New Folder Name Optical Distortion

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Optical distortion in Gsänger Pockels cells

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ABSTRACT

We have looked at the optical distortion produced by a Pockels cell phase modulator that is driven at high frequencies and modulation depths. An effective technique to lower the distortion is to use two modulators in series, mounted so that the linear component (and all odd components) of the phase distortion is compensated. In short, modulation-produced beam distortion should not be a problem for LIGO.

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Introduction

The modulation topology for LIGO calls for phase modulating the input beam(s) at a relatively high modulation depth(s) and at relatively high rf-frequencies. This exposes the thermal aberrations in the phase modulating Pockels cells in a regime in which their performance has not been well studied, at least to our knowledge.

The non-zero loss tangent in the rf-band of electro-optic materials used for Pockels cells leads to a fraction of the rf modulation power being absorbed in the cell. This leads to temperature gradients, which in turn result in index gradients and finally the effect in which we are interested, distortions of an optical beam passing through the cell. The deleterious effect of these distortions is that they remove power from the TEM_{00} mode of laser beam, thus reducing the useful power available for the interferometer. A useful way of characterizing the distortion is thus in terms of the power which is removed from the incoming TEM_{00} mode. This characterization is made in terms of the modulation frequency and depth, the number of Pockels cells, and the beam size in the cells.

Properties of the Gsänger cells

The Gsänger PM-25 phase modulators consist of a pair of y-cut ADP crystals with Brewster angle input and output faces, as shown in figure 1. They are mounted so that they are heat sunk on one face to a ceramic 3/4–cylinder holder.

The modulation index, Γ, is given by

$$\Gamma = \frac{2^{3/2}\pi}{\lambda} \, \frac{n_e^3 n_o^3}{(n_e^2 + n_o^2)^{3/2}} \; r_{41} \frac{l}{d} \, V$$

where V is the applied voltage and the other parameters are defined in figure 1 and Table 1, which shows the optical and thermal properties of ADP.

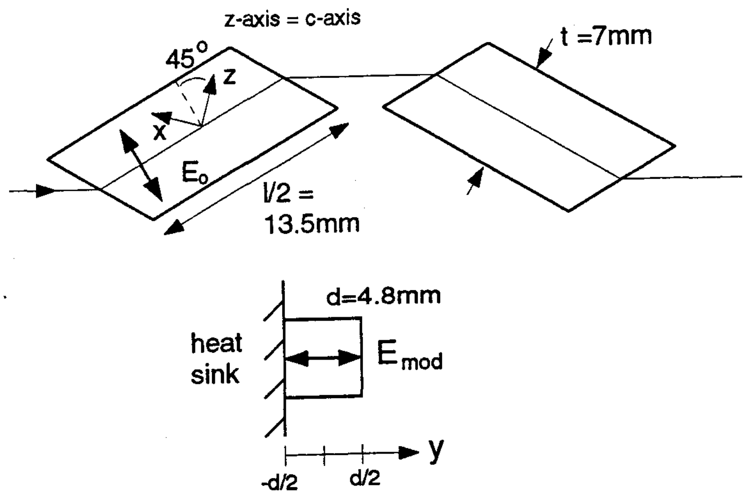

Figure 1: Gsanger PM-25 Pockels cell

n_o	ordinary ray index (515nm)	1.5276
n_e	extraordinary ray index	1.4832
ϵ_{11}/ϵ_0 ϵ_{33}/ϵ_0	dielectric constant at room temp (relatively ind. of freq.)	56 14
r_{41}	electro-optic coefficient used in Gsanger cell (515 nm)	24.5 x 10^{-12} m/V
dn_o/dT dn_e/dT	temp. dependence of o- and e-ray indices	4 x 10^{-5}/°K 1 x 10^{-6}/°K
K_a K_c	thermal conductivity along a- and c-axes	1.25 W/m-°K 0.71 W/m-°K
α_a α_c	thermal expansion coefficient along a- and c-axes	3.2 x 10^{-5}/°K 4.2 x 10^{-6}/°K
$\tan\delta_a$(10 MHz) $\tan\delta_a$(100 MHz)	loss tangent along a-axis (axis of applied modulation field)	$< 5 \times 10^{-4}$ $< 5 \times 10^{-4}$
g(n)	$\frac{2\sqrt{2}\, n_e^3 n_o^3}{(n_e^2 + n_o^2)^{3/2}}$	3.41

Thermal modeling

The loss tangent leads to a uniform deposition of heat in the crystal. We thus need the solution to the steady state heat equation in the crystal:

$$K\nabla^2 T + Q = 0$$

where T is the temperature distribution, Q is the heat generation, and K is the thermal conductivity of the material. The boundary conditions are determined by the way the crystal is mounted. The phase modulators presently in use, type PM-25 manufactured by Gsänger, consist of a pair of Brewster-angle cut crystals mounted with contact on one side to a ceramic holder. The heat loss through the unmounted surfaces (free-convective and radiative losses, of the order 1 mW/°C per crystal) is much smaller than the conductive loss through the holder (of the order 100 mW/°C per crystal), so that we neglect heat loss through all crystal faces other than the one in contact with the holder, and the problem becomes one-dimensional. Refering to figure 1, the boundary condition at the surface y = d/2 is $\partial T/\partial y = 0$. This model may be an oversimplification of the boundary conditions, but at least it will give an idea of how the distortion depends on the parameters involved.

The solution is $T(y) = T_0 + (Q/2K)(dy - y^2) \equiv T_0 + T(y)$. The heat generation is given by

$$QV = Qltd = P_{ab} = \pi f_{rf} C\, V_{rf}^2 \tan\delta$$

where $C = \varepsilon_0 \varepsilon_{11} lt/d$ is the capacitance of the cells and tanδ is the loss tangent of ADP at the rf-frequency. In terms of the modulation index, the power absorbed in the cell is

$$P_{ab} = \frac{f_{rf}\varepsilon_0\lambda^2}{\pi}\,\frac{td}{Nl}\,\frac{\varepsilon_{11}\tan\delta}{g^2(n) r_{41}^2}\,\Gamma^2 = 75\,\mathrm{mW}\left(\frac{f_{rf}}{10\,\mathrm{MHz}}\right)\left(\frac{\tan\delta}{10^{-3}}\right)\frac{\Gamma^2}{N}$$

where N, the number of Gsanger cells, has been added to account for the possibility of more than one cell mounted in series (i.e., for a given modulation index, the absorbed power goes down as 1/N).

The temperature at the heat-sunk surface, $T_s = T_0 - 3Qd^2/8K$, is determined by the complicated thermal properties of the crystals' connection to the ceramic holder, the holder's connection to the Al housing, and the housing's thermal environment. We'll approximate all this by assuming that the Al housing is a heat reservoir at room temperature, and that the thermal resistance of the connections between the cells and the ceramic, and between the ceramic and the housing are negligible. The balance of power flow then says that $P_{ab} = lt\lambda[T_s - T_{RT}]$, where λ is the heat transfer coefficient of the ceramic holder and T_{RT} is the room temp.

One limit to the maximum allowed temperature of the crystal is the temperature at which it begins to emit ammonia; this occurs above about 90°C. Given the estimated heat transfer coefficient to the case (0.1W/°C), the crystals should be safely under this limit for any conditions under which we would be operating them.

Stress distribution. The exact form of the stress distribution is certainly quite complicated, and we're not attempting to do a good job of modeling it. However, under the assumption of a one dimensional heat flow the geometry looks quite similar to uniformly pumped slab lasers, and we can compare much of the analysis of slab lasers to our problem.

The elements of the stress tensor are denoted by σ_{ij}. In the slab problem, the y-direction is the direction of heat flow, as is the case here, the z-direction is along the length of the slab, and the x-direction the other transverse direction. The yy component of stress, σ_{yy}, is zero everywhere, and by symmetry $\sigma_{xx} = \sigma_{zz}$, which is: $\sigma_{xx} = [E\alpha/(1-\nu)]T(y)$. Here E is the Young's modulus, α the thermal expansion coefficient, and ν the Poisson's ratio of the material; T(y) is defined such that $\int_{-d/2}^{d/2} T(y)dy = 0$.

Heat generation by the optical beam. The material ADP also absorbs some of the optical beam, contributing a non-uniform heating of the crystal. We've been unable to find a number for the actual absorption coefficient of ADP at 515 nm, but Gsänger claims that the PM-25 transmits >99% of the power. This would mean that it requires >10 W of optical power before the heat absorbed from the beam becomes comparable to the heat from the modulation field.

We'll neglect the effects of absorption of the optical beam, but at some power level it will become important, and we will want to know the absorption coefficient of ADP.

Optical modeling

As shown in figure 1, the optical beam propagates through the y-cut cell with a polarization axis in the *xz* plane at a 45° angle to the x and z axes. The effective index seen by the beam is $\overline{n} = \sqrt{2}\, n_e n_o / \left(n_o^2 + n_e^2\right)^{1/2} = 1.505$.

Under the application of thermal stress, the indices of refraction become second rank tensors whose components are given by

$$n_{ij_{e,o}} = n_{e,o} + \frac{dn_{e,o}}{dT} T + \sum_{kl} B_{ijkl} \sigma_{kl}$$

where the B_{ijkl} are the components of the stress optic tensor.

In the ADP crystal this must be applied to the different crystal axes, and the whole thing becomes very complicated. We are saved by comparing the magnitudes of the change of index due directly to the temperature change to that due to the stress induced by the temperature gradient. The particular material properties of ADP result in the stress induced changes being about a factor of ten smaller than the direct change due to the temperature. So we happily drop all discussion of stress-optic effects. The change in effective index is then:

$$\Delta\overline{n} = \frac{d\overline{n}}{dT} T(y) = \left[\frac{\overline{n}}{n_e}\frac{dn_e}{dT}\left(1 - \frac{\overline{n}^2}{2n_o^2}\right) + \frac{\overline{n}}{n_o}\frac{dn_o}{dT}\left(1 - \frac{\overline{n}^2}{2n_e^2}\right)\right] T(y) \approx \frac{1}{2}\frac{dn_o}{dT} T(y) \,.$$

The optical path distortion due to thermal index changes after passing through the cell is, to first order:

$$\psi(y) = \frac{\pi}{\lambda}\frac{dn_o}{dT}\, l\, T(y) \equiv c_1 \left(y - y^2/d\right)$$

where $c_1 = f_{rf} \varepsilon_0 \lambda (d/Nl) \left(\varepsilon_{11} \tan\delta / 2K_a g^2 r_{41}^2\right) (dn_o/dT)\, \Gamma^2$.

We are interested in what happens to a TEM_{00} beam after it passes through the modulator. The question is how much power is left in (or removed from) the TEM_{00} mode after the modulator, which is

$$\left|\langle 0 | e^{i\psi(y)} | 0 \rangle\right|^2 = \frac{e^{-\left(\frac{(c_1 \omega_0/2)^2}{1+\left(c_1\omega_0^2/2d\right)^2}\right)}}{\sqrt{1 + \left(c_1 \omega_0^2 / 2d\right)^2}} \equiv P_{00}$$

where the calculation is done at the beam waist, ω_0, and assuming negligible change of the beam size as it traverses the cell (the result also applies if the cell is not at the beam waist and there is negligible change in beam size through the cell). We've also assumed that the beam passes through the center of the cell, i.e. that the beam is centered at y = 0.

Since the beam size ω_0 is at least several times smaller than the cell width d, the above expression is dominated by the numerator of the exponential: $P_{00} \approx e^{-(c_1\omega_0/2)^2}$. For small distortions the power lost from the fundamental is

$$1 - P_{00} \approx (c_1\omega_0/2)^2 = 6.7 \times 10^{-2}\left(\frac{\tan\delta\left(f_{rf}\right)}{10^{-3}}\right)^2\left(\frac{f_{rf}}{10\,\mathrm{MHz}}\right)^2\left(\frac{\omega_0}{0.5\mathrm{mm}}\right)^2\frac{\Gamma^4}{N^2}\,.$$

The phase distortion over the beam size is dominated by the linear term, corresponding to a change in the angle of propagation of the beam, or equivalently an excitation of the TEM_{01} mode for small angles. This angle can be corrected for, leaving a phase distortion having just the quadratic term: $\psi = c_1 y^2/2$. The power lost from the fundamental mode in this case is

$$1 - P_{00} = 1 - \frac{1}{\sqrt{1 + \left(c_1\omega_0^2/2d\right)^2}} \approx \frac{c_1^2\omega_0^4}{8d^2}$$

$$= 3.6 \times 10^{-4}\left(\frac{\tan\delta\left(f_{rf}\right)}{10^{-3}}\right)^2\left(\frac{f_{rf}}{10\,\mathrm{MHz}}\right)^2\left(\frac{\omega_0}{0.5\mathrm{mm}}\right)^4\frac{\Gamma^4}{N^2}\,.$$

Measurements

We've made measurements of the Pockels cell induced beam distortion by passing a beam through a modulated cell(s), and analyzing the output beam using a scanning Fabry-Perot cavity (figure 2). We monitored the power in the first four higher order modes (n+m=1 —> 4) as a measure of the power removed from fundamental. For each modulation index, the fractional power in these modes was corrected for the static mode coupling defect: the fractional power in the n+m=1 —> 4 modes for no modulation. The static mode coupling defect was 1.7–2 %.

The power lost to these higher order modes was measured both without realigning the cavity to compensate for the perturbed beam direction, and after realigning the cavity. The definition of 'realignment' is that the power in the n+m=1 modes is made insignificant (< 10^{-3} fractional power).

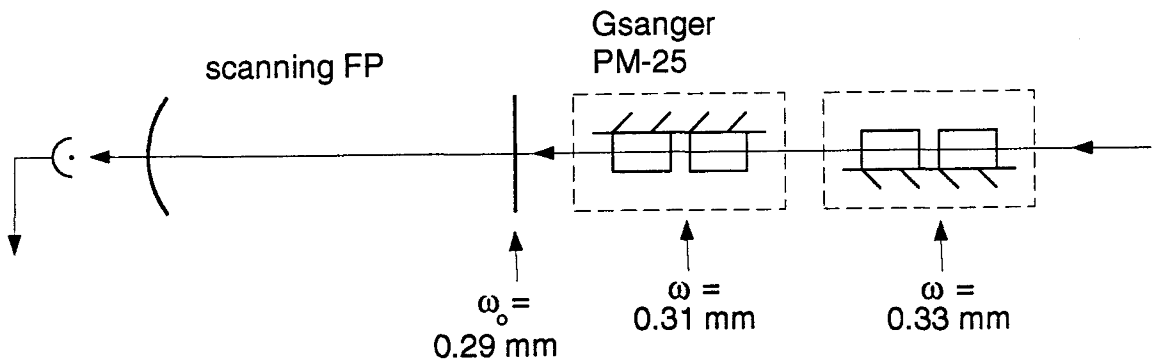

Figure 2: measurement setup

Measurements were made at two modulation frequencies, 25 MHz and 50 MHz. At each frequency, measurements were made with one and two PM-25 cells. When two cells were used, they were mounted so that their heat sinking was on opposite sides, so that the linear phase distortion of the first cell was compensated to some degree by an oppositely signed linear distortion in the second.

The results of these measurements are shown in figures 3 (single PM-25) and 4 (two PM-25s). Even with a single cell, the loss can be low if the angular change is corrected for. In the two cell case, the angle is not completely compensated and some of the 'lost power' can be recovered by realigning. The two cells were driven with separate step-up transformers; the cells were driven out of phase, and the relative drive amplitudes and phases trimmed to null the modulation. The amplitude of the modulation was thus the same for each cell. A nice long term solution would be to connect the cells in parallel and drive them from the same step-up transformer.

By looking at just the angle change of the beam with a quadrant photodiode, we determined that for a given modulation index, the induced output angle using two cells was a factor of 30–50 smaller than the induced angle using a single cell.

This measurement also gave us a measure of the thermal time constant for a cell; it is approximately 10 seconds.

Scalings. The solid lines in figure 3 shows the prediction of (1 – equation 9), fitting for the coefficient of Γ^4. The data show reasonably well the Γ^4 scaling predicted for the lower modulation depths. For the 25 MHz data, the fit gives $(c_1\omega_0/2)^2/\Gamma^4 = 0.0865 \equiv a$; the prediction of eq. 10 is $a = 0.04$ for a loss tangent of $5\text{x}10^{-4}$. For the 50 MHz data, the fit gives$a$ = 1.79, compared with $a = 0.16$ predicted by eq. 10. The discrepancy is not understood, but could possibly be explained by larger values of the loss tangent, and deviations from the assumption that the beam passes through the center of the cell(s).

The scaling with frequency is predicted to go as f^2 at low modulation depths if the loss tangent is independent of frequency. The data for the single Pockels cell, before realignment measurements show roughly a factor of ten difference between 25 and 50 MHz, but closer to a factor of five difference after realignment.

The beam size in the cells was ω=0.31 mm; we did not investigate how the power loss scaled with beam size. If the angle is corrected for, or two cells are used, the loss is predicted to scale as ω^4. The Rayleigh range for the size used here is $z_R = 0.55$ m, so that the beam size is essentially constant as it traverses the cell. A smaller beam size could be used resulting in even smaller loss. A larger beam could also be used depending on the modulation depth, frequency, and number of cells.

With only two frequencies and one beam size, we have not extensively studied the scaling of all the parameters, but we hope there is enough information here to allow one to determine a workable modulation setup for a given set of requirements.

References

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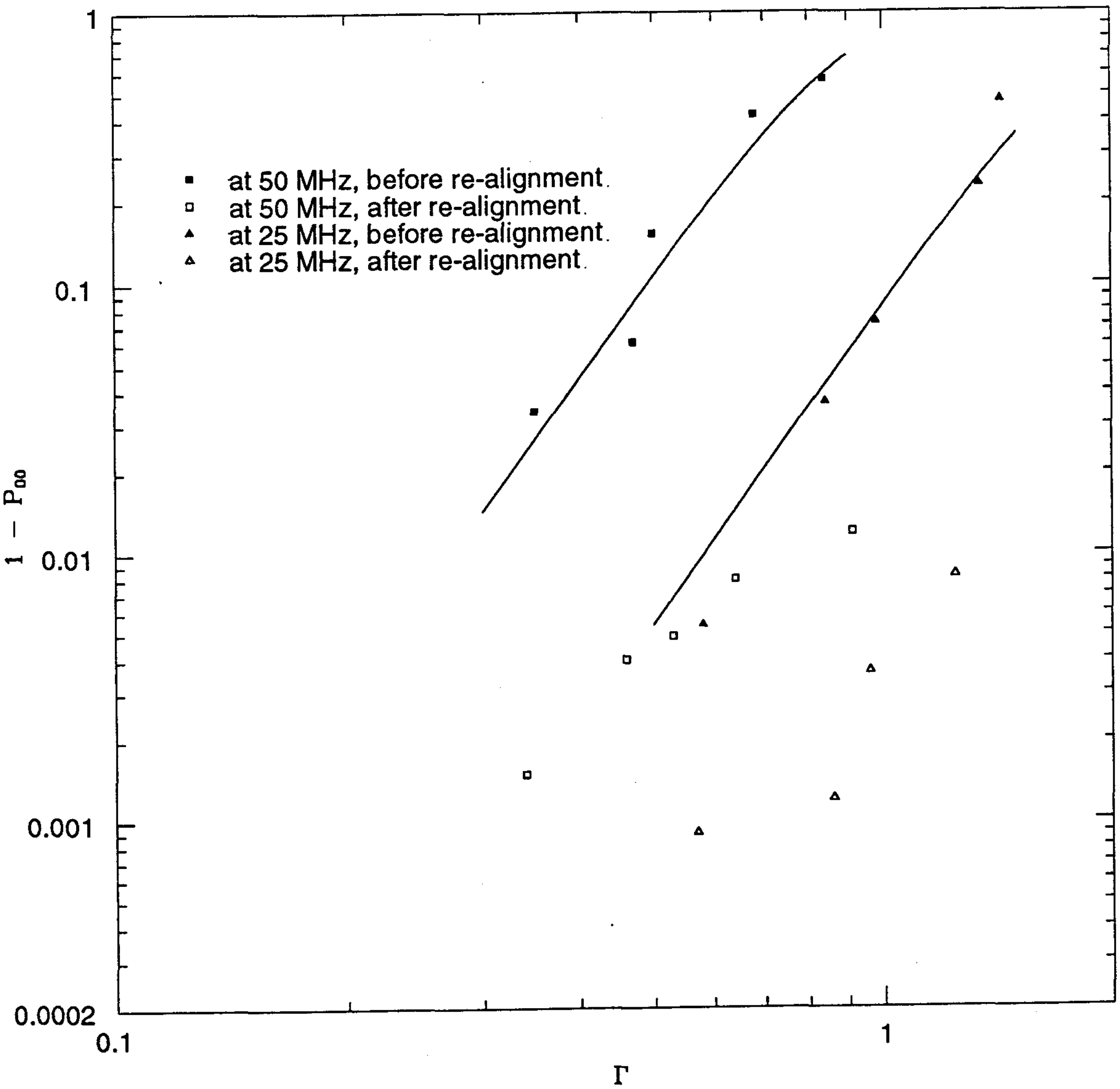

Figure 3. Measurements of the power lost from the fundamental mode as a function of the modulation depth for a single PM-25 and two modulation frequencies. The solid curves are fits to the data, using equation 9 (see text).

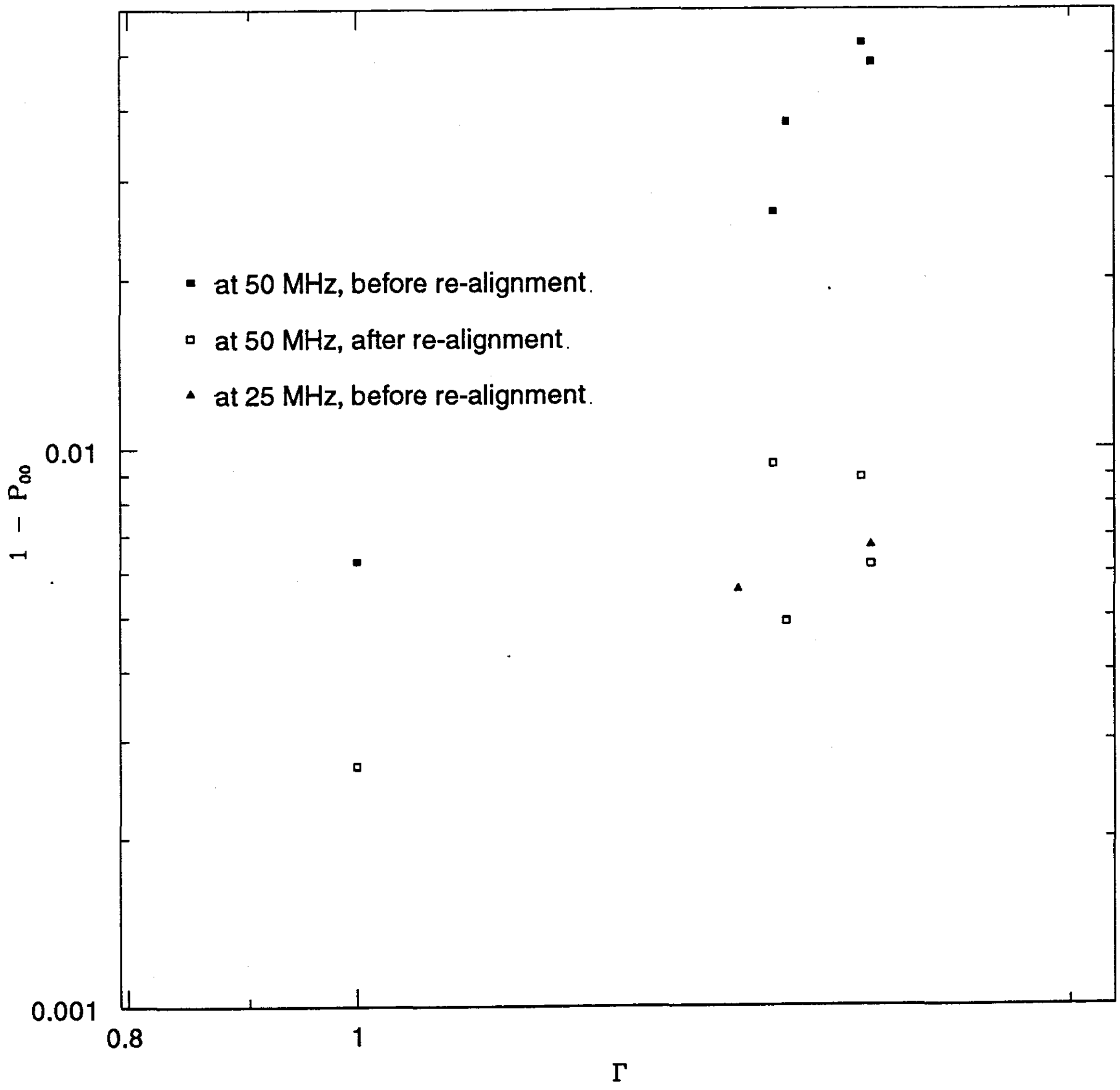

Figure 4. Measurements of the power lost from the fundamental mode as a function of the modulation depth for two PM-25's and two modulation frequencies.