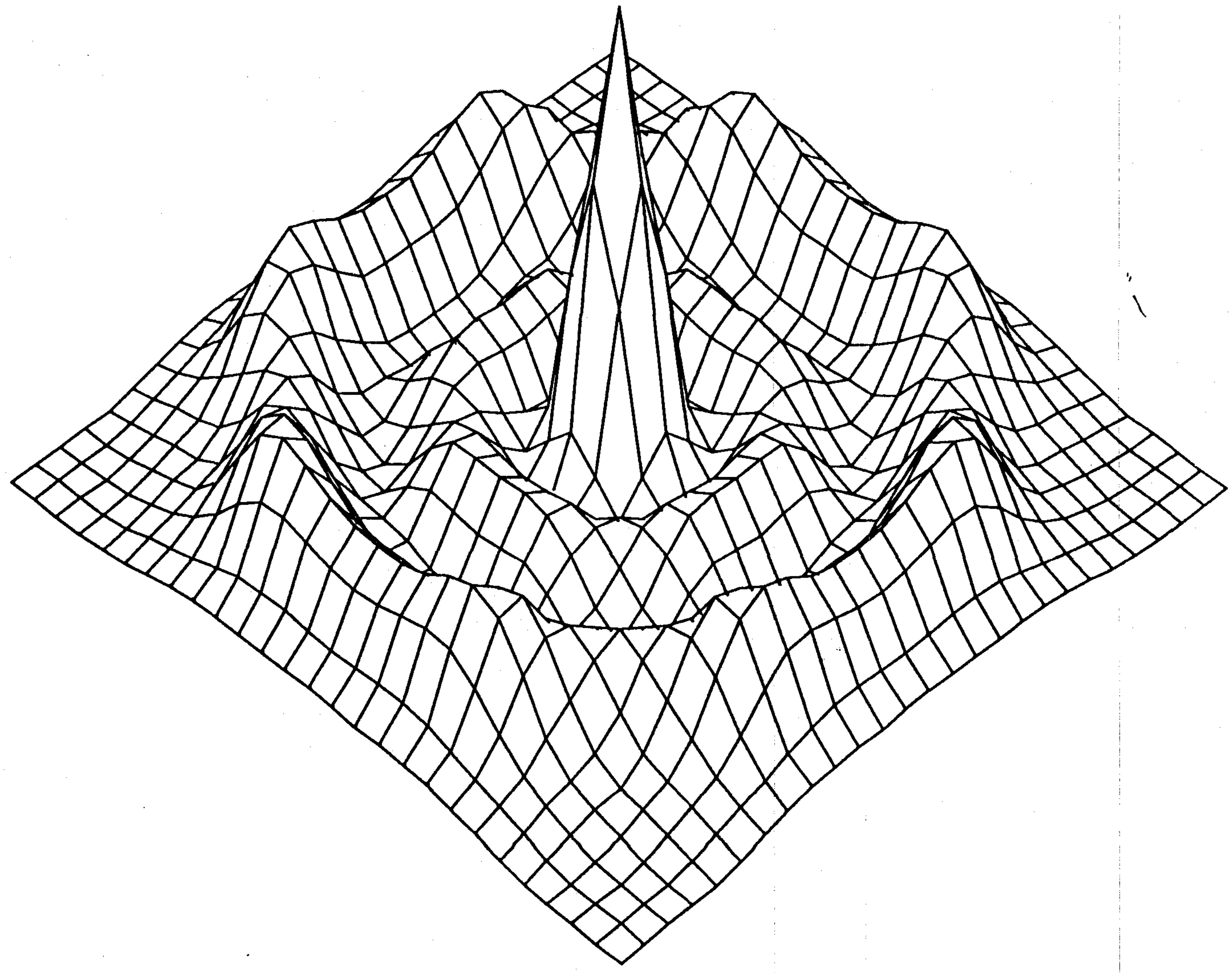
PLOT LIMITS
(X AND Y IN CM)

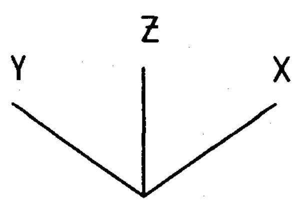
	MIN	MAX
X	-0.533	0.533
Y	-0.533	0.533
Z	8.24E-08	1.76E-04

INTENSITY

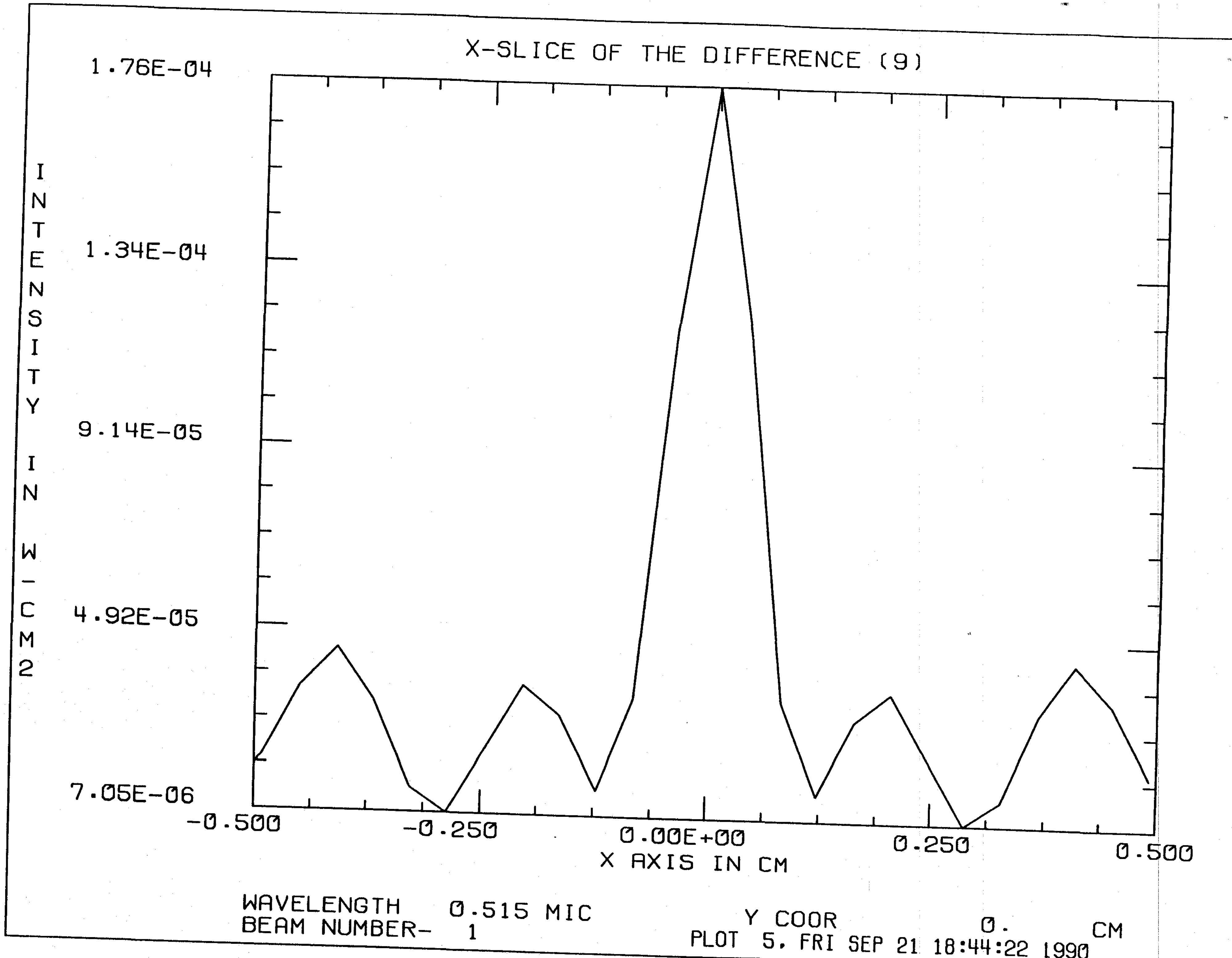
BEAM NO. 1

WAVELEN = 0.515 MIC





PLOT 4, FRI SEP 21 18:44:21 1990



X-SLICE OF THE DIFFERENCE (9)
INTENSITY IN W-CM2
1.76E-04
1.34E-04
9.14E-05
4.92E-05
7.05E-06
-0.500
-0.250
0.00E+00
0.250
0.500
X AXIS IN CM
WAVELENGTH 0.515 MIC
BEAM NUMBER- 1
Y COOR 0. CM
PLOT 5, FRI SEP 21 18:44:22 1990

```
  energy    = 0.298984     real register no.  1 =  0.298984
  Real register no.  1 changed to  0.999999

  real register no.  1 changed to -1.490116E-06

  real register no.  1 changed to  1.490116E-06

  energy    = 0.298984     real register no.  2 =  0.298984
PLOT  2, Fri Sep 21 19:09:47 1990
     chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)    Beam Radius (cm)    Field Radius (cm)
  no.    Xg         Yg         Zg         Xb         Yb         Xb         Yb
   1     0.         0.     8.040E+05 0.371      0.371       2.43       2.43
                circular aperture
                 Beam Half Widths      Coord. of Apt. Center
 Beam No.   X          Y          X          Y     Units X    Units Y
   1    0.371      0.371       0.         0.       7.849E-02  7.849E-02

   mirror  with curvature radii = -6100.     in x  -6100.     in y

 beam n. Tilt x  Tilt y       Radx       Rady       Unitsx     Unitsy
    1      0.         0.     6100.      6100.      7.8492E-02 7.8492E-02

PLOT  3, Fri Sep 21 19:09:56 1990
     chief  Ray Propagation of    4000.     (cm).

 Beam Beam Center Coordinates (cm)    Beam Radius (cm)    Field Radius (cm)
  no.    Xg         Yg         Zg         Xb         Yb         Xb         Yb
   1     0.         0.     8.080E+05 0.218      0.218       1.27       1.27
                circular aperture
                 Beam Half Widths      Coord. of Apt. Center
 Beam No.   X          Y          X          Y     Units X    Units Y
   1    0.218      0.218       0.         0.       4.097E-02  4.097E-02

   mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

 beam n. Tilt x  Tilt y       Radx       Rady       Unitsx     Unitsy
    1      0.         0.   -5.0000E+14-5.0000E+14 4.0968E-02 4.0968E-02

 accumulating into beam no.    1 unitsx      unitsy
                               4.097E-02   4.097E-02
 beam no.   2 accumulated
  beam no.   Fac
      1     0.25000

                circle   gaussian fluence distribution
  beam no.  I (w/cm2) R0 x       R0 y       Sgxp  Sgyp   Dec x      Dec y
       2    1.00       0.218      0.218      1.0   1.0     0.         0.

  Beam No.  Unitsx    Unitsy     Fieldx  Fieldy
     2     4.097E-02 4.097E-02  1.27      1.27
   Kbeam    Mbeam
      1       2
 correlation, real-imaginary                   9.999976E-01        -6.329788E-04

 correlation, magnitude-angle(deg) =           9.999977E-01     -0.0363

PLOT  4, Fri Sep 21 19:10:09 1990
PLOT  5, Fri Sep 21 19:10:11 1990
Fri Sep 21 19:10:11 1990
```

DIFF. BETWEEN PERFECT AND CONVERGED BEAMS (10)

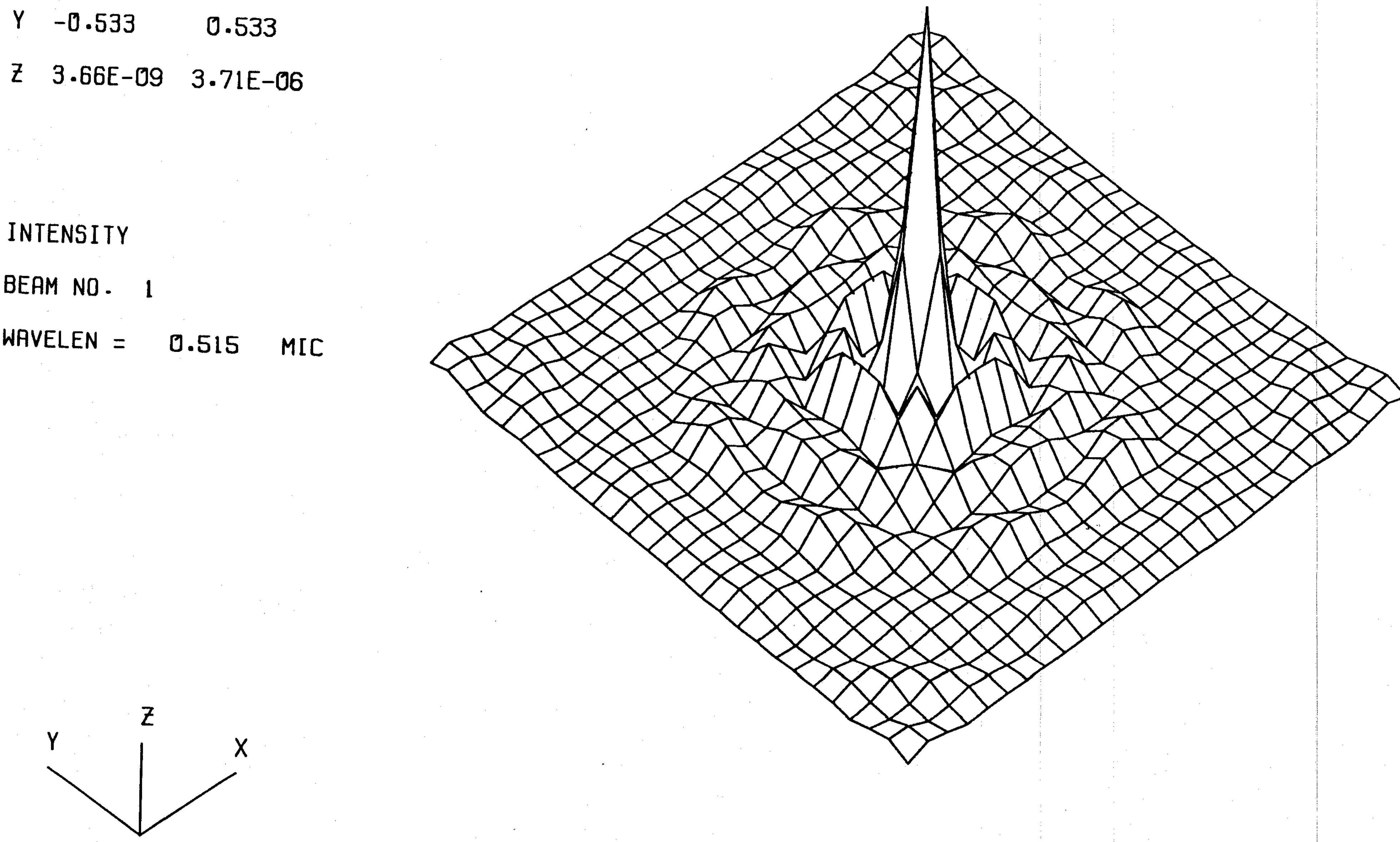
PLOT LIMITS
(X AND Y IN CM)

	MIN	MAX
X	-0.533	0.533
Y	-0.533	0.533
Z	3.66E-09	3.71E-06

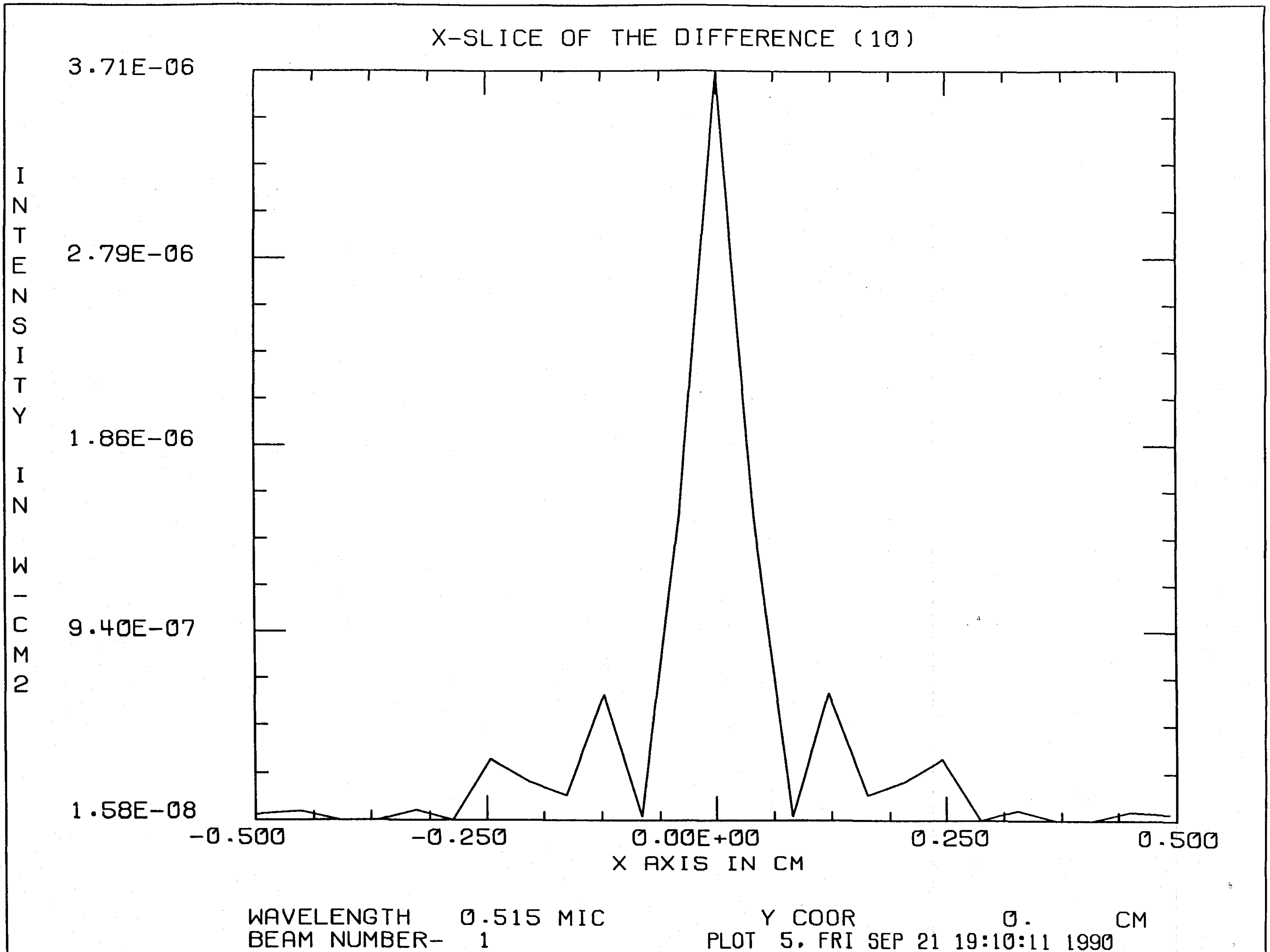
INTENSITY

BEAM NO. 1

WAVELEN = 0.515 MIC



PLOT 4, FRI SEP 21 19:10:09 1990


X-SLICE OF THE DIFFERENCE (10)
3.71E-06
2.79E-06
1.86E-06
9.40E-07
1.58E-08
INTENSITY IN W-CM2
-0.500
-0.250
0.00E+00
0.250
0.500
X AXIS IN CM
WAVELENGTH 0.515 MIC
BEAM NUMBER- 1
Y COOR 0. CM
PLOT 5, FRI SEP 21 19:10:11 1990

Appendix_VI_a


```
energy    = 0.299020      real register no.  1 =  0.299020
Real register no.  1 changed to  0.999982

real register no.  1 changed to -1.811981E-05

real register no.  1 changed to  1.811981E-05

energy    = 0.299020      real register no.  2 =  0.299020
PLOT  2, Fri Sep 21 22:02:21 1990
    chief  Ray Propagation of    4000.     (cm).

Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg         Yg        Zg        Xb        Yb        Xb        Yb
 1      0.         0.      2.040E+05 0.371     0.371      2.43      2.43
                circular aperture
                 Beam Half Widths      Coord. of Apt. Center
Beam No.   X          Y           X          Y      Units X    Units Y
 1     0.371      0.371          0.         0.     7.849E-02  7.849E-02

mirror  with curvature radii = -6100.     in x  -6100.     in y

beam n. Tilt x  Tilt y        Radx      Rady       Unitsx     Unitsy
 1        0.        0.        6100.     6100.     7.8492E-02 7.8492E-02
PLOT  3, Fri Sep 21 22:02:32 1990
    chief  Ray Propagation of    4000.     (cm).

Beam Beam Center Coordinates (cm)   Beam Radius (cm)   Field Radius (cm)
 no.    Xg         Yg        Zg        Xb        Yb        Xb        Yb
 1      0.         0.      2.080E+05 0.218     0.218      1.27      1.27
                circular aperture
                 Beam Half Widths      Coord. of Apt. Center
Beam No.   X          Y           X          Y      Units X    Units Y
 1     0.218      0.218          0.         0.     4.097E-02  4.097E-02

mirror  with curvature radii = 1.0000E+15 in x  1.0000E+15 in y

beam n. Tilt x  Tilt y        Radx      Rady       Unitsx     Unitsy
 1        0.        0.    -5.0000E+14-5.0000E+14 4.0968E-02 4.0968E-02

accumulating into beam no.   1 unitsx     unitsy
                             4.097E-02   4.097E-02
beam no.  2 accumulated
 beam no.  Fac
   1     0.25000


            circle   gaussian fluence distribution
beam no.  I (w/cm2) R0 x      R0 y       Sgxp  Sgyp   Dec x     Dec y
    2     1.00      0.218     0.218      1.0   1.0     0.        0.

Beam No.  Unitsx    Unitsy    Fieldx  Fieldy
  2     4.097E-02 4.097E-02  1.27     1.27
 Kbeam    Mbeam
   1        2
correlation, real-imaginary                  9.999917E-01      -6.247396E-04

correlation, magnitude-angle(deg) =          9.999918E-01     -0.0358

PLOT  4, Fri Sep 21 22:02:42 1990
PLOT  5, Fri Sep 21 22:02:44 1990
Fri Sep 21 22:02:44 1990
```

DIFF. BETWEEN PERFECT AND CONVERGED BEAMS (11)

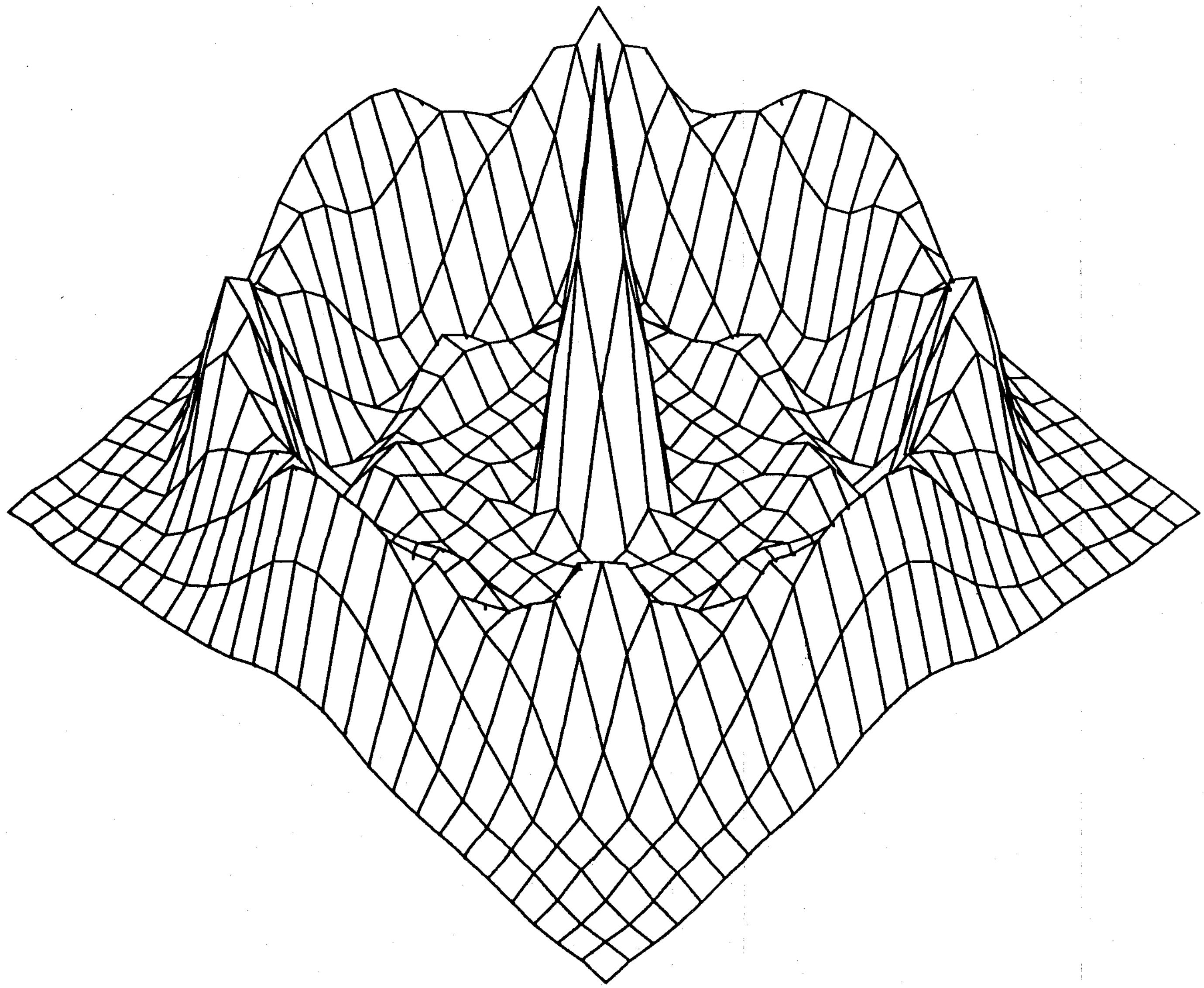
PLOT LIMITS
(X AND Y IN CM)

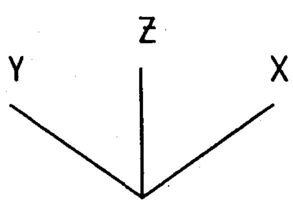
	MIN	MAX
X	-0.533	0.533
Y	-0.533	0.533
Z	5.40E-08	2.51E-05

INTENSITY

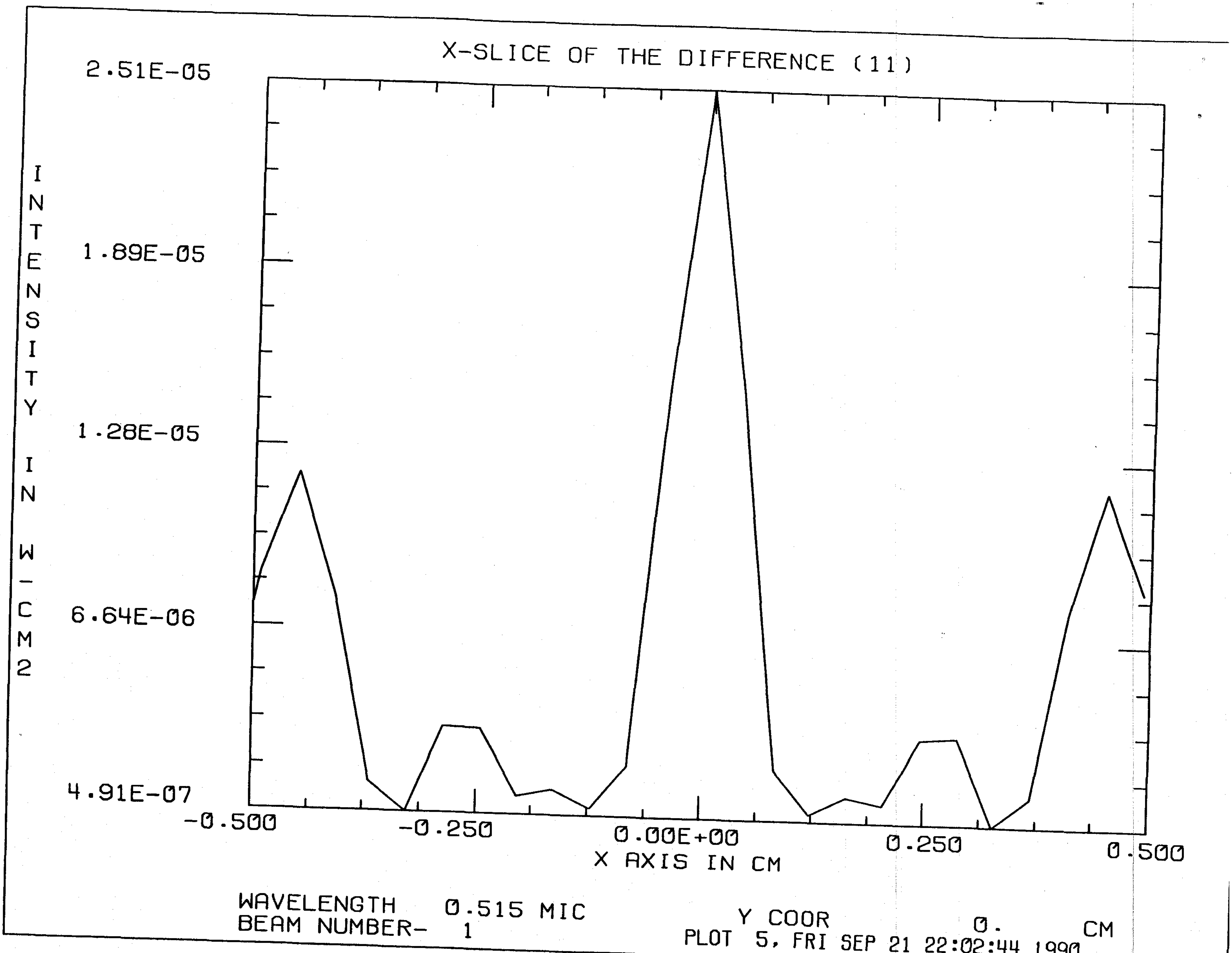
BEAM NO. 1

WAVELEN = 0.515 MIC





PLOT 4, FRI SEP 21 22:02:42 1990


X-SLICE OF THE DIFFERENCE (11)
2.51E-05
1.89E-05
1.28E-05
6.64E-06
4.91E-07
INTENSITY IN W-CM2
-0.500
-0.250
0.00E+00
0.250
0.500
X AXIS IN CM
WAVELENGTH 0.515 MIC
BEAM NUMBER- 1
Y COOR 0. CM
PLOT 5, FRI SEP 21 22:02:44 1990

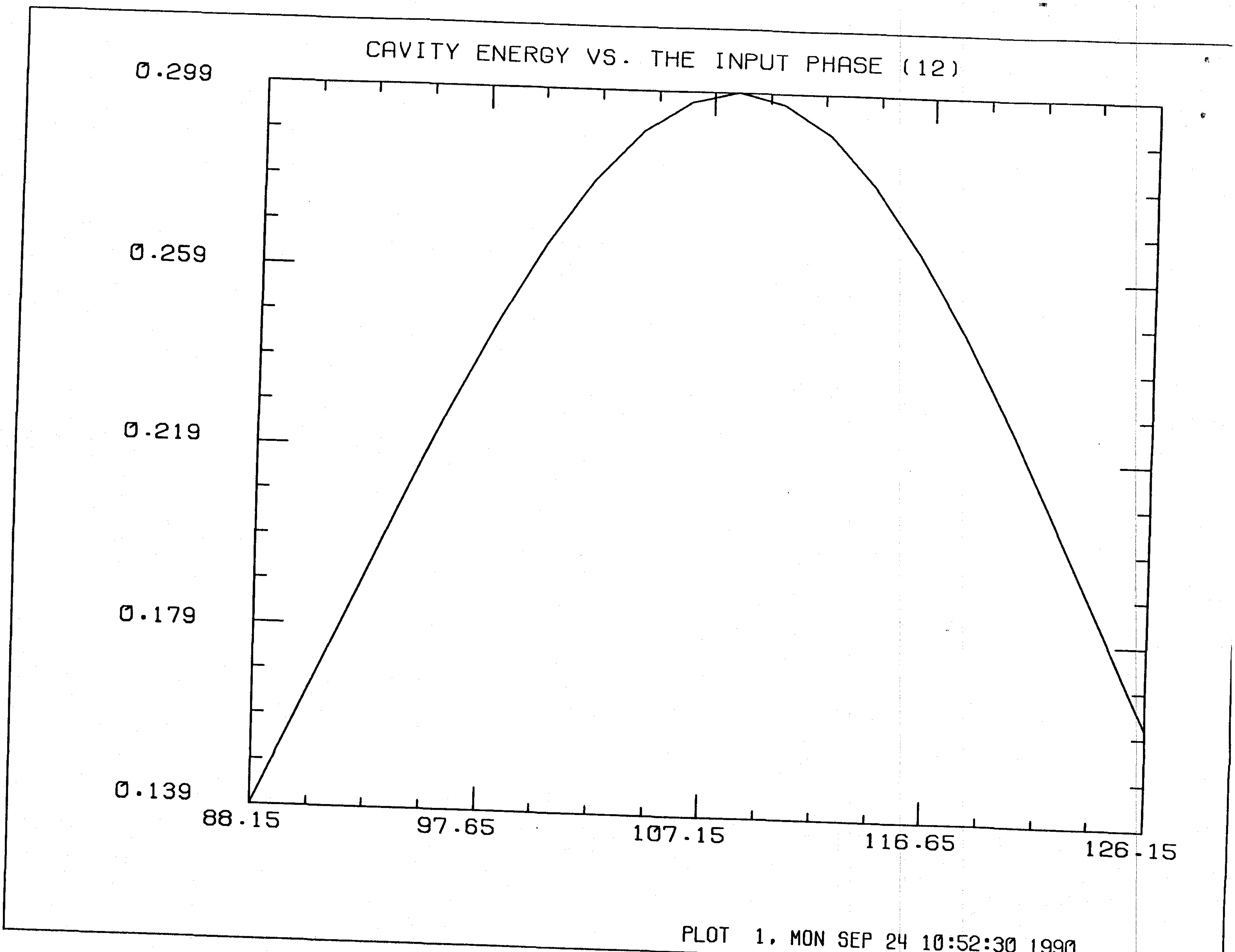
Appendix_VII_a


```
C  The simulation of 40 meter cavity with scanning input light frequency
C  and flat (incorrect) mode  shape.  This one tries paraxial coordinate system.
C  The units are auto adjusting and the array size is 64 by 64.
C
C  Define the cavity iteration macro:
C
macro/def cavity/overwrite
C
   prop 4000.0                            # Propagate 40 m. forward
C
   clap/cir        1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph      1 -6100.0              # Mirror of 61 m. radius
C
   prop 4000.0                            # Propagate 40 m. backwards
C
   clap/cir        1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph      1 1.0e15               # Flat mirror
C
   phase/piston    1 %r2                  # Add the scanning phase (mode
                                          # frequency selection).
C
   add/coh/con     1 2                    # Add beam 2 to beam 1 coherently
                                          # (interference with the incoming beam)
C
   mult/scalar     1 .25000000            # Normalize it by multiplying
                                          # the intensity by .25000000.
C
   copy/con        1 2                    # Copy the resultant beam to
                                          # the accumulator beam
C
macro/end                                 # End of macro definition.
C
C
C  Define the frequency scanning macro:
C
macro/def   scanfreq/overwrite
C
   register/add/int 1 1                   # Increment pass counter
C
   clear 1 1                              # Make initial beam a plane
                                          # wave (beam 1)
   clear 2 1                              # Make initial beam a plane
                                          # wave (beam 2)
C
   macro/run  cavity/25                   # Run the initial waveform
                                          # through the cavity at most
                                          # 25 times.
C
   register/set/param 3 1 energy/real     # Put the energy of the beam 1
                                          # in real register 3.
C
   udata/set %i1 %r2 %r3                  # Save the current phase angle and
                                          # resulting energy in the user storage
                                          # indexed by the pass number.
C
   register/add/real 2 %r1                # Increment the phase angle (real
                                          # register 2) by the contents of
                                          # real register 1.
C
macro/end
C
C
```

```
C  Main program starts:
C
nbeam 2                               # Define two beams; beam 2 is the
                                      # accumulating beam.
C
array/set 1 64  64  0                 # Set the array dimensions for beam 1
array/set 2 64  64  0                 # Set the array dimensions for beam 2
C
color 1 0.5145                        # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                        # Set the mean wavelength for beam 2 (microns)
C
units/set 1 0.04 0.04                 # The distance between the array elements
                                      # for beam 1 (cm)
units/set 2 0.04 0.04                 # The distance between the array elements
                                      # for beam 2 (cm)
C
units/field 1 1.27 1.27               # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27               # Set the field radius for beam 2 (cm)
C
zbound/set 1 .217865054304            # Set the waist size for beam 1 (cm)
                                      # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304            # Set the waist size for beam 2 (cm)
                                      # (ideal, defines the Rayleigh range)
C
C  Ready to start. Initialize variables:
C
beams/all/off                         # Turn all beams off (no propagation)
C
beams/on 1                            # Beam 1 propagates, beam 2 accumulates
C
status/p                              # Print the status
C
register/set/int 1 0                  # Macro pass integer register
C
register/set/real 1 2.0               # Set the step size in scanning
                                      # to 2.0 degrees (real register 1).
C
register/set/real 2 88.14816805       # Set the initial Guoy phase 20
                                      # degrees before real resonance
                                      # (real register 2).
C
C  Start running the scanning macro:
C
mac/run scanfreq/20                   # Run the scanning macro 20
                                      # times for 40 degree coverage
                                      # around the real resonance.
C
C  Scan is complete.
C
C  Plot the energy versus the phase angle:
C
title
Cavity energy vs. the input phase (12)
plot/disk  energy_phase.12
plot/udata first=1 last=1
C
udata/list/disk energy_ph_lst.12      # Make a list of the results
                                      # as well.
C
C  This is it:
C
end
```

Appendix_VII_b

1	8.8148170471E+01	1.3948339224E-01	0.0000000000E+00	0.0000000000E+00
2	9.0148170471E+01	1.6121517122E-01	0.0000000000E+00	0.0000000000E+00
3	9.2148170471E+01	1.8354260921E-01	0.0000000000E+00	0.0000000000E+00
4	9.4148170471E+01	2.0580491424E-01	0.0000000000E+00	0.0000000000E+00
5	9.6148170471E+01	2.2726698220E-01	0.0000000000E+00	0.0000000000E+00
6	9.8148170471E+01	2.4715708196E-01	0.0000000000E+00	0.0000000000E+00
7	1.0014817047E+02	2.6471003890E-01	0.0000000000E+00	0.0000000000E+00
8	1.0214817047E+02	2.7921226621E-01	0.0000000000E+00	0.0000000000E+00
9	1.0414817047E+02	2.9004982114E-01	0.0000000000E+00	0.0000000000E+00
10	1.0614817047E+02	2.9675203562E-01	0.0000000000E+00	0.0000000000E+00
11	1.0814817047E+02	2.9902049899E-01	0.0000000000E+00	0.0000000000E+00
12	1.1014817047E+02	2.9675489664E-01	0.0000000000E+00	0.0000000000E+00
13	1.1214817047E+02	2.9005777836E-01	0.0000000000E+00	0.0000000000E+00
14	1.1414817047E+02	2.7922669053E-01	0.0000000000E+00	0.0000000000E+00
15	1.1614817047E+02	2.6473692060E-01	0.0000000000E+00	0.0000000000E+00
16	1.1814817047E+02	2.4720385671E-01	0.0000000000E+00	0.0000000000E+00
17	1.2014817047E+02	2.2734549642E-01	0.0000000000E+00	0.0000000000E+00
18	1.2214817047E+02	2.0593699813E-01	0.0000000000E+00	0.0000000000E+00
19	1.2414817047E+02	1.8375878036E-01	0.0000000000E+00	0.0000000000E+00
20	1.2614817047E+02	1.6156291962E-01	0.0000000000E+00	0.0000000000E+00


CAVITY ENERGY VS. THE INPUT PHASE (12)
0.299
0.259
0.219
0.179
0.139
88.15
97.65
107.15
116.65
126.15
PLOT 1, MON SEP 24 10:52:30 1990

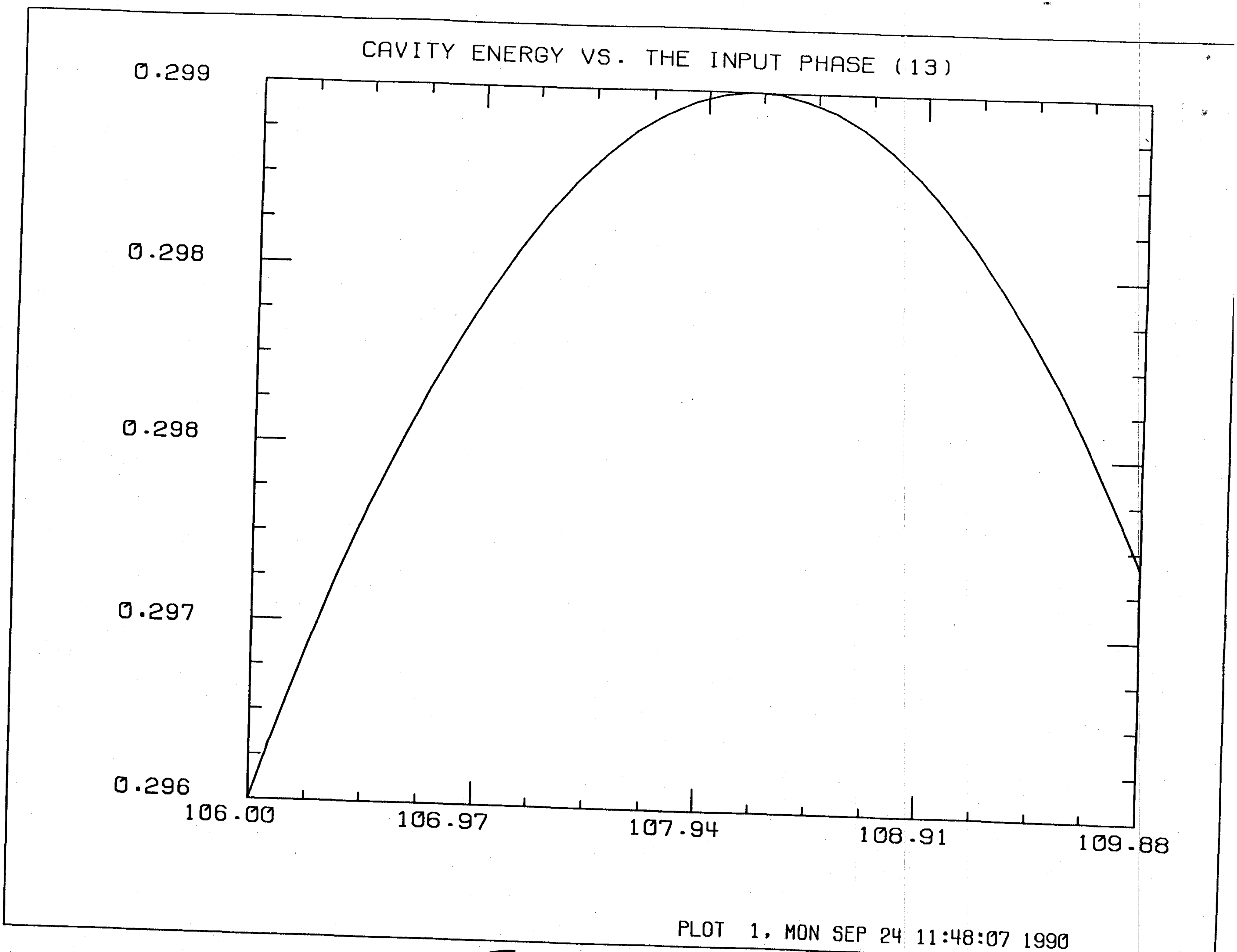
Appendix_VIII_a


```
C  The simulation of 40 meter cavity with scanning input light frequency
C  and flat (incorrect) mode  shape.  This one tries paraxial coordinate system.
C  The units are auto adjusting and the array size is 64 by 64.
C
C  Define the cavity iteration macro:
C
macro/def cavity/overwrite
C
   prop 4000.0                           # Propagate 40 m. forward
C
   clap/cir       1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph     1 -6100.0              # Mirror of 61 m. radius
C
   prop 4000.0                           # Propagate 40 m. backwards
C
   clap/cir       1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph     1 1.0e15               # Flat mirror
C
   phase/piston   1 %r2                  # Add the scanning phase (mode
                                         # frequency selection).
C
   add/coh/con    1 2                    # Add beam 2 to beam 1 coherently
                                         # (interference with the incoming beam)
C
   mult/scalar    1 .25000000            # Normalize it by multiplying
                                         # the intensity by .25000000.
C
   copy/con       1 2                    # Copy the resultant beam to
                                         # the accumulator beam
C
macro/end                                # End of macro definition.
C
C
C  Define the frequency scanning macro:
C
macro/def   scanfreq/overwrite
C
   register/add/int 1 1                  # Increment pass counter
C
   clear 1 1                             # Make initial beam a plane
                                         # wave (beam 1)
   clear 2 1                             # Make initial beam a plane
                                         # wave (beam 2)
C
   macro/run  cavity/25                  # Run the initial waveform
                                         # through the cavity at most
                                         # 25 times.
C
   register/set/param 3 1 energy/real    # Put the energy of the beam 1
                                         # in real register 3.
C
   udata/set %i1 %r2 %r3                 # Save the current phase angle and
                                         # resulting energy in the user storage
                                         # indexed by the pass number.
C
   register/add/real 2 %r1               # Increment the phase angle (real
                                         # register 2) by the contents of
                                         # real register 1.
C
macro/end
C
C
```

```
C  Main program starts:
C
nbeam 2                                  # Define two beams; beam 2 is the
                                         # accumulating beam.
C
array/set 1 64  64  0                    # Set the array dimensions for beam 1
array/set 2 64  64  0                    # Set the array dimensions for beam 2
C
color 1 0.5145                           # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                           # Set the mean wavelength for beam 2 (microns)
C
units/set 1 0.04 0.04                    # The distance between the array elements
                                         # for beam 1 (cm)
units/set 2 0.04 0.04                    # The distance between the array elements
                                         # for beam 2 (cm)
C
units/field 1 1.27 1.27                  # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27                  # Set the field radius for beam 2 (cm)
C
zbound/set 1 .217865054304               # Set the waist size for beam 1 (cm)
                                         # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304               # Set the waist size for beam 2 (cm)
                                         # (ideal, defines the Rayleigh range)
C
C  Ready to start. Initialize variables:
C
beams/all/off                            # Turn all beams off (no propagation)
C
beams/on 1                               # Beam 1 propagates, beam 2 accumulates
C
status/p                                 # Print the status
C
register/set/int 1 0                     # Macro pass integer register
C
register/set/real 1 0.125                # Set the step size in scanning
                                         # to 0.125 degrees (real register 1).
C
register/set/real 2 106.0000000          # Set the initial Guoy phase some
                                         # degrees before real resonance
                                         # (real register 2).
C
C  Start running the scanning macro:
C
mac/run scanfreq/32                      # Run the scanning macro 32
                                         # times for 4 degree coverage
                                         # around the real resonance.
C
C  Scan is complete.
C
C  Plot the energy versus the phase angle:
C
title
Cavity energy vs. the input phase (13)
plot/disk  energy_phase.13
plot/udata first=1 last=1
C
udata/list/disk energy_ph_lst.13         # Make a list of the results
                                         # as well.
C
C  This is it:
C
end
```

Appendix_VIII_b

1	1.0600000000E+02	2.9640492797E-01	0.0000000000E+00	0.0000000000E+00
2	1.0612500000E+02	2.9669916630E-01	0.0000000000E+00	0.0000000000E+00
3	1.0625000000E+02	2.9697617888E-01	0.0000000000E+00	0.0000000000E+00
4	1.0637500000E+02	2.9723551869E-01	0.0000000000E+00	0.0000000000E+00
5	1.0650000000E+02	2.9747790098E-01	0.0000000000E+00	0.0000000000E+00
6	1.0662500000E+02	2.9770284891E-01	0.0000000000E+00	0.0000000000E+00
7	1.0675000000E+02	2.9790991545E-01	0.0000000000E+00	0.0000000000E+00
8	1.0687500000E+02	2.9809868336E-01	0.0000000000E+00	0.0000000000E+00
9	1.0700000000E+02	2.9827088118E-01	0.0000000000E+00	0.0000000000E+00
10	1.0712500000E+02	2.9842546582E-01	0.0000000000E+00	0.0000000000E+00
11	1.0725000000E+02	2.9856151342E-01	0.0000000000E+00	0.0000000000E+00
12	1.0737500000E+02	2.9868018627E-01	0.0000000000E+00	0.0000000000E+00
13	1.0750000000E+02	2.9878118634E-01	0.0000000000E+00	0.0000000000E+00
14	1.0762500000E+02	2.9886460304E-01	0.0000000000E+00	0.0000000000E+00
15	1.0775000000E+02	2.9893031716E-01	0.0000000000E+00	0.0000000000E+00
16	1.0787500000E+02	2.9897838831E-01	0.0000000000E+00	0.0000000000E+00
17	1.0800000000E+02	2.9900783300E-01	0.0000000000E+00	0.0000000000E+00
18	1.0812500000E+02	2.9902070761E-01	0.0000000000E+00	0.0000000000E+00
19	1.0825000000E+02	2.9901516438E-01	0.0000000000E+00	0.0000000000E+00
20	1.0837500000E+02	2.9899153113E-01	0.0000000000E+00	0.0000000000E+00
21	1.0850000000E+02	2.9895079136E-01	0.0000000000E+00	0.0000000000E+00
22	1.0862500000E+02	2.9889151454E-01	0.0000000000E+00	0.0000000000E+00
23	1.0875000000E+02	2.9881480336E-01	0.0000000000E+00	0.0000000000E+00
24	1.0887500000E+02	2.9872098565E-01	0.0000000000E+00	0.0000000000E+00
25	1.0900000000E+02	2.9860931635E-01	0.0000000000E+00	0.0000000000E+00
26	1.0912500000E+02	2.9847884178E-01	0.0000000000E+00	0.0000000000E+00
27	1.0925000000E+02	2.9833182693E-01	0.0000000000E+00	0.0000000000E+00
28	1.0937500000E+02	2.9816654325E-01	0.0000000000E+00	0.0000000000E+00
29	1.0950000000E+02	2.9798430204E-01	0.0000000000E+00	0.0000000000E+00
30	1.0962500000E+02	2.9778346419E-01	0.0000000000E+00	0.0000000000E+00
31	1.0975000000E+02	2.9756554961E-01	0.0000000000E+00	0.0000000000E+00
32	1.0987500000E+02	2.9732999206E-01	0.0000000000E+00	0.0000000000E+00



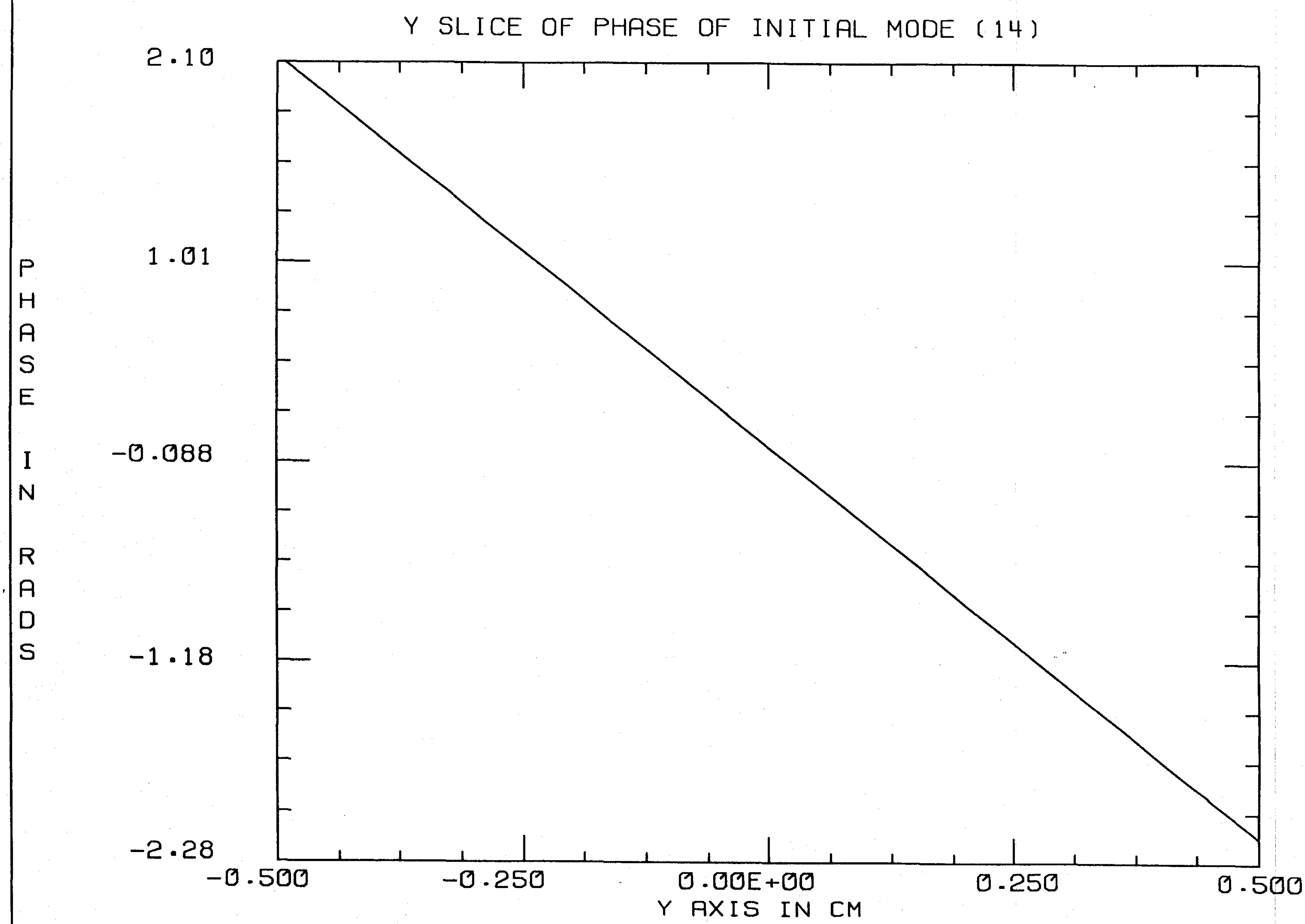
CAVITY ENERGY VS. THE INPUT PHASE (13)
0.299
0.298
0.298
0.297
0.296
106.00
106.97
107.94
108.91
109.88
PLOT 1, MON SEP 24 11:48:07 1990

```
C  The simulation of 40 meter cavity with correct input light frequency
C  and correct but tilted  mode  shape.  This is a comparison between the input
C  mode and what comes out.  This one tries the paraxial coordinate system with
C  the energy ratios as the convergence criterion (1e-4).  The units are auto
C  adjusting and the array size is 64 by 64.
C
C  Define the iteration macro:
C
macro/def cavity/overwrite
C
   register/add/int 1 1                  # Increment pass counter
C
   copy/con       1 2                    # Copy the resultant beam to
                                         # the accumulator beam
C
   prop 4000.0                           # Propagate 40 m. forward
C
   clap/cir       1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph     1 -6100.0              # Mirror of 61 m. radius
C
   prop 4000.0                           # Propagate 40 m. backwards
C
   clap/cir       1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph     1 1.0e15               # Flat mirror
C
   phase/piston 1 108.14816805           # Add the correct Guoy phase for
                                         # TEM00 mode (mode frequency
                                         # selection).
C
   add/coh/con    1 2                    # Add beam 2 to beam 1 coherently
                                         # (interference with the incoming beam)
C
   mult/scalar    1 .25000000            # Normalize it by multiplying
                                         # the intensity by .25000000.
C
   register/set/param  1 1 energy/real   # Put the energy of the Beam 1 in
                                         # real register number 1.
C
   register/div/real   1 %r1 %r2         # Calculate the ratio of the current
                                         # and the previous energies.
C
   register/sub/real 1 1.0               # Subtract 1.0 from it.
C
C  If it is negative, change its sign:
C
   if %r1 < 0.0 register/mul/real 1 %r1 -1.0
C
   if %r1 <= 0.0001 macro/exit           # Exit from macro on convergence
C
   register/set/param 2 1 energy/real    # Otherwise, put the current energy
                                         # in real register number 2.
C
macro/end                                # End of macro definition.
C
nbeam 2                                  # Define two beams; beam 2 is the
                                         # accumulating beam.
C
array/set 1 64  64  0                    # Set the array dimensions for beam 1
array/set 2 64  64  0                    # Set the array dimensions for beam 2
C
color 1 0.5145                           # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                           # Set the mean wavelength for beam 2 (microns)
```

```
C
units/set 1 0.04 0.04                          # The distance between the array elements
                                               # for beam 1 (cm)
units/set 2 0.04 0.04                          # The distance between the array elements
                                               # for beam 2 (cm)
C
units/field 1 1.27 1.27                        # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27                        # Set the field radius for beam 2 (cm)
C
gaussian/cir/con 1 1.0 .217865054304           # Make beam 1 the correct mode
C
abr/tilt 1 0.68 0.0 1.0 0.0 0.0 1              # Add 0.002 degree phase tilt (0.68 waves
                                               # at 1 cm radius, beam 1)
C
zbound/set 1 .217865054304                     # Set the waist size for beam 1 (cm)
                                               # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304                     # Set the waist size for beam 2 (cm)
                                               # (ideal, defines the Rayleigh range)
C
register/set/param 2 1 energy/real             # Put the energy of the beam 1
                                               # in real register number 2.
C
C  Plot the initial mode shape (beam 1):
C
title
initial mode shape (14)
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  initial_mode.14
plot/isometric first=1 last=1
C
C  Plot the phase of the initial mode:
C
title
y slice of phase of initial mode (14)
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  initial_phase.14
plot/yslice/phase 1 0.0 -0.5 0.5
C
C  Ready to start. Initialize variables:
C
beams/all/off                                  # Turn all beams off (no propagation)
C
beams/on 1                                     # Beam 1 propagates, beam 2 accumulates
C
status/p                                       # Print the status
C
register/set/int 1 0                           # Macro pass integer register
C
C  Start running the macro:
C
mac/run cavity/50                              # Run the cavity macro for 50
                                               # round trips (maximum)
C
C  Either 50 trips have taken place or the mode is converged:
C
C
C  Plot the converged mode shape at the flat mirror:
C
title
converged mode shape at flat (14)
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
```

```
plot/disk  final_flat.14
plot/isometric first=1 last=1
C
C
C  Propagate beam forward for 40 m to the other mirror:
C
copy/con 1 2                            # Copy the resultant beam
                                        # to the accumulator beam.
C
prop 4000.0                             # Propagate 40 m.
C
clap/cir     1 1.0                      # 1.0 cm. radius aperture
C
mirror/sph 1 -6100.0                    # Mirror of 61 m. radius
C
C  Plot the mode shape at the other mirror:
C
title
converged mode shape at curved (14)
set/density 64 64
set/window/abs -1.28 1.28 -1.28 1.28
plot/disk final_curved.14
plot/isometric first=1 last=1
C
C  Propagate the beam 40 m backwards to the flat mirror:
C
prop 4000.0
C
clap/cir      1 1.0                     # 1.0 cm. radius aperture
C
mirror/sph    1 1.0e15                  # Flat mirror
C
phase/piston 1 108.14816805             # Add the correct Guoy phase for
                                        # TEM00 mode (mode frequency
                                        # selection).
C
add/coh/con   1 2                       # Add beam 2 to beam 1 coherently
                                        # (interference with the incoming beam)
C
mult/scalar   1 .25000000               # Normalize it by multiplying
                                        # the intensity by .25000000.
C
C Reconstruct the input mode in beam 2:
C
gaussian/cir/con 2 1.0 .217865054304    # Make beam 2 the correct mode
C
abr/tilt 2 0.68 0.0 1.0 0.0 0.0 2       # Add 0.002 degree phase tilt (0.68 waves
                                        # at 1 cm radius, beam 2)
C
mult/mode/orthogonal  2 1               # Compute the orthogonal component to the
                                        # resulting mode in the input mode.
C
C  Plot the intensity of the orthogonal component:
C
title
Diff. between input and converged beams (14)
set/density 64 64
set/window/abs -0.5 0.5 -0.5 0.5
plot/disk  final_flat_diff.14
plot/isometric first=2 last=2
C
C  Plot the x-slice of the orthogonal intensity:
title
y-slice of the difference (14)
```

```
plot/disk final_fl_df_y.14
plot/yslice/intensity 2 0.0 -0.5 0.5
C
C  This is it:
C
end
```



WAVELENGTH 0.515 MIC X COOR 0. CM
BEAM NUMBER- 1 PLOT 2, MON SEP 24 19:37:58 1990

PLOT LIMITS
(X AND Y IN CM)

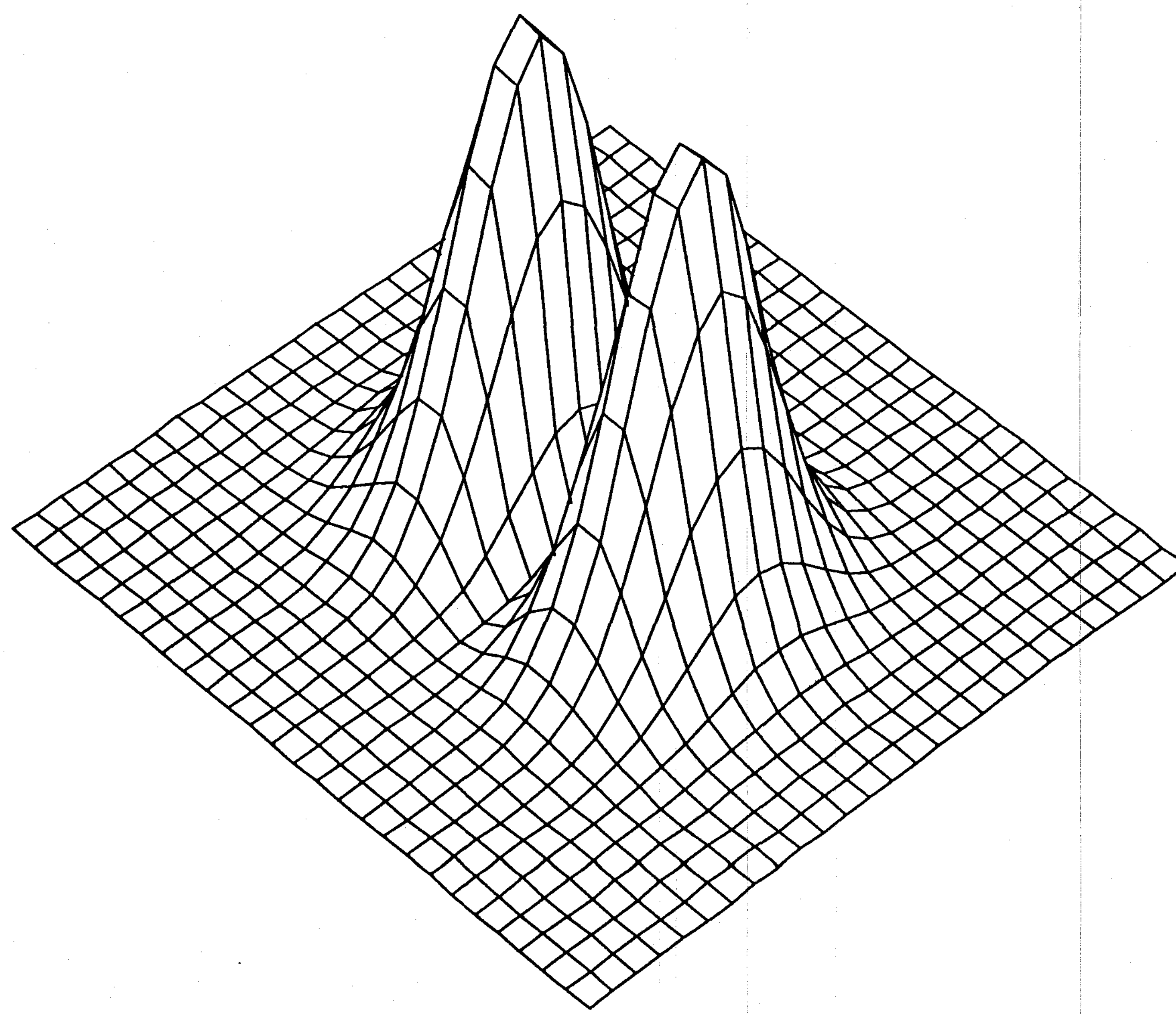
	MIN	MAX
X	-0.533	0.533
Y	-0.533	0.533
Z	1.73E-08	0.144

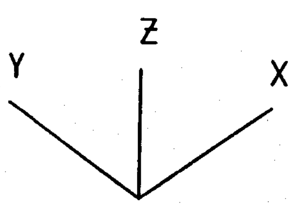
DIFF. BETWEEN INPUT AND CONVERGED BEAMS (14)

INTENSITY

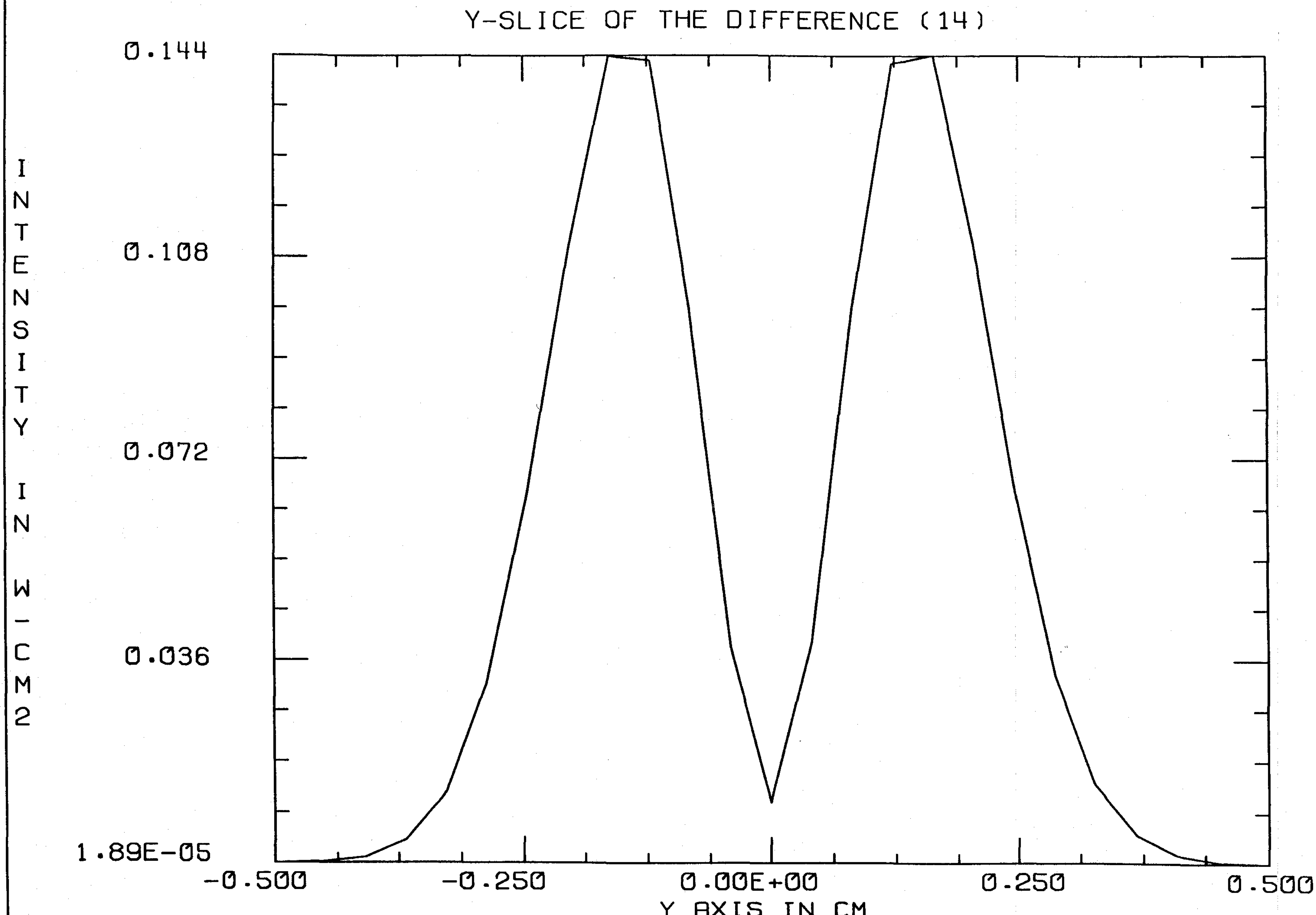
BEAM NO. 2

WAVELEN = 0.515 MIC





PLOT 5, MON SEP 24 19:38:51 1990



WAVELENGTH 0.515 MIC X COOR 0. CM
BEAM NUMBER- 2 PLOT 6, MON SEP 24 19:38:51 1990

Appendix_X_a

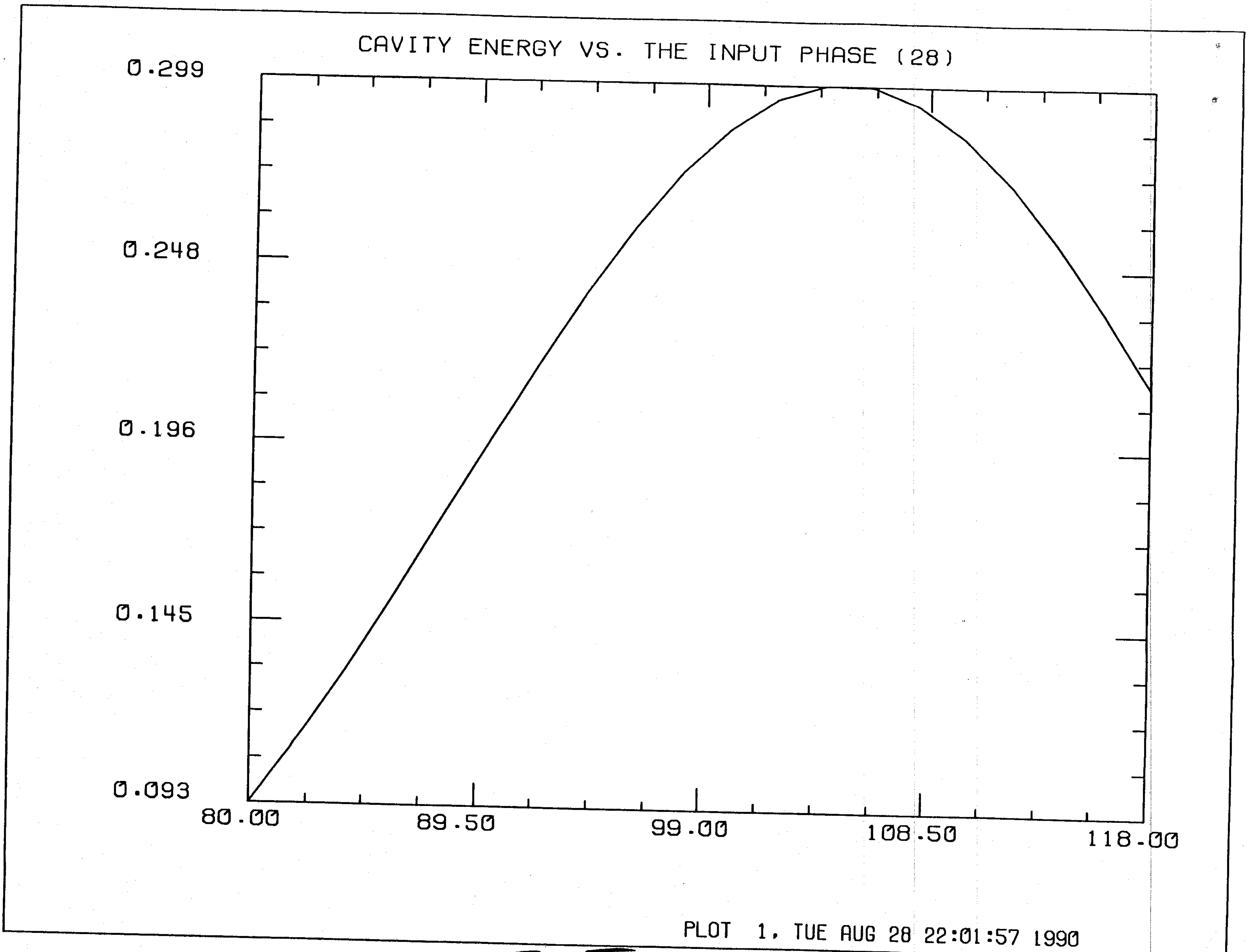

```
C  The simulation of 40 meter cavity with scanning input light frequency
C  and flat (incorrect) mode  shape.  Lambda/20 random figure errors are
C  introduced at each mirror.  The units are auto adjusting and the array
C  size is 64 by 64.  This one has correlation lengths above 0.5 cm.
C  The figure errors are peak-to-peak as opposed to RMS.
C
C  Define the cavity iteration macro:
C
macro/def cavity/overwrite
C
   prop 4000.0                              # Propagate 40 m. forward
C
   mult/beam        1 4                     # Add the phase errors.
C
   clap/cir         1 1.0                   # 1.0 cm. radius aperture
C
   mirror/sph       1 -6100.0               # Mirror of 61 m. radius
C
   prop 4000.0                              # Propagate 40 m. backwards
C
   mult/beam        1 3                     # Add the phase errors.
C
   clap/cir         1 1.0                   # 1.0 cm. radius aperture
C
   mirror/sph       1 1.0e15                # Flat mirror
C
   phase/piston     1 %r2                   # Add the scanning phase (mode
                                            # frequency selection).
C
   add/coh/con      1 2                     # Add beam 2 to beam 1 coherently
                                            # (interference with the incoming beam)
C
   mult/scalar      1 .25000000             # Normalize it by multiplying
                                            # the intensity by .25000000.
C
   copy/con         1 2                     # Copy the resultant beam to
                                            # the accumulator beam
C
macro/end                                   # End of macro definition.
C
C
C  Define the frequency scanning macro:
C
macro/def   scanfreq/overwrite
C
   register/add/int 1 1                     # Increment pass counter
C
   clear 1 1                                # Make initial beam a plane
                                            # wave (beam 1)
   clear 2 1                                # Make initial beam a plane
                                            # wave (beam 2)
C
   macro/run  cavity/25                     # Run the initial waveform
                                            # through the cavity at most
                                            # 25 times.
C
   register/set/param 3 1 energy/real       # Put the energy of the beam 1
                                            # in real register 3.
C
   udata/set %i1 %r2 %r3                    # Save the current phase angle and
                                            # resulting energy in the user storage
                                            # indexed by the pass number.
C
   register/add/real 2 %r1                  # Increment the phase angle (real
```

```
                                   # register 2) by the contents of
                                   # real register 1.
C
macro/end
C
C
C  Main program starts:
C
nbeam 4                            # Define four beams; beam 2 is the
                                   # accumulating beam; beams 3 and 4
                                   # are phase error beams.
C
array/set 1 64  64  0              # Set the array dimensions for beam 1
array/set 2 64  64  0              # Set the array dimensions for beam 2
array/set 3 64  64  0              # Set the array dimensions for beam 3
array/set 4 64  64  0              # Set the array dimensions for beam 4
C
color 1 0.5145                     # Set the mean wavelength for beam 1 (microns)
color 2 0.5145                     # Set the mean wavelength for beam 2 (microns)
color 3 0.5145                     # Set the mean wavelength for beam 3 (microns)
color 4 0.5145                     # Set the mean wavelength for beam 4 (microns)
C
units/set 1 0.04 0.04              # The distance between the array elements
                                   # for beam 1 (cm)
units/set 2 0.04 0.04              # The distance between the array elements
                                   # for beam 2 (cm)
units/set 3 0.04 0.04              # The distance between the array elements
                                   # for beam 3 (cm)
units/set 4 0.04 0.04              # The distance between the array elements
                                   # for beam 4 (cm)
C
units/field 1 1.27 1.27            # Set the field radius for beam 1 (cm)
units/field 2 1.27 1.27            # Set the field radius for beam 2 (cm)
units/field 3 1.27 1.27            # Set the field radius for beam 3 (cm)
units/field 4 1.27 1.27            # Set the field radius for beam 4 (cm)
C
zbound/set 1 .217865054304         # Set the waist size for beam 1 (cm)
                                   # (ideal, defines the Rayleigh range)
zbound/set 2 .217865054304         # Set the waist size for beam 2 (cm)
                                   # (ideal, defines the Rayleigh range)
zbound/set 3 .217865054304         # Set the waist size for beam 3 (cm)
                                   # (ideal, defines the Rayleigh range)
zbound/set 4 .217865054304         # Set the waist size for beam 4 (cm)
                                   # (ideal, defines the Rayleigh range)
C
C  Ready to start. Initialize variables:
C
beams/all/off                      # Turn all beams off (no propagation)
C
beams/on 1                         # Beam 1 propagates, beam 2 accumulates;
                                   # beams 3 and 4 will sit around.
C
status/p                           # Print the status
C
register/set/int 1 0               # Macro pass integer register
C
register/set/real 1 2.0            # Set the step size in scanning
                                   # to 2.0 degrees (real register 1).
C
register/set/real 2 80.00000000    # Set the initial Guoy phase 20
                                   # degrees before real resonance
                                   # (real register 2).
C
clear 3 1                          # Set beam 3 magnitude to 1.
```

```
C
clear 4 1                                # Set beam 4 magnitude to 1.
C
phase/random/rec 3 -0.05 0.5 0.6         # Add lambda/20 random phase error
                                         # to beam 3.
phase/random/rec 4 -0.05 0.65 0.7        # Add lambda/20 random phase error
                                         # to beam 4.
C
C  Start running the scanning macro:
C
mac/run scanfreq/20                      # Run the scanning macro 20
                                         # times for 40 degree coverage
                                         # around the real resonance.
C
C  Scan is complete.
C
C  Plot the energy versus the phase angle:
C
title
Cavity energy vs. the input phase (28)
plot/disk  energy_phase.28
plot/udata first=1 last=1
C
udata/list/disk energy_ph_lst.28         # Make a list of the results
                                         # as well.
C
C  This is it:
C
end
```

Appendix_X_b

1	8.0000000000E+01	9.3413665891E-02	0.0000000000E+00	0.0000000000E+00
2	8.2000000000E+01	1.1149115115E-01	0.0000000000E+00	0.0000000000E+00
3	8.4000000000E+01	1.3136711717E-01	0.0000000000E+00	0.0000000000E+00
4	8.6000000000E+01	1.5263901651E-01	0.0000000000E+00	0.0000000000E+00
5	8.8000000000E+01	1.7478577793E-01	0.0000000000E+00	0.0000000000E+00
6	9.0000000000E+01	1.9717614353E-01	0.0000000000E+00	0.0000000000E+00
7	9.2000000000E+01	2.1909500659E-01	0.0000000000E+00	0.0000000000E+00
8	9.4000000000E+01	2.3977600038E-01	0.0000000000E+00	0.0000000000E+00
9	9.6000000000E+01	2.5843986869E-01	0.0000000000E+00	0.0000000000E+00
10	9.8000000000E+01	2.7434116602E-01	0.0000000000E+00	0.0000000000E+00
11	1.0000000000E+02	2.8681534529E-01	0.0000000000E+00	0.0000000000E+00
12	1.0200000000E+02	2.9532361031E-01	0.0000000000E+00	0.0000000000E+00
13	1.0400000000E+02	2.9948985577E-01	0.0000000000E+00	0.0000000000E+00
14	1.0600000000E+02	2.9912650585E-01	0.0000000000E+00	0.0000000000E+00
15	1.0800000000E+02	2.9425045848E-01	0.0000000000E+00	0.0000000000E+00
16	1.1000000000E+02	2.8507924080E-01	0.0000000000E+00	0.0000000000E+00
17	1.1200000000E+02	2.7201661468E-01	0.0000000000E+00	0.0000000000E+00
18	1.1400000000E+02	2.5562375784E-01	0.0000000000E+00	0.0000000000E+00
19	1.1600000000E+02	2.3658140004E-01	0.0000000000E+00	0.0000000000E+00
20	1.1800000000E+02	2.1564407647E-01	0.0000000000E+00	0.0000000000E+00


CAVITY ENERGY VS. THE INPUT PHASE (28)
0.299
0.248
0.196
0.145
0.093
80.00
89.50
99.00
108.50
118.00
PLOT 1, TUE AUG 28 22:01:57 1990

Appendix_XI_a


```
C  The simulation of 40 meter cavity with scanning input light frequency
C  and flat (incorrect) mode  shape.  Lambda/20 random figure errors are
C  introduced at each mirror.  The units are auto adjusting and the array
C  size is 64 by 64.   This one has correlation lengths above 0.5 cm.
C  The figure errors are peak-to-peak as opposed to RMS.  High
C  resolution scan.
C
C  Define the cavity iteration macro:
C
macro/def cavity/overwrite
C
   register/add/int 2 1                  # Increment pass counter
C
   copy/con       1 2                    # Copy the resultant beam to
                                         # the accumulator beam
C
   prop 4000.0                           # Propagate 40 m. forward
C
   mult/beam      1 4                    # Add the phase errors.
C
   clap/cir       1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph     1 -6100.0              # Mirror of 61 m. radius
C
   prop 4000.0                           # Propagate 40 m. backwards
C
   mult/beam      1 3                    # Add the phase errors.
C
   clap/cir       1 1.0                  # 1.0 cm. radius aperture
C
   mirror/sph     1 1.0e15               # Flat mirror
C
   phase/piston   1 %r2                  # Add the scanning phase (mode
                                         # frequency selection).
C
   add/coh/con    1 2                    # Add beam 2 to beam 1 coherently
                                         # (interference with the incoming beam)
C
   mult/scalar    1 .25000000            # Normalize it by multiplying
                                         # the intensity by .25000000.
C
macro/end                                # End of macro definition.
C
C
C  Define the frequency scanning macro:
C
macro/def   scanfreq/overwrite
C
   register/add/int 1 1                  # Increment pass counter
C
   clear 1 1                             # Make initial beam a plane
                                         # wave (beam 1)
C
   register/set/int 2 0                  # Re-initialize pass counter
                                         # for the "cavity" macro
C
   macro/run  cavity/25                  # Run the initial waveform
                                         # through the cavity at most
                                         # 25 times.
C
   register/set/param 3 1 energy/real    # Put the energy of the beam 1
                                         # in real register 3.
C
   udata/set %i1 %r2 %r3                 # Save the current phase angle and
```

```
                                        # resulting energy in the user storage
C                                       # indexed by the pass number.
   register/add/real 2 %r1
                                        # Increment the phase angle (real
                                        # register 2) by the contents of
C                                       # real register 1.
macro/end
C
C
C  Main program starts:
C
nbeam 4
                                        # Define four beams; beam 2 is the
                                        # accumulating beam; beams 3 and 4
C                                       # are phase error beams.
array/set 1 64  64  0
array/set 2 64  64  0                   # Set the array dimensions for beam 1
array/set 3 64  64  0                   # Set the array dimensions for beam 2
array/set 4 64  64  0                   # Set the array dimensions for beam 3
C                                       # Set the array dimensions for beam 4
color 1 0.5145
color 2 0.5145                          # Set the mean wavelength for beam 1 (microns)
color 3 0.5145                          # Set the mean wavelength for beam 2 (microns)
color 4 0.5145                          # Set the mean wavelength for beam 3 (microns)
C                                       # Set the mean wavelength for beam 4 (microns)
units/set 1 0.04 0.04
                                        # The distance between the array elements
units/set 2 0.04 0.04                   # for beam 1 (cm)
                                        # The distance between the array elements
units/set 3 0.04 0.04                   # for beam 2 (cm)
                                        # The distance between the array elements
units/set 4 0.04 0.04                   # for beam 3 (cm)
                                        # The distance between the array elements
C                                       # for beam 4 (cm)
units/field 1 1.27 1.27
units/field 2 1.27 1.27                 # Set the field radius for beam 1 (cm)
units/field 3 1.27 1.27                 # Set the field radius for beam 2 (cm)
units/field 4 1.27 1.27                 # Set the field radius for beam 3 (cm)
C                                       # Set the field radius for beam 4 (cm)
zbound/set 1 .217865054304
                                        # Set the waist size for beam 1 (cm)
zbound/set 2 .217865054304              # (ideal, defines the Rayleigh range)
                                        # Set the waist size for beam 2 (cm)
zbound/set 3 .217865054304              # (ideal, defines the Rayleigh range)
                                        # Set the waist size for beam 3 (cm)
zbound/set 4 .217865054304              # (ideal, defines the Rayleigh range)
                                        # Set the waist size for beam 4 (cm)
C                                       # (ideal, defines the Rayleigh range)
C  Ready to start. Initialize variables:
C
beams/all/off
C                                       # Turn all beams off (no propagation)
beams/on 1
                                        # Beam 1 propagates, beam 2 accumulates;
                                        # beams 3 and 4 will sit around.  They are
C                                       # the beams carrying the distortions.
status/p
C                                       # Print the status
register/set/int 1 0
C                                       # Scanning macro pass integer register
register/set/real 1 0.25
                                        # Set the step size in scanning
C                                       # to 0.25 degrees (real register 1).
```

```
register/set/real 2 100.0                # Set the initial Guoy phase some
                                         # degrees before real resonance
                                         # (real register 2).
C
clear 3 1                                # Set beam 3 magnitude to 1.
C
clear 4 1                                # Set beam 4 magnitude to 1.
C
phase/random/rec 3 -0.05 0.5 0.6         # Add lambda/20 random phase error
                                         # to beam 3.
phase/random/rec 4 -0.05 0.65 0.7        # Add lambda/20 random phase error
                                         # to beam 4.
C
C  Start running the scanning macro:
C
mac/run scanfreq/32                      # Run the scanning macro 32
                                         # times for 8 degree coverage
                                         # around the real resonance.
C
C  Scan is complete.
C
C  Plot the energy versus the phase angle:
C
title
Cavity energy vs. the input phase (29)
plot/disk  energy_phase.29
plot/udata first=1 last=1
C
udata/list/disk energy_ph_lst.29         # Make a list of the results
                                         # as well.
C
C  This is it:
C
end
```