New Folder Name Diffraction Loss

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Diffraction Loss and Minimum Mirror Size

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1. Unapertured Spot Sizes

Consider a cavity formed from a pair of mirrors separated by d. If the mirrors are identical, in the absence of diffraction the spot size at each mirror surface is [1]

$$w_S{}^2 = 2\lambda d \left(1 - g^2\right)^{-1/2} \tag{1}$$

where $g = 1 - d/R_S$, with d the separation between mirrors with curvature R_S, and $\lambda = \lambda/2\pi$. As usual, w is the radius where the field amplitude is e^{-1} its value at the spot center.

For a plane-curve cavity of length d', curved-mirror radius R_S, and $g' = 1 - d'/R_S$, the spot sizes are

$$w_F{}^2 = 2\lambda d' \left[\frac{g'}{1 - g'}\right]^{1/2} \tag{2}$$

and

$$w_C{}^2 = 2\lambda d' \left[\frac{1}{g'(1 - g')}\right]^{1/2} \tag{3}$$

on the flat (w_F) and curved (w_C) mirrors, respectively. Note that $w_F/w_C = \sqrt{g'}$. The plane-curve cavity is equivalent to a symmetric cavity of twice the length and the same curvature on the curved mirror: $d = 2d'$ and $g = 2g' - 1$. Constraining $d = d'$ and adjusting the mirror curvature, the relative spot sizes for equivalent cavities are

$$w_C = \sqrt{2} w_S \tag{4}$$

and

$$w_F = w_S \sqrt{1 + g} \tag{5}$$

In the absence of diffraction, the spots would have a perfect gaussian distribution, with a fraction of the energy α in the spot beyond the radius a:

$$\alpha = e^{-2\pi N \sqrt{1 - g^2}} \tag{6}$$

where N is the fresnel number:

$$N = \frac{a^2}{\lambda d}. \tag{7}$$

2. The Effect of Diffraction

Diffraction from a finite-diameter mirror perturbs the field near the edge into a nongaussian distribution, and the loss due to limited mirror size, $\alpha(a)$, is usually calculated by numerical integration [2]. I was unable to find calculations in the literature appropriate to supermirrors; published plots don't go below $\alpha \approx 10^{-3}$. Fig. 1 extends the calculation of energy loss per reflection α for the TEM00 mode down to $\alpha = 10^{-6}$. The diffraction loss calculated by numerical integration approaches the gaussian estimate as the mirror size increases. At $\alpha = 10^{-6}$ it agrees with the estimate of Eq. 6 within 10%, for $|g| \geq .5$; it agrees to higher accuracy with the analytical approximation for the degenerate case of $g = 0$. [2]. Table 1 is based on the calculations shown in Fig. 1. For the symmetric cavity the required mirror diameter follows directly from the Fresnel number N. For the plane-curved cavity, the approximation is made that the mirror diameters scale as the unapertured spot sizes (Equations 4 and 5.)

$g = 1 - \frac{d}{R_S}$	$N = \frac{a_S^2}{\lambda d}$	SYMMETRIC CAVITY		PLANE-CURVED CAVITY		
		R_S (km)	$2a_S$ (cm)	R_C (km)	$2a_F$(cm)	$2a_C$ (cm)
0.00	1.54	4.00	11.25	8.00	11.25	15.92
0.10	2.36	4.44	13.92	8.89	14.60	19.68
0.20	2.36	5.00	13.94	10.00	15.27	19.71
0.30	2.35	5.71	13.90	11.43	15.85	19.66
0.50	2.28	8.00	13.69	16.00	16.77	19.37
0.60	2.91	10.00	15.47	20.00	19.57	21.88
0.70	3.00	13.33	15.71	26.67	20.48	22.21
0.80	3.80	20.00	17.68	40.00	23.72	25.00
0.90	5.25	40.00	20.78	80.00	28.64	29.39
-0.10	2.36	3.64	13.92	7.27	13.20	19.68
-0.20	2.36	3.33	13.94	6.67	12.47	19.71
-0.30	2.35	3.08	13.90	6.15	11.63	19.66
-0.50	2.28	2.67	13.69	5.33	9.68	19.37
-0.60	2.91	2.50	15.47	5.00	9.78	21.88
-0.70	3.00	2.35	15.71	4.71	8.60	22.21
-0.80	3.80	2.22	17.68	4.44	7.91	25.00
-0.90	5.25	2.11	20.78	4.21	6.57	29.39

Table 1: Miminum mirror diameter required for diffraction loss of $\alpha = 10^{-6}$, as function of g-parameter and Fresnel number N; $\lambda = 514$ nm, $d = 4$km. The entries $2a$ are the mirror diameters, in two configurations: symmetrical ($2a_S$) and plane-curve ($2a_F$ and $2a_C$). To make the plane-curved cavity equivalent to the symmetric cavity, the radii of curvature satisfy $R_C = 2R_S$.

References

[1] Kortz, H.P. and Weber, H., *Appl. Optics* **20**, 11, 1936–1940 (1981). Beware of typos.

[2] Kogelnik, H., *BSTJ* **44**,455 (1965).

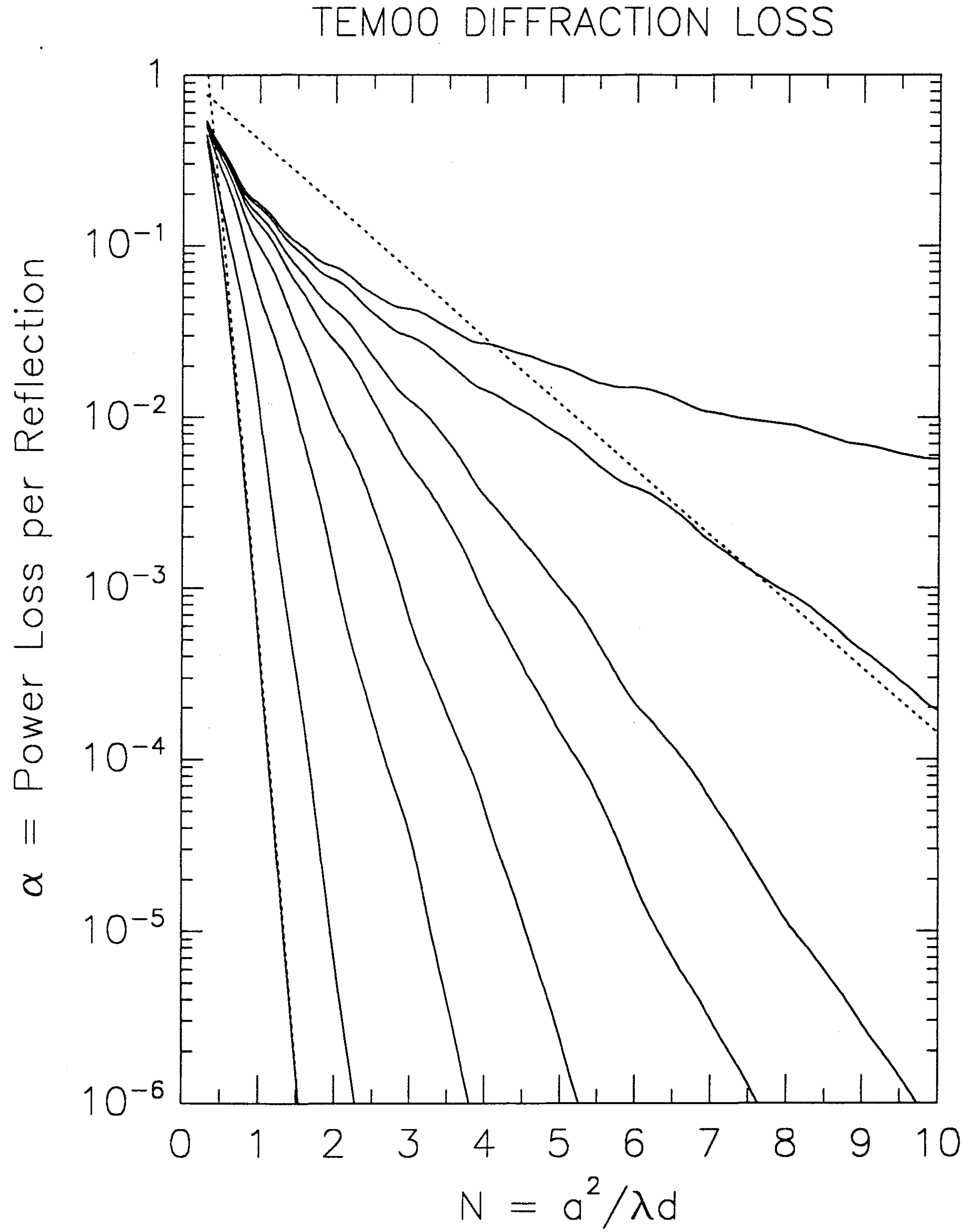

Figure 1: The loss per reflection α due to diffraction as a function of mirror size a and g-parameter. $|g|$ runs left to right: (0 .5 .8 .9 .95 .97 .99 .999). The dotted curve at $g = 0$ (overlapping with the solid curve) is the estimate $\alpha = 16\pi^2 e^{-4\pi N}$, Ref. [2]. The dotted curve at $g = .99$ represents the gaussian estimate (Eq. 6).

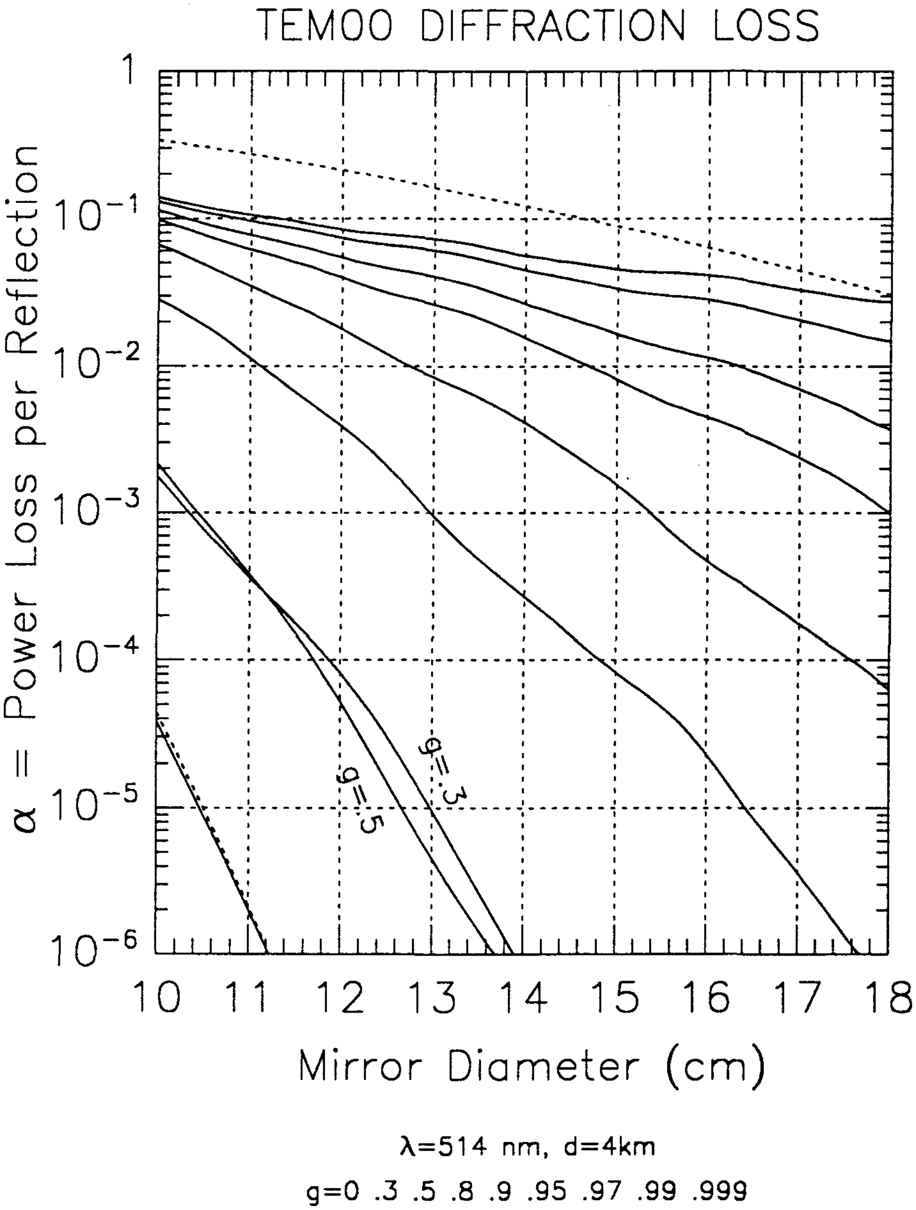

Figure 2: Similar to Fig. 1, but plotted in terms of mirror diameter for a symmetric cavity.