New Folder Name *SHOT Noise Comparison*

Comparison of Non-recombined and Recombined Interferometer Shot Noise Sensitivity

R. E. Spero

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Non-recombined Fabry-Perot

The displacement sensitivity for the system in its present, non-recombined configuration is[1]

$$\tilde{x}_{\text{NR}}(f) = \frac{L}{\pi \tau_e} \sqrt{\frac{3}{8}} \sqrt{\mathcal{M}} \left[\frac{\lambda h}{cP} \left(1 + [f/f_k]^2 \right) \right]^{1/2} \tag{1}$$

with the modulation function defined by

$$\mathcal{M} = \frac{1}{3} \left[\frac{M^{-1} + A^2 J_0^2 - 2AJ_0^2 + 2AJ_0 J_2}{M A^2 J_0^2 J_1^2} \right]$$

Here λ is the optical wavelength, P is the total bright-fringe power, f = gravity wave signal frequency, M is the mode matching fraction (in the absence of modulation and mirror coupling mismatches), A represents the field inside the cavity and is given by $A = 2/(1 + \mathcal{L}/T_1)$, L is the length of one arm, $\tau_t = L/c$, τ_e is the cavity energy storage time, $f_k \equiv 1/(4\pi\tau_e)$, with $\tau_e = 2\tau_t [T_1 + \mathcal{L}]^{-1}$, and the Bessel functions J_0, J_1, J_2 are evaluated at the modulation index.

Recombined Sensitivity

The sensitivity for recombined but not recycled interferometers in Fabry-Perot and delay-line configurations is[2]

$$S_{h_{\text{FR}}}(f) = \frac{2\hbar c\lambda}{P} \left(\frac{1}{2BL} \right)^2 \left[1 + (2\pi BLf/c)^2 \right] \tag{2}$$

$$S_{h_{\text{DL}}}(f) = \frac{2\hbar c\lambda}{P} \left(\frac{1}{2BL} \right)^2 \left[\frac{2\pi BLf/c}{\sin(2\pi BLF/c)} \right]^2 \tag{3}$$

where λ = (optical wavelength)$/2\pi$, $\tilde{x}(f) = L\sqrt{S_h(f)}$, $2\pi BL/c \equiv 1/f_k$. For the Fabry-Perot, $1/2BL = 1/4c\tau_e$, and for the delay-line, B is the number of round trips in each arm.

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[1] *Shot Noise in the Caltech Gravitational Wave Detector—The mid-1984 Configuration,*, Stanley E. Whitcomb. There is a copy on file with annotations by R.S., draft dated 23 January 1990. The dependence on modulation index and mode matching is in agreement with a paper to be published by Niebauer et. al., Phys. Rev. A

[2] From the Article by Kip S. Thorne in *300 Years of Gravitation*, eds. S.W. Hawking and W. Israel, Cambridge University Press, p. 330 (1987) page 424, Equation 115.