New Folder Name PENDULUM SUSPENSIONS

Ultra High Q Pendulum Suspensions for Gravitational Wave Detectors

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Abstract

We show that pendulum suspensions in gravitational wave detectors can be designed to achieve Q factors approaching 10^{10}. This should enable a 3km terrestrial laser interferometer detector to achieve strain sensitivity of $10^{-22}/\sqrt{Hz}$ at frequencies as low as 10Hz.

Introduction

The sensitivity of terrestrial laser interferometer gravitational wave detectors (LIDs) is limited at low frequencies by a combination of Brownian motion noise from the suspension and seismic noise. Figure 1 shows schematically the typical limiting noise sources which define the overall sensitivity of a LID. High frequency sensitivity is limited chiefly by photon shot noise in the laser interferometer, which scales as $f^{3/2}$ [1](This may be modified by using various recycling schemes).

The Brownian motion noise amplitude for any normal mode at frequency ω_0 is given by $\Delta x^2 = \frac{4kT\omega_0}{MQ\omega^4}$, where k is the Boltzman constant, T the temperature, M the mass of the system and Q the Q-factor of the system. Once the seismic noise cut off is lowered sufficiently through use of high performance vibration isolators, this will become a critical problem.

Giazotto et al[2] have demonstrated vibration isolators with cut off frequency ~5Hz using air spring systems. Elements with similar frequency responses have also been developed both at the University of Western Australia and INFN Pisa based on all metal cantilever springs[3,4]. Thus it appears that isolators with cut off frequency ≤ 10Hz are possible, and Brownian motion noise in the suspension becomes of critical importance. Figure 2 shows a theoretical isolator response for which the pendulum Q-factor is 10^6. The Brownian motion noise, marked in bold, is clearly a limiting factor. It reduces substantially and gives significant improvement in performance as the Q is increased to 10^{10}.

The Q factor of a pendulum suspension can in principle be very high since the energy storage is predominantly in the effectively lossless gravitational field. However some elastic energy must always be stored in the flexure which supports the pendulum. The Q is limited by the losses in this element. Most researchers have assumed that wires are necessary, and have assumed reasonable Q factors for the wire to give a maximum $Q \sim 10^8$ [5]. Robertson et al[6] has demonstrated a pendulum Q factor $\sim 10^7$.

We show here that significant improvements can be achieved if the pendulum is replaced by a one dimensional compound pendulum supported by a thin membrane. Using known materials, and assuming that Q is independent of frequency, Q-factors approaching 10^{10} can be achieved.

Pendulum Q Factor

Consider a physical pendulum of mass M with length L on which the mirror of the interferometer detector is attached as show in Fig.3. This pendulum is supported by a thin

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membrane of thickness d, width a, length *l*, Young's modulus E and yield strength Y. Using the angular deflection θ as variable, the angular spring constant due to gravity κ_g for the pendulum is

$$\kappa_g = \frac{1}{2} MgL . \tag{1}$$

The angular spring constant of the supporting membrane (treated as a cantilever) is

$$\kappa_s = \frac{E\, a\, d^3}{4\, l_{eff}} , \tag{2}$$

where l_{eff} is the length of the membrane over which the deformation is distributed. When a thin sheet is bending under large tension only a small length is in flexure. Most of the membrane is held flat by tension. In the limit of a thin membrane and large tension it is a reasonable approximation that the effective length for flexure $l_{eff} \sim 3d$. This result indicates that there is no advantage in making the membrane longer than a few times the membrane thickness. We now estimate the Q-factor of this pendulum.

We assume that the intrinsic Q-factor of the membrane Q_0 is independent of frequency. This is commonly observed in most materials. The energy lost per cycle w_d is independent of frequency

$$w_d = \alpha X^2 \tag{3}$$

where α is a constant and X is the amplitude. The effective damping is

$$c_{eff} = \frac{\alpha}{\pi\, \omega} \tag{4}$$

Modelling the system as a single linear mass-spring system with a pair of parallel springs with total spring constant $k = k_1 + k_2$, and damping in one spring giving by eq.(4), the Q-factor of the whole pendulum can be expressed as

$$Q = \frac{M\omega_0}{c_{eff}} = \frac{\pi k}{\alpha} = \frac{\pi(k_1+k_2)}{\alpha} = \frac{\pi k_2}{\alpha}\,(1+\frac{k_1}{k_2}) = Q_0\,(1+\frac{k_1}{k_2}) , \tag{5}$$

where k_1 and k_2 are the spring constants of the physical pendulum and the membrane respectively. If $k_1 \gg k_2$, then

$$Q \approx Q_0 \frac{k_1}{k_2} . \tag{6}$$

Using the angular variable and eqs. (1) and (2) above, the Q-factor of the pendulum is

$$Q = Q_0 \frac{\kappa_g}{\kappa_s} = 2\, Q_0\, (\frac{Y}{E})\, (\frac{l}{d})\, \frac{L}{d} . \tag{7}$$

The membrane must satisfy a strength requirement $Mg \leq Y\, a\, d$. Using this result, it follows that

$$Q = 2\, Q_0\, (\frac{Y^2}{E})\, (\frac{l}{d})\, \frac{L\, a}{Mg} = R\, Q_0 , \tag{8}$$

where $$R = 2\, (\frac{Y^2}{E})\, (\frac{l}{d})\, \frac{L\, a}{Mg} .$$

This result assumes that the membrane is stressed to its yield strength. If a safety margin is needed, so that $Y\, a\, d = q\, M\, g$, where $q > 1$, then

$$Q \approx \frac{R\, Q_0}{q^2} \tag{9}$$

We note that if the damping constant is frequency independent, so that the intrinsic Q of the membrane is frequency dependant, then Eq.(6) is replaced by

$$Q \approx Q_0 \sqrt{\frac{k_1}{k_2}} . \tag{10}$$

In designing a practical pendulum suspension, it is important that the pendulum size be such that internal mechanical resonances are above the interferometer pass band. This sets limits on the maximum length of the pendulum, and also relates the pendulum length to its mass, since the pendulum diameter must be such that bending modes are high enough.

A practical design could have length 1m, diameter 0.2m and mass 100kg, constructed from aluminium. If the length were to be increased to 2 metres, the diameter also would have to be doubled to maintain the internal bending modes frequency high enough. Due to the increased mass, there would be no advantage in this increased size. Internal resonances in practice limit the pendulum length to about 1.5 metres.

Now considering the above values for the pendulum suspension, $M = 100\text{kg}$, $L = 1\text{m}$, $a = 0.2\text{m}$, $l = l_{eff} = 3d$, then

$$Q \approx 1.22 \times 10^{-3} \left(\frac{Y^2}{E}\right) Q_0 \ .$$

Choice of Membrane Material

We have surveyed the properties of a range of metals, alloys and other possible materials. From equation (8), it is necessary to find a material which gives the maximum value for $Q_o Y^2/E$. Unfortunately there is little data available on maximum intrinsic Q factors at low frequencies. Table I lists the relevant properties of selected promising materials, excluding Q-factor data. Column 5 gives the ratio Y^2/E. Column 6 gives the R-value based on the parameters chosen above and column 7 gives the required membrane thickness. Properties depend on the annealing state and are given only for nominally hard and soft forms[7].

Amongst the metals, we note the exceptional R-values for Re, Nb, Ta and V. Amongst the alloys, TiAlV is highest, but various steels, beryllium copper and phosphor bronze all have high values for R. Finally, we note that many glassy alloys have exceptionally high tensile strengths, giving R-values up to 3.2×10^4.

While sapphire and quartz are not exceptional in their R-values their Q-factors could be high. In particular sapphire has been shown to have very high Q at ultrasonic frequencies[8].

Q-factor for Niobium membrane pendulum

We go on to discuss the Q-factor of Nb which has been characterised for use in resonant bar gravity wave detectors[9]. Studies show that depending on the material processing Q-factors up to 4.6×10^6 are achievable at room temperature[10]. Results show that the Q-factor is roughly frequency independent between 700Hz and 5kHz. Table II lists the Q-factor of Nb samples at room temperature taken from references [9] and [10]. To obtain high Q-factors surface etching is always necessary. Q values are highest for annealed ingot material, or for ingots themselves. These have very large crystals (typically 50mm). While the Q-factor of cold tolled plate is an order of magnitude lower, the increased yield strength significantly compensates for this.

If the pendulum length is increased to 1.5m, and the membrane is widened to 0.5m, the pendulum Q-factor would rise to 7.7×10^9 for the best material properties in Table II. This represents a pendulum with a ring down time about 80 years.

Table I

Material		Young's modulus E (GPa)	tensile strength (MPa)	yield Strength Y (Mpa)	$\frac{Y^2}{E}$ (10^5 N.m^{-2})	R(10^3)	d (µm)
(1) Pure Metals							
Be	soft hard	318.0	310 550	240 345	1.8 3.7	0.2 0.5	20.4 14.2
Co	soft hard	211.0	760 1135	345-485	5.6-11.1	0.9-1.4	14.2-10.1
Hf	soft hard	141.0	445 745	240 365	2.7 9.4	0.3 1.2	20.4 13.0
Nb	soft Hard	104.9	330 585	240 550	5.5 28.8	0.7 3.5	20.4 8.9
Ni	soft hard	199.5	400 660	150 480	1.1 11.0	0.1 1.3	12.25 10.2
Re	soft hard	466.0	1125 2225	315 2150	2.1 99.1	0.3 12.1	15.6 2.2
Ta	soft hard	185.7	310-485 760	310-380 705	5.2-7.8 26.8	0.6-1.0 3.3	15.8-12.9 7.0
Ti	annealed	120.2	230-460	140-250	1.6-5.2	0.2-0.6	19.6
Mo	soft hard	324.8	485-550 620-690	415-450 550	5.3-6.2 9.3	0.6-0.8 1.1	11.8-10.9 8.9
U	soft hard	175.8	385 580	190 250	2.1 3.6	0.3 0.4	25.8 19.6
V	soft hard	127.8	260-585 530-730	170-450 515-690	2.3-15.8 20.6-37.3	0.3-1.9 2.5-4.6	28.8-10.9 9.5-7.1
W	soft hard	411.0	550-620 1920	550	7.3	0.9	8:9
		modulus of elasticity E (GPa)	tensile strength T (MPa)		$\frac{T^2}{E}$ (10^5 N.m^{-2})	R(10^3)	d (µm)
(2) Sapphire	Al_2O_3	344.5	276-413		2.2-5.0	0.3-0.6	17.8-11.8
(3) Quartz *	SiO_2	71.7	48.2		0.32	0.04	
(4) Alloys							
Cu63/Zn37		95-110	330-550		11.5-27.5	1.4-3.4	14.8-8.9
Cu94/Sn6(phosphor -bronze)		90-120	320-740		11.4-45.6	1.4-5.6	15.3-6.6
Cu98/Be2		120-160	500-1300		20.8-105.6	2.5-12.9	9.8-2.6
Fe/Cr18/Ni8		190-210	510-1100		13.9-57.6	1.7-7.1	9.6-4.5
Fe/Cr18/Ni8/Ti		190-210	500-1500		13.9-107.1	1.7-13.1	9.8-3.3
Fe/Cr17/Ni7		214	1020-1550		48.6-112.2	6.0-13.7	4.8-3.2
Ni58/Cr19/Co14/Mo/Ti/Al/Fe		221	550-1210		13.7-66.2	1.7-8.1	8.9-4.1
Ni72/Cr16/Fe8		157	600-1200		22.3-91.7	2.7-11.2	8.2-12.3
Ni75/Cr20/Al25/Cu2.5		221	1210		66.2	8.1	8.9
Ti90/Al6/V4		106-114	1035-1410		101.1-174.4	12.4-21.6	4.7-3.5
Ti94.5/Al3/V2.5		103.5	830		66.6	8.2	5.9
Glassy Alloys		tensile Modulus E (GPa)	tensile strength T (MPa)				
group 1*		57-61	>700		>80	>9.8	7.0
group 2**		150	1500-2000		150.0-266.7	18.4-32.7	3.3-2.5

* clear fused.

*Group1	Co66/Si15/B14/Fe4/Ni1	Co69/B12/Si12/Fe4/Mo2/Ni1	Fe40/Ni38/B18/Mo4	Fe78/B13/Si9	Fe81/B13.5/C2
**Group2	Co66/Si16/B12/Fe4/Mo2	Co70/(Si+B)23/Mn5(Fe+Mo)2	Fe79/Bi16/Si5	Ni40/Fe40/(Si+B)19/Mo1-2	Ni78/B14/Si8

Table II

Nb sample	Temperature (K)	Frequency (Hz)	Q (10^5)
100μm etched from surface (hard)	295 ± 5	700 1600	2.3 2.6
Annealed ingot material, etched 100μm (soft)	295 ± 5	1300	5.5
Annealed ingot material, etched 320μm (soft)	295 ± 5	1300 3000	32.0 23.0
Annealed for 2hrs, etched 100μm (soft)	295 ± 5	700	3.0
Annealed for 8hrs, etched 20μm (soft)	295 ± 5	700	2.5
Annealed for 2hrs, etched 10μm (soft)	295 ± 5	700	1.6
1.5 tonne bar (~5cm crystals size) (soft)	295 ± 5	700	46.0

Conclusion

Many materials appear to be promising candidates for high Q pendulum pivots. Nb can be expected to allow pendulum Q-factor up approaching 10^{10} assuming that frequency independent Q-factors are maintained down to 1Hz. We intend to investigate the Q-factor of other materials at low frequencies to assess their suitability for this application.

We note that the thermoelastic effect which can contribute significant dissipation to many elastic systems is negligible for such thin membrane as we are considering here. The Q-factor of Nb at 4.2K is greater than $2x10^8$ [10]. At this temperature pendulum Q-factors could be raised to $3x10^{11}$.

Finally, we point out that the one-dimensional flexure proposed here is well suited to gravitational wave detectors, and can allow easy beam steering. However a method of bonding mirrors to the test mass needs to be found that does not degrade the internal mode Q-factors or otherwise degrade performance.

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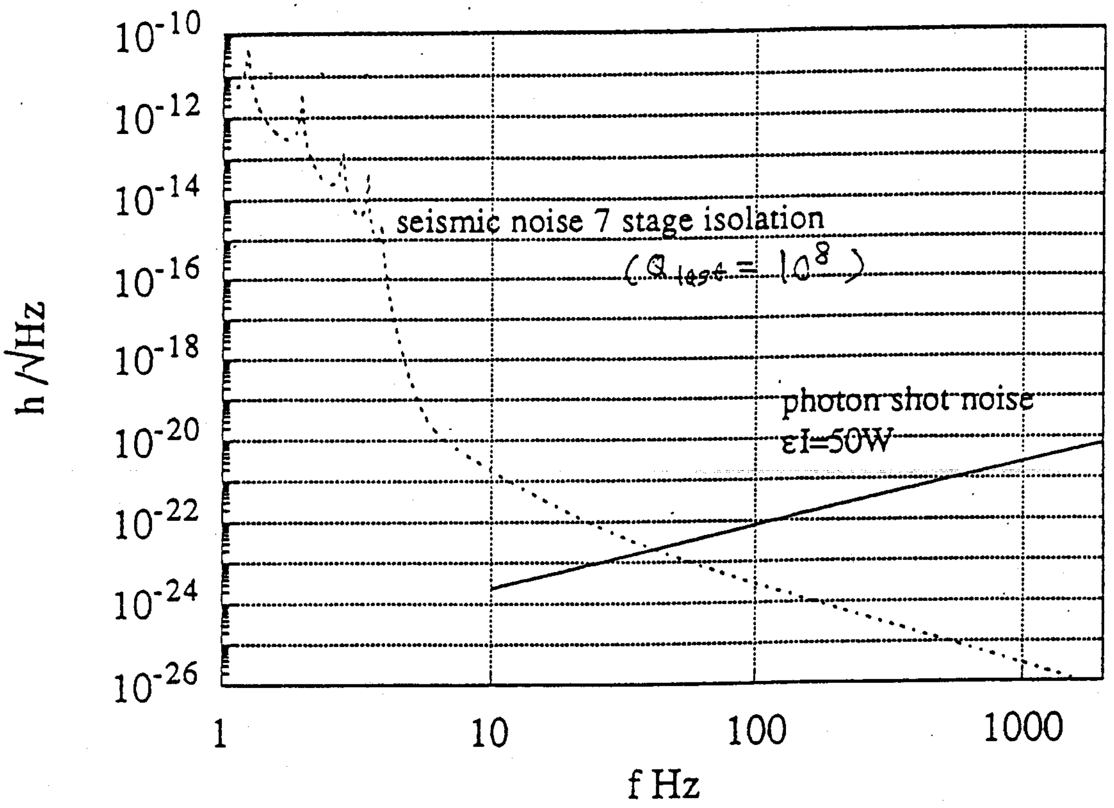
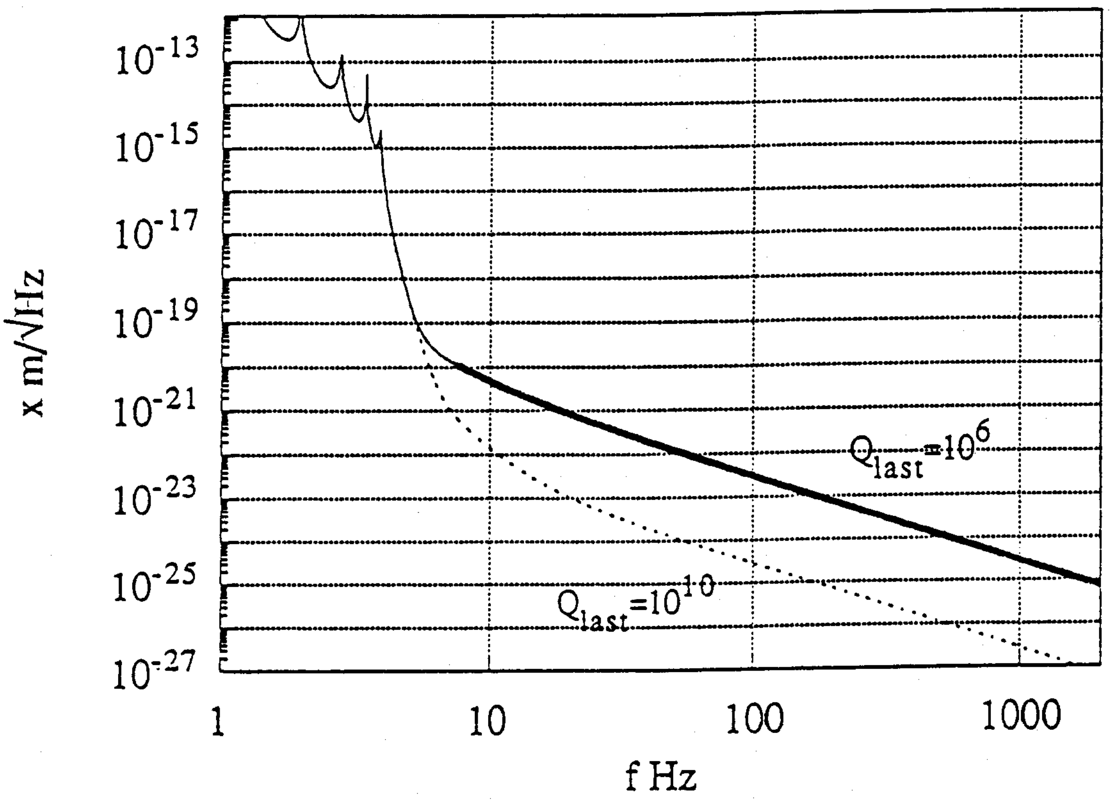


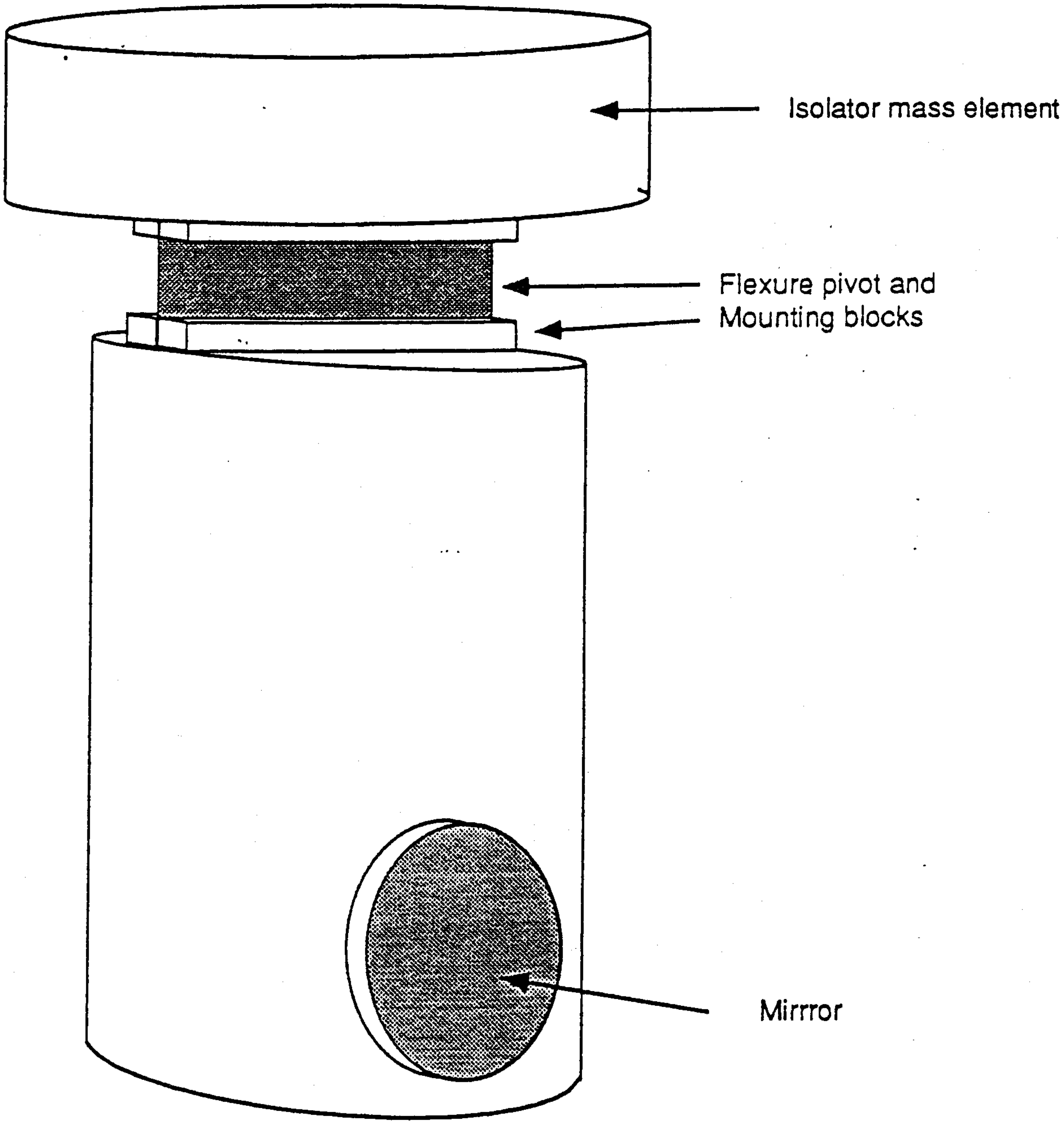
10^-10
10^-12
10^-14
10^-16
10^-18
10^-20
10^-22
10^-24
10^-26
h /√Hz
seismic noise 7 stage isolation
(Q_last = 10^8)
photon shot noise
εI=50W
1
10
100
1000
f Hz

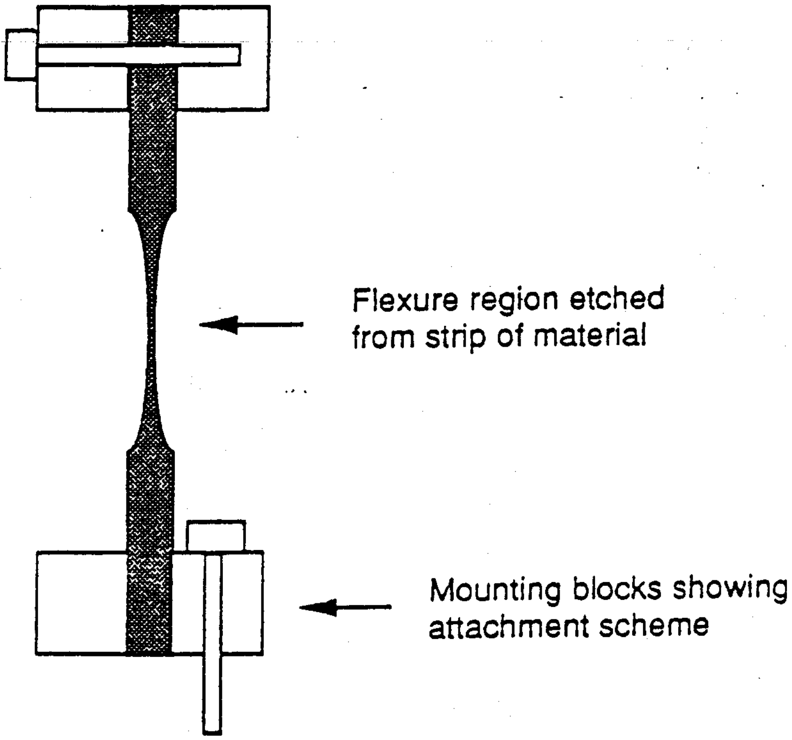
Fig 1



10^-13
10^-15
10^-17
10^-19
10^-21
10^-23
10^-25
10^-27
x m/√Hz
Q_last=10^6
Q_last=10^10
1
10
100
1000
f Hz

Fig 2


Isolator mass element
Flexure pivot and
Mounting blocks
Mirrror


Flexure region etched from strip of material
Mounting blocks showing attachment scheme