New Folder Name Design of Suspension Systems

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Design of Suspension Systems for Measurement of High Q Pendulums

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Abstract

Laser interferometer gravitational wave detectors require the use of massive ultra high Q test mass pendulums to minimise thermal noise. The verification of ultra high Q-factors in pendulums is difficult due to the coupling of support structure recoil losses into the pendulum stage. We present a numerical analysis of three different pendulum suspension systems to assess their performance. We show that conventional two stage and three stage suspension systems cannot be expected to verify Q-factors beyond 10^8 due to recoil losses, but that a twin pendulum system can allow Q-factors of 10^{10} to be observed.

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1. Introduction

Laser interferometer gravitational wave detectors use widely separated nearly free suspended test masses mirrors to form a Fabry-Perot Michelson interferometer[1-3]. The target strain sensitivity h is of an order of $10^{-23}/\sqrt{Hz}$. Once the seismic noise is lowered sufficiently through the use of high performance vibration isolators, Brownian motion noise at low frequency becomes a critical problem. This arises both from internal modes in the test mass, and from the pendulum mode of the suspension system. In this paper we address the specifically the problem of verification of pendulum losses. The Brownian motion noise amplitude of a simple pendulum at angular frequency ω, for any normal mode at frequency $\omega_0 << \omega$ is given by $\Delta x^2 = 4k_BT\omega_0^2 / MQ\omega^5$ [4], where k_B is the Boltzmann constant, T the temperature, M the effective mass of the system and Q is the quality factor. Of the various parameters in this equation the quality factor is the one which is most easily optimised.

It has been shown that significant improvement of the Q-factor of the test mass pendulum can be achieved if the pendulum is a one dimensional compound pendulum supported by a thin membrane rather than a simple pendulum[5,6]. Using known materials and assuming that Q is independent of frequency, reference [5] shows theoretically that Q-factors exceeding 10^{10} can be achieved. Using wire suspensions Q-factors $>10^8$ should be achieved[7].

Such high pendulum Q-factors are very difficult to measure, because of recoil losses in the pendulum support structure[4]. Relatively heavy pendulums are required for test masses (10kg-100kg) for minimising thermal noise and also for reducing the fundamental uncertainty principle limiting noise. To verify a high Q-factor, it is necessary to have a massive, high Q and high resonant frequency support structure. This is hard to achieve in practice because the support is strongly coupled to the very lossy ground.

Before modelling the losses it is worth elaborating on this problem of the support structure. Our analysis was motivated by observations of resonant mode frequencies and Q-factors of massive steel blocks used as support elements for pendulums. The recoil effect limits the observable pendulum mode Q-factor to a value roughly given by the product of the Q-value of the support, the mass ratio between the support and the pendulum and the frequency ratio between the stages[10]. We quickly learnt that large masses invariably had low resonant modes. We discovered that a concrete floor bedded onto sand in a basement laboratory typically had resonant modes below 50Hz, and any mass placed on this floor was characterised by an even lower resonant frequency. Clearly special facilities, such as massive hard rock foundation or enormous steel structures could be utilised to obtain some advantage but we believe that the limits presented here will not be easy to exceed.

We used a 400kg steel block as a support mass. Although the lowest internal mode frequency was ~ 500Hz, we found no means of mounting it on the laboratory floor where the lowest mode frequency was higher than 30Hz. The Q-factor was about 20. To achieve this it was bedded into wet concrete. With changes in geometry and thicker floors we believe that this frequency could be increased to ~ 100Hz.

In this paper, we investigate various designs for the pendulum support structure. We show that with the above practical constraints it is very difficult to realise a support for a single massive pendulum which will exhibit a Q-factor above 10^8, but that a twin pendulum system can be used to test practical high Q pendulum designs for Q-factors as high as 10^{10}. In section 3 we present preliminary tests on 1kg and 10kg pendulums that are consistent with the theoretical predictions.

2. Practical Design of Suspension Systems for Q Measurement

In this section we model three different pendulum suspension systems. In all cases we assume that the pendulums are compound pendulums supported by thin membranes of negligible length compared with the length of the pendulums, as illustrated in figure 1. It is assume that the support is infinitely stiff in the vertical (z) direction and the membrane hinge provides high rigidity in the transverse (y) direction. The structure is well suited to a one dimensional analysis. The vertical coupling of a pendulum has been analysed by Pitre *et al*[8]. The magnitude of vertical centrifugal losses is negligible compared with the horizontal losses considered here. Only horizontal recoil coupling along x direction is considered.

(a) Two Stage Suspension System

We first consider a pendulum supported by a single stage support. The support can be modelled as a mass spring resonator. The system is shown schematically in figure 2. Assuming the damping mechanism is structural damping, that is, the equivalent damping force $F = (ik/Q)x$ (k is the stiffness, Q is the Q-factor)[9], the linearized equations of the system are ,

$$(m_1+m_0)\ddot{x}_1+m_0L\ddot{\theta}+k_1(1+i/Q_1)x_1=0$$
$$m_0L\ddot{x}_1+(I_c+m_0L^2)\ddot{\theta}+m_0gL(1+i/Q_0)\theta=0 \qquad (1)$$

Here m_0 is the mass of the compound pendulum, L is the distance between the centre of mass and the supporting point of the compound pendulum, I_c is the moment of inertia of the pendulum about the axis along x direction through its centre of mass, m_1 is the equivalent mass of the support and k_1 is its stiffness, Q_0 and Q_1 are the intrinsic Q-factor of the pendulum and the support respectively. Assuming the pendulum is a cylinder, the eigenvalue equation of this system can be obtained from equation (1),

$$\left(1-\frac{1}{(1+\alpha)(4/3+\beta^2/4)}\right)\lambda^4+\left(\omega_0^2(1+i/Q_0)+\frac{\alpha\omega_1^2}{1+\alpha}(1+i/Q_1)\right)\lambda^2 + \frac{\alpha\omega_1^2\omega_0^2}{1+\alpha}(1+i/Q_1)(1+i/Q_0)=0 \quad (2)$$

Here $\alpha = m_1/m_0$, $\beta = r/L$ (r is radius of the pendulum), $\omega_0 = 2\pi f_0 = \sqrt{m_0 gL/(I_c+m_0L^2)}$ and $\omega_1 = 2\pi f_1 = \sqrt{k_1/m_1}$.

The suspension system is characterised by two normal modes. If the natural frequencies ω_1, ω_0 are widely separated, the normal mode frequencies are relatively unperturbed from the natural frequencies, but the normal mode Q-factors are altered significantly. We are concerned specifically with the Q-factor of the pendulum mode. The mode Q-factor is the observable Q-factor of the pendulum and is denoted by Q_p.

By solving λ in equation (2), the pendulum mode Q-factor, Q_p and the pendulum frequency can be obtained. The value of Q_p depends on the frequency and Q-factor of the supporting structure (f_1 and Q_1) and on the mass ratio (α) of the supporting structure and the pendulum. Figure 3 shows Q_p as a function of f_1 for various values of Q_1. It can be seen that increasing the frequency of the supporting structure is most effective for obtaining a high pendulum mode Q-factor, Q_p. Increasing the intrinsic Q-factor of the support is less effective. Numerical results also show that increasing the mass ratio is least effective. The mode Q, Q_p, increases almost linearly with mass ratio as shown in figure 4. This result agrees with results for a simple two stage spring mass system[4,10].

Clearly, to obtain a high value of Q_p requires the highest possible Q_1, high resonant frequency of the support and high mass ratio. However as discussed in section 1, if m_1 is large and α is relatively large (α~40), it is difficult to achieve f_1 > 100Hz and Q_1 >20. The low value of Q_1 arises due to vibrational losses into the floor. For a support with f_1 = 100Hz and Q_1 = 20, and α = 40, the highest attainable value of Q_p is 10^7 as shown in figure 3. If we can reduce the mass of the pendulum to 1kg so that α = 400, Q_p can be increased to 6×10^7 as shown in figure 5. Presumably with large scale engineering, or on a vertical site where massive rock structures are available for a foundation, higher values of Q_1 and f_1 could be achieved.

(b) Three Stage Suspension System

We now investigate a pendulum with two support stages. This is significant for two reasons: (i) To find whether any advantage can be gained using an additional stage, and (ii) because the ground (eg. a building foundation or floor) in practice represents an additional resonant suspension stage. The model for a 3 stage system consisting of two supports and a pendulum is shown in figure 6. One can consider the first stage to represent the lowest normal mode of the foundation, and the second to represent the supporting stage. Again assuming structural damping, the linearized equations of the system are now,

$$
\begin{aligned}
&m_1\ddot{x}_1+(k_1(1+i/Q_1)+k_2(1+i/Q_2))x_1-k_2(1+i/Q_2)x_2=0\\
&(m_2+m_0)\ddot{x}_2+m_0L\ddot{\theta}-k_2(1+i/Q_2)x_1+k_2(1+i/Q_2)x_2=0\\
&m_0L\ddot{x}_2+(I_c+m_0L^2)\ddot{\theta}+m_0gL(1+i/Q_0)\theta=0
\end{aligned}
\qquad (3)
$$

Here m_0 is the pendulum mass, I_c is the moment of inertia of the pendulum about its centre of mass, m_1 and m_2 are the equivalent masses of the first and second stages respectively and k_1 and k_2 are their stiffness coefficients. The intrinsic Q-factors of the first and second supporting stages and the pendulum are Q_1, Q_2 and Q_0 respectively. For a cylindrical pendulum, the eigenvalue equation of system can be obtained from equation (3),

$$
\begin{aligned}
&(\lambda^2+\omega_1^2(1+i/Q_1)+\frac{\alpha_2}{\alpha_1}\omega_2^2(1+i/Q_2))(\lambda^2(1+1/\alpha_2)+\omega_2^2(1+i/Q_2))\bullet\\
&(\lambda^2+\omega_0^2(1+i/Q_0))-\frac{1/\alpha_2}{4/3+\beta^2/4}\lambda^4(\lambda^2+\omega_1^2(1+i/Q_1)+\frac{\alpha_2}{\alpha_1}\omega_2^2(1+i/Q_2))\\
&+\frac{\alpha_2}{\alpha_1}\omega_2^4(1+i/Q_2)^2(\lambda^2+\omega_0^2(1+i/Q_0))=0
\end{aligned}
\qquad (4)
$$

where $\alpha_1=m_1/m_0$, $\alpha_2=m_2/m_0$, $\beta=r/L$, $\omega_0=2\pi f_0=\sqrt{m_0gL/(I_c+m_0L^2)}$, $\omega_1=2\pi f_1=\sqrt{k_1/m_1}$, and $\omega_2=2\pi f_2=\sqrt{k_2/m_2}$.

By solving equation (4), we can investigate the effect of the frequencies and Q-factors on the mode Q of the pendulum, Q_p.

Solutions of equation (4) are given in figures 7 and 8. Results show that changing the parameters of the second stage alone cannot raise the limit of the mode Q-factor of the pendulum, Q_p, very much, as shown in figure 7. However increasing the first stage frequency is more effective in obtaining a higher value of Q_p as show in figure 8. A higher value of Q_1 will result in a higher Q_p but this is not as effective as increasing f_1.

To attain a high value of Q_p we again require the highest possible Q-factors and resonant frequencies of the supporting structures. Assuming a 10kg pendulum and a supporting system with $Q_1 = 20$, $Q_2 = 100$, $f_1 = 100$Hz, $f_2 = 100$Hz, $\alpha_1 = 40$ and $\alpha_2 = 10$, the highest value for Q_p is 7×10^6 as shown in figure 8.

Making the mass ratio α_1 and α_2 larger can increase Q_p. If the pendulum mass is reduced to 1kg, $\alpha_1 = 400$ and $\alpha_2 = 100$, $Q_1 = 20$, $Q_2 = 100$, $f_1 = 100$Hz, $f_2 = 100$Hz, pendulum mode Q of

4×10^7 can be obtained as shown in figure 9. These results show that the three mode system is generally inferior to the two mode system. The floor will always contribute at least one additional mode, and in general it will have a set of normal modes. Hence one can expect pendulum Q-factors to be generally lower than predicted by simple lumped parameter models of the sort used here.

Considering that pendulums ~100kg need to be tested for gravitational wave detectors, the above results emphasise the difficulty in testing such high Q systems. Much better results can be achieved, however, if a twin pendulum system is used.

(c) Twin Pendulum System

To observe high pendulum Q-factors, we propose the use of a twin pendulum as shown in figure 10. The differential mode of two identical pendulums will have very small losses coupled through the supporting structure. Suppose two pendulums are suspended on a support as shown in figure 10. Assuming structural damping, the linearized equations of the system are,

$$\begin{aligned} &(m_1 + m_{01} + m_{02})\ddot{x}_1 + m_{01}L_1\ddot{\theta}_1 + m_{02}L_2\ddot{\theta}_2 + k_1(1+i/Q_1)x_1 = 0 \\ &m_{01}L_1\ddot{x}_1 + (I_{c1} + m_{01}L_1^2)\ddot{\theta}_1 + m_{01}gL_1(1+i/Q_{01})\theta_1 = 0 \\ &m_{02}L_2\ddot{x}_1 + (I_{c2} + m_{02}L_2^2)\ddot{\theta}_2 + m_{02}gL_2(1+i/Q_{02})\theta_2 = 0 \end{aligned} \qquad (5)$$

Here, m_1 is the equivalent mass and k_1 the stiffness of the supporting structure, and Q_1 is its intrinsic Q-factor. If the pendulums are identical, we have pendulum masses $m_{01} = m_{02} = m_0$, distances from supporting points to the centres of mass $L_1 = L_2 = L$, moment of inertias about the centres of mass of the pendulums $I_{c1} = I_{c2} = I_c$, and the intrinsic Q factors of the pendulums $Q_{01} = Q_{02} = Q_0$. The eigenvalue equation of the system can be obtained from equation (5),

$$\begin{aligned} &(((m_1+2m_0)\lambda^2+k_1(1+i/Q_1))\,((I_c+m_0L^2)\lambda^2+m_0gL(1+i/Q_0))-2m_0^2\,L^2\lambda^4) \bullet \\ &((I_c+m_0L^2)\lambda^2+m_0gL(1+i/Q_0)) = 0 \end{aligned} \qquad (6)$$

From equation (6), the following equation can be obtained,

$$(I_c+m_0L^2)\lambda^2+m_0gL(1+i/Q_0) = 0 \qquad (7)$$

Equation (7) shows that one mode of the twin pendulum suspension can have a Q-factor equal to Q_0, the intrinsic Q of the pendulums. This mode is the differential mode, with Q-factor denoted Q_-.

In practice the intrinsic Q of the support is much less than that of the pendulum. So when the coupling between pendulum and support is very strong the common mode of the system will decay more quickly than the differential mode. This is useful in quickly eliminating the signal from the common mode. Generally, a lower support frequency and a smaller mass ratio of support and pendulum leads to stronger coupling. Note however that from a thermal noise point of view, the low Q common mode will strongly couple thermal noise from previous stages.

In reality the pendulums will not be exactly identical. We have investigated numerically the effect of differences between the two pendulums on the value of Q_-. The results show that there is a negligible effect from differences in mass ratios ($\Delta\alpha$), ratios of radius to length ($\Delta\beta$) and intrinsic Q-factors (ΔQ) of the two pendulums. This result follows from conservation of linear momentum. However differences between the resonant frequencies of the two pendulums can significantly degrade Q_-. This result is illustrated in figure 11.

For reasonable parameters and frequency differences in the range of 0.01% ~ 1% f_0, it is easy to achieve Q_- up to 10^9, or even 10^{10} when the intrinsic Q is 10^{10}. Specifically, for a support structure with frequency f_1=2Hz, intrinsic Q-factor $Q_1 = 2000$ and mass ratio m_1/m_0=1, we expect to obtain $Q_- \sim 10^9$ when the intrinsic Q-factor of the pendulum $Q_0 = 10^{10}$ and Δf=0.1%f_0 as shown in figure 11. A ten-fold increase in the mass ratio allows Q-factors ~10^{10} to be observed.

There exist other practical problems. For example, suppose the axes of two pendulums are not exactly parallel. This will cause a vibration component perpendicular to the direction of pendulum vibration, introducing extra energy losses. It can be shown that the differential mode Q of the pendulum is expected to be limited to 6×10^5 for nonparallelism of 0.002rad and frequency difference of 0.1% f_2 when f_1 = 1Hz, Q_1 = 2000, $Q_0 = 10^{10}$ and m_1/m_0 =1. Another mechanical problem is the resonant seismic excitation of the pendulum, and the long decay time. High resolution amplitude measurements are needed to resolve a Q-factor of 10^9 in a reasonable period of time, since the decay time is 30 years.

3. Experimental results and comparison with theoretical analysis

We now report experimental results on a single compound pendulum supported by a massive second stage. The experimental set up is shown in figure 12 . The pendulum was suspended by a foil which is held by two pairs of brass clamps. One clamp was attached to the top of the pendulum and the other was attached to an aluminium disc which is clamped to the steel support. The support was built into the concrete of the floor. The Q-factor of the supporting stage is about 20.

A simple optical transducer was used to monitor the pendulum motion. A razor blade was fixed to the pendulum (parallel to the direction of swing). The blade cut the light beam between a small diode laser and photodiode. The signal from the photodiode was amplified and analysed by a computer.

The mode Q of a 9kg pendulum using three different foils are listed in table I. The mode Q-factors of the pendulum with brass and titanium foils are far below the expected intrinsic Q-factors, Q_0. The intrinsic Q-factor of these pendulums in column 6 have been calculated assuming an intrinsic loss in the foil, based on the analysis of reference [5]. Although the expected intrinsic Q-factors are quite different, the measured mode Q-factors are very similar because the measured mode Q-factors are limited by the recoil effects discussed here. Only the low Q mylar foil is lossy enough for the observed Q_p to be comparable to the expected intrinsic pendulum Q-factor Q_0.

Table I Comparison of theoretical prediction and experimental results for various pendulums

mass (kg)	foil material	thickness (mm)	observed Q_p	predicted limiting Q_p 2 stage	predicted limiting Q_p 3 stage	Intrinsic pendulum Q_0	Assumed material Q
9	mylar	0.3	2.4×10^4	4.4×10^5	3.9×10^4	2.5×10^4	10^2
9	brass	0.07	8.8×10^4			1.3×10^7	10^4
9	titanium	0.025	9.2×10^4			2.6×10^8	10^4
1	titanium	0.025	3.5×10^5		5.5×10^5	2.9×10^7	10^4

The expected theoretical limiting mode Q-factors of the pendulum (column 5) using two and three stages models assumes negligible intrinsic pendulum losses ($Q_0 = 10^{10}$). It appears that the two stage model significantly under estimate the recoil losses, while the three stage model over estimates the losses. The latter probably arises because of the difficulty in determining the appropriate lumped parameter by which to model to laboratory floor.

The dependence of Q-factor on the mass ratio is also shown in table I. A 1kg pendulum, with a suspension foil of 0.025mm titanium was used instead of the 9kg pendulum. As expected, the mode Q-factor of this small pendulum is greater than that of a 9kg pendulum. The observed Q was 3.5×10^5, somewhat below the 5.5×10^5 Q-factor expected from the 3 stage model.

4. Conclusion

We have shown that to obtain a high measurable mode Q-factor of a single compound pendulum, it is necessary to have the highest possible Q-factors and resonant frequencies of the supporting structures. Due to recoil losses, a massive single compound pendulum is unlikely to achieve a Q-factor above 10^7.

Adding an additional supporting stage to a two stage pendulum suspension further degrades the mode Q-factor of the pendulum.

Experimental results confirm that increasing the mass ratio between the supporting stage and the pendulum increases the mode Q of the pendulum. Experimental results also confirm that a three stage suspension model, for which the first stage models the resonant properties of the floor structure, is more consistent with observation than a model which ignores the compliance of the foundation.

The twin pendulum suspension has been.shown to allow the observation of much higher Q-factors. For pendulums tuned within $0.1\%f_0$, Q-factors exceeding 10^9 should be observable. However the resolution of such a high Q-factor in a reasonable period of time may be difficult.

In future we intend to investigate the twin pendulum experimentally with view to determining the high intrinsic Q-factor of pendulums with various suspension foils.

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Figure Captions

Fig.1 Schematic diagram of a compound pendulum. A thin metal foil supports the pendulum and acts as a low stiffness hinge to obtain single axis oscillator.

Fig.2 Model of a two stage suspension system consisting of a mass-spring support stage and a compound pendulum. Structural damping is assumed in this system.

Fig.3 Mode Q-factor of a pendulum (Q_p) as a function of the support stage frequency (f_1) for a range of support stage Q-factors, $Q_1 = 5, 20, 100, 400$, with $\alpha = m_1/m_0 = 40$, $Q_0=10^{10}$, $f_0=1$Hz.

Fig.4 Mode Q-factor of a pendulum (Q_p) as a function of the pendulum intrinsic Q-factor (Q_0) for a range of the mass ratios, $\alpha = m_1/m_0 = 10, 20, 30, 40$, with $Q_1 = 20$, $f_1 = 20$Hz, $f_0=1$Hz.

Fig.5 Mode Q-factor of a pendulum (Q_p) as a function of the supporting stage frequency (f_1) for a range of supporting stage Q-factors, $Q_1 = 5, 20, 100, 400$, with a larger mass ratio $\alpha=m_1/m_0 = 400$, and $Q_0=10^{10}$, $f_0 = 1$Hz.

Fig.6 Model of a three stage suspension system consisting of two mass-spring supporting stages and a compound pendulum. Structural damping is assumed.

Fig.7 Mode Q-factor of a pendulum (Q_p) (in a three stage system) as a function of the second support stage frequency (f_2) for a range of the second supporting stage Q-factors, $Q_2 = 5, 20, 100, 400$, with $\alpha_1 = m_1/m_0 = 40$, $\alpha_2 = m_2/m_0 = 10$, $Q_1 = 20$, $f_1 = 20$Hz.

Fig.8 Mode Q-factor of a pendulum (Q_p) (in a three stage system) as a function of the first support stage frequency (f_1) for a range of the first supporting stage Q-factors, $Q_1 = 5, 20, 100, 400$, with $\alpha_1 = m_1/m_0 = 40$, $\alpha_2 = m_2/m_0 = 10$, $Q_2 = 100$, $f_2 = 100$Hz.

Fig.9 Mode Q-factor of a pendulum (Q_p) as a function of the first support stage frequency (f_1) for a range of the first supporting stage Q-factors, $Q_1 = 5, 20, 100, 400$, with larger mass ratio $\alpha_1=m_1/m_0=400$, $\alpha_2 = m_2/m_0= 100$, and $Q_2 = 100$, $f_2 = 100$Hz.

Fig.10 Model of a twin pendulum suspension system consisting of a mass-spring support stage and two compound pendulums. Structural damping is assumed.

Fig.11 Differential mode Q-factor (Q_-) of the twin pendulum as a function of the frequency difference of the pendulums (Δf) for a range of the support stage Q-factors, $Q_1 = 20, 200, 800, 2000$, and $m_1/m_0=1$, $Q_0 = 10^{10}$, $f_0 = 1$Hz, $f_1 = 2$Hz.

Fig. 12 Experimental set up for pendulum Q-factor measurements. A massive steel block (400kg) supported by three strong legs is supported on an Al plate which is bedded into a concrete foundation cast onto a concrete floor laid on sand. The pendulum was mounted from heavy steel brackets on the side of the steel block. Motion was monitored by an optical shadow sensor.

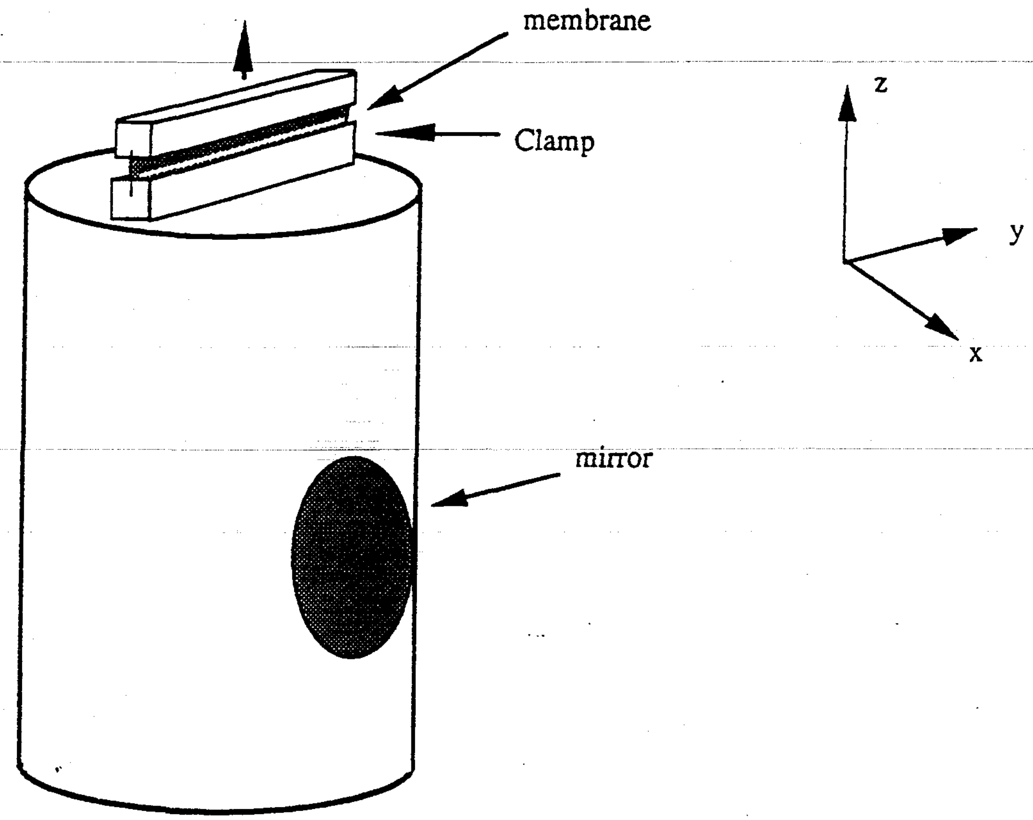
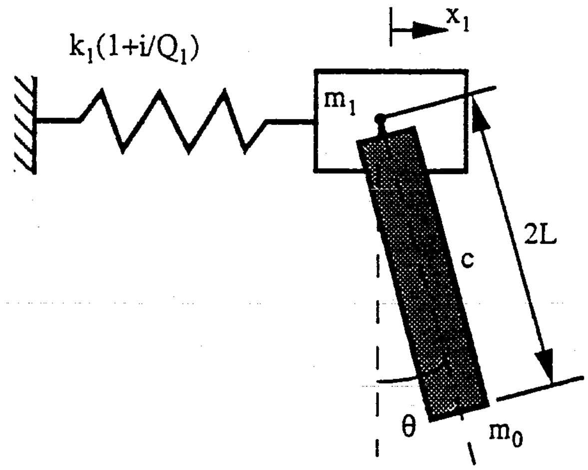
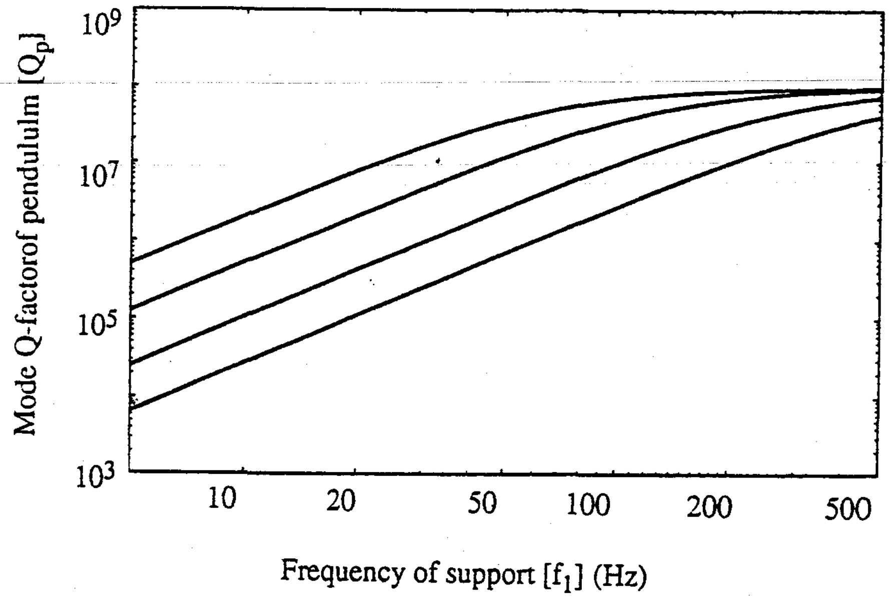


Fig. 1 compound pendulum

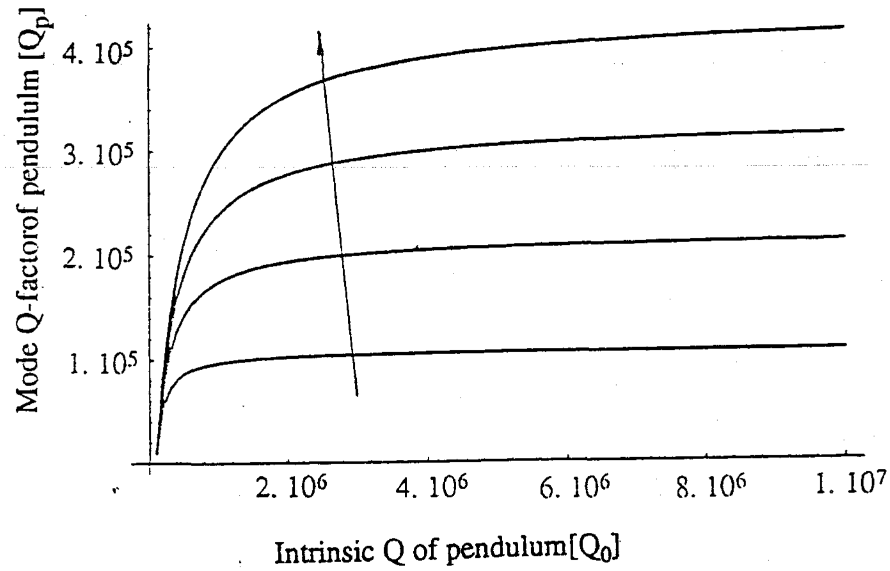


x1
k1(1+i/Q1)
m1
2L
c
θ
m0

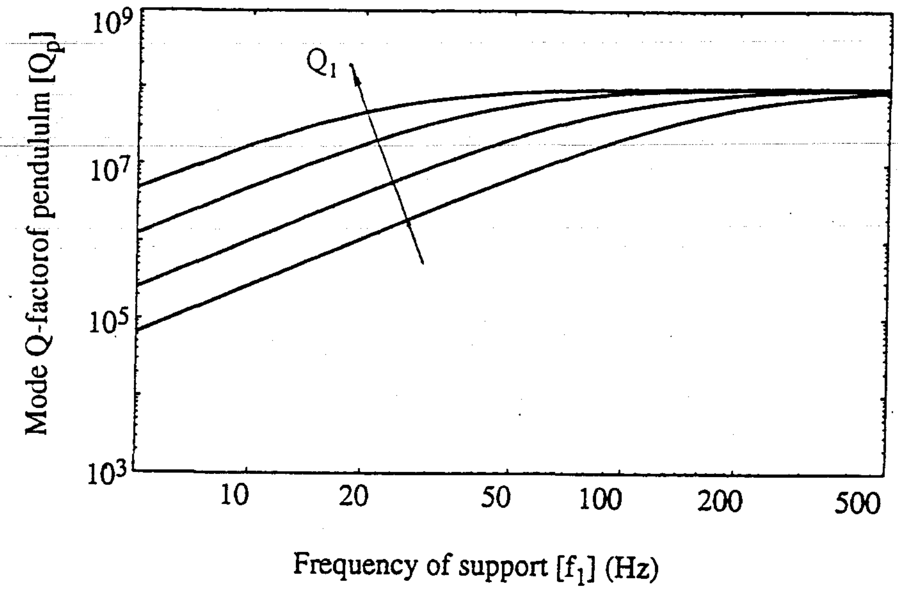


Mode Q-factorof pendululm [Q_p]
10^9
10^7
10^5
10^3
10
20
50
100
200
500
Frequency of support [f_1] (Hz)

Fig 3

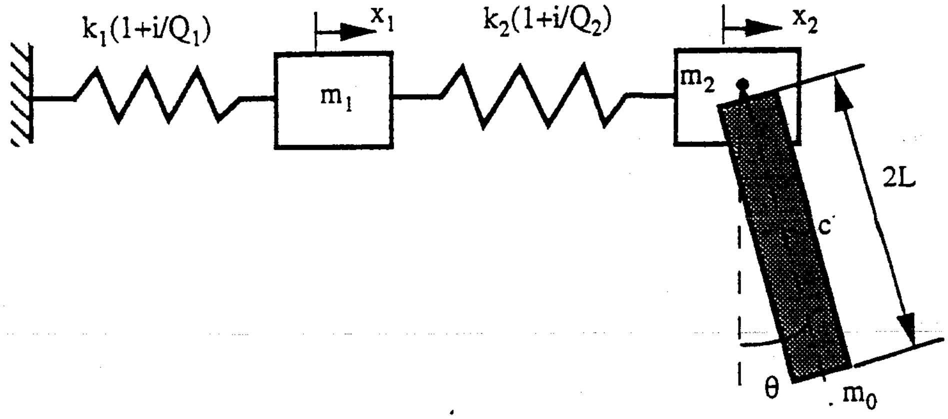


Mode Q-factorof pendululm [Qp]
4. 10^5
3. 10^5
2. 10^5
1. 10^5
2. 10^6
4. 10^6
6. 10^6
8. 10^6
1. 10^7
Intrinsic Q of pendulum[Q0]

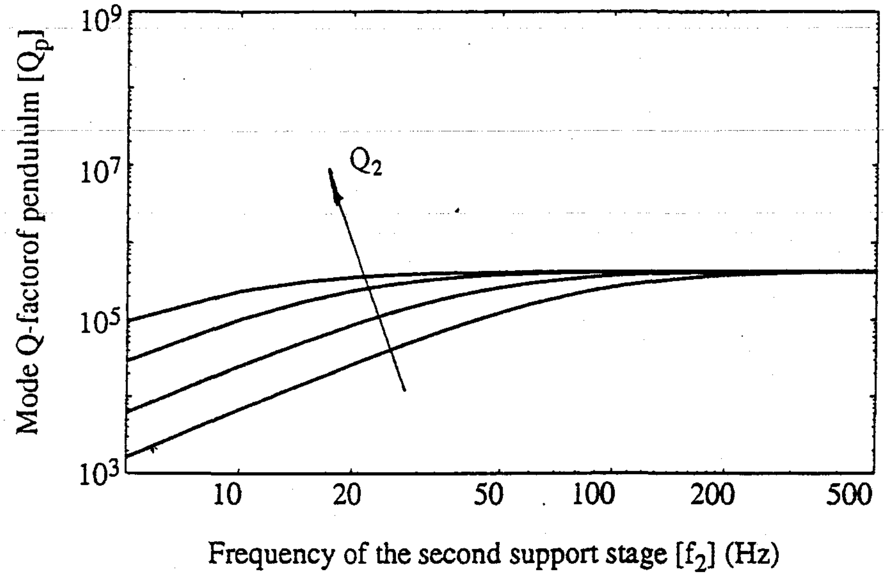


Mode Q-factorof pendululm [Q_p]
10^9
10^7
10^5
10^3
Q_1
10
20
50
100
200
500
Frequency of support [f_1] (Hz)

Fig 5

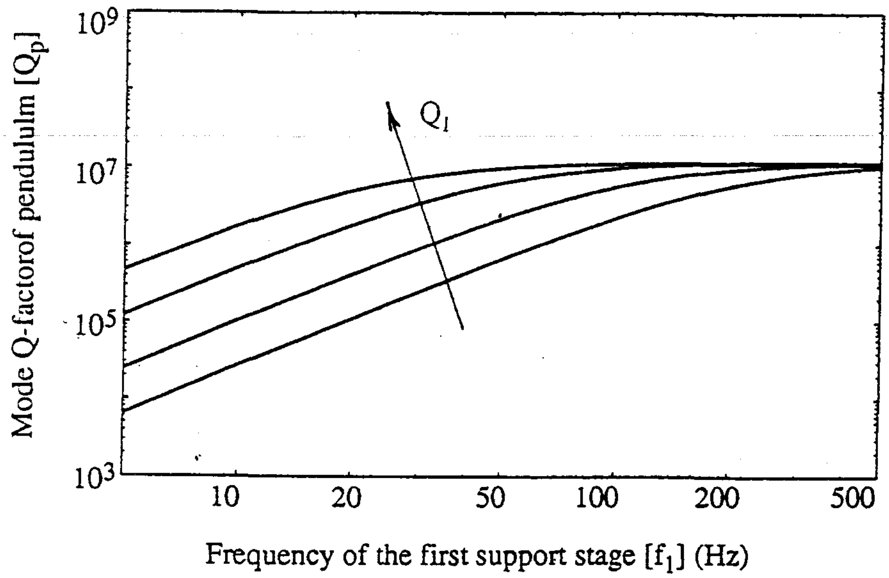


$k_1(1+i/Q_1)$
x_1
m_1
$k_2(1+i/Q_2)$
x_2
m_2
2L
c
θ
m_0



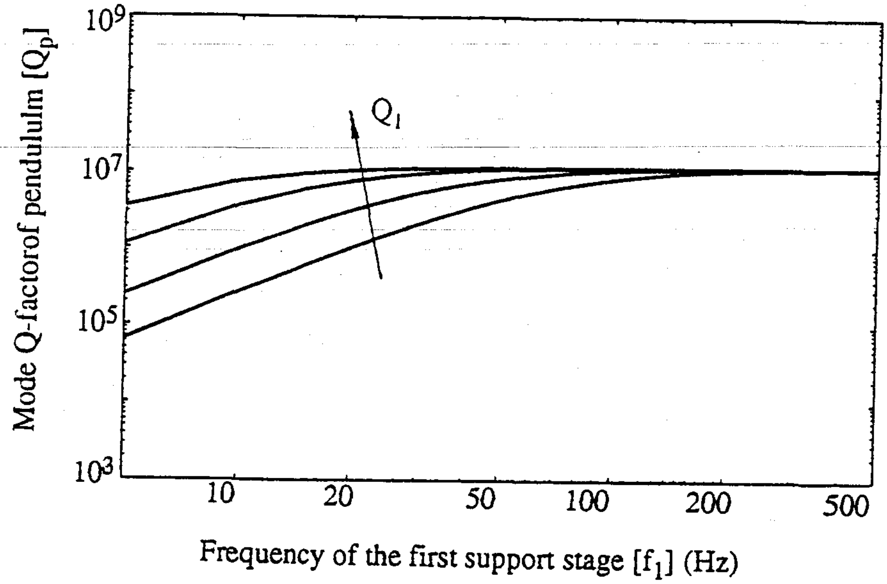
Mode Q-factorof pendulum [Q_p]
10^9
10^7
10^5
10^3
Q_2
10
20
50
100
200
500
Frequency of the second support stage [f_2] (Hz)

Fig 7



Mode Q-factorof pendululm [Qp]
10^9
10^7
10^5
10^3
Q_l
10
20
50
100
200
500
Frequency of the first support stage [f1] (Hz)

Fig 8



Mode Q-factorof pendululm [Q_p]
10^9
10^7
10^5
10^3
Q_l
10
20
50
100
200
500
Frequency of the first support stage [f_1] (Hz)

Fig 9

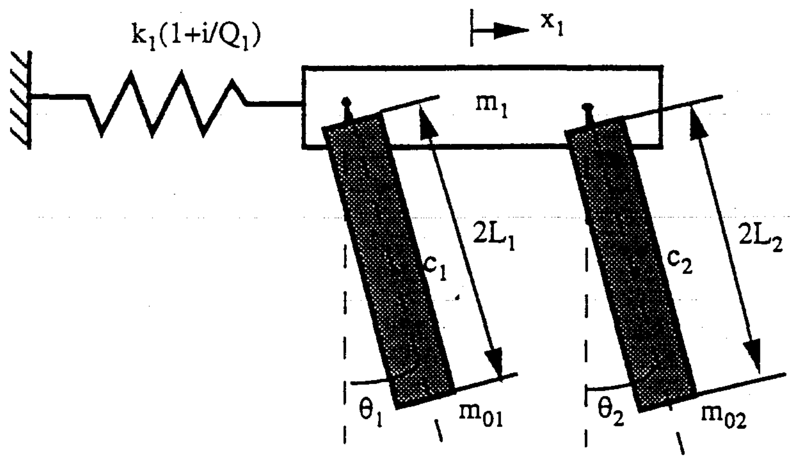


Fig 10

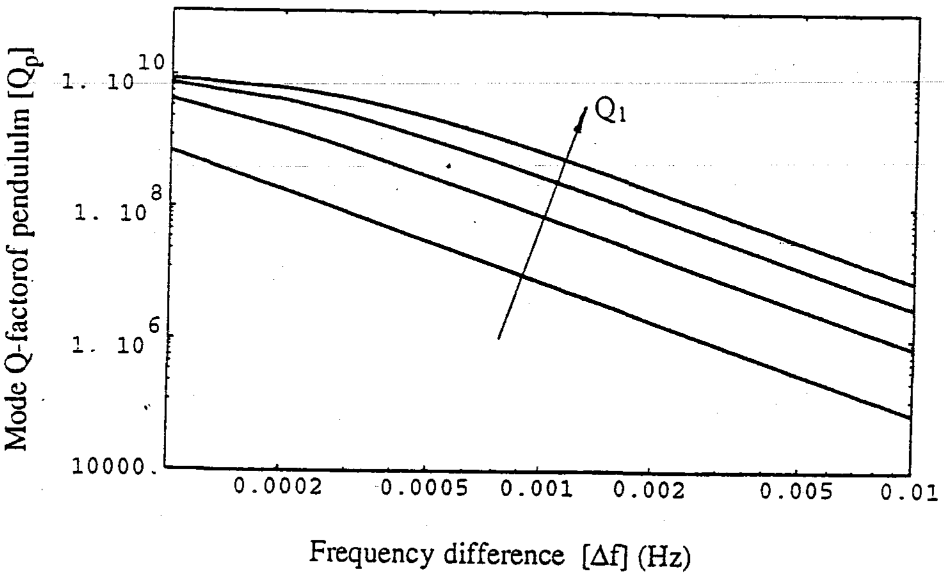

Mode Q-factorof pendululm [Q_p]
1. 10^10
1. 10^8
1. 10^6
10000.
0.0002
0.0005
0.001
0.002
0.005
0.01
Q_1
Frequency difference [Δf] (Hz)

Fig 1c

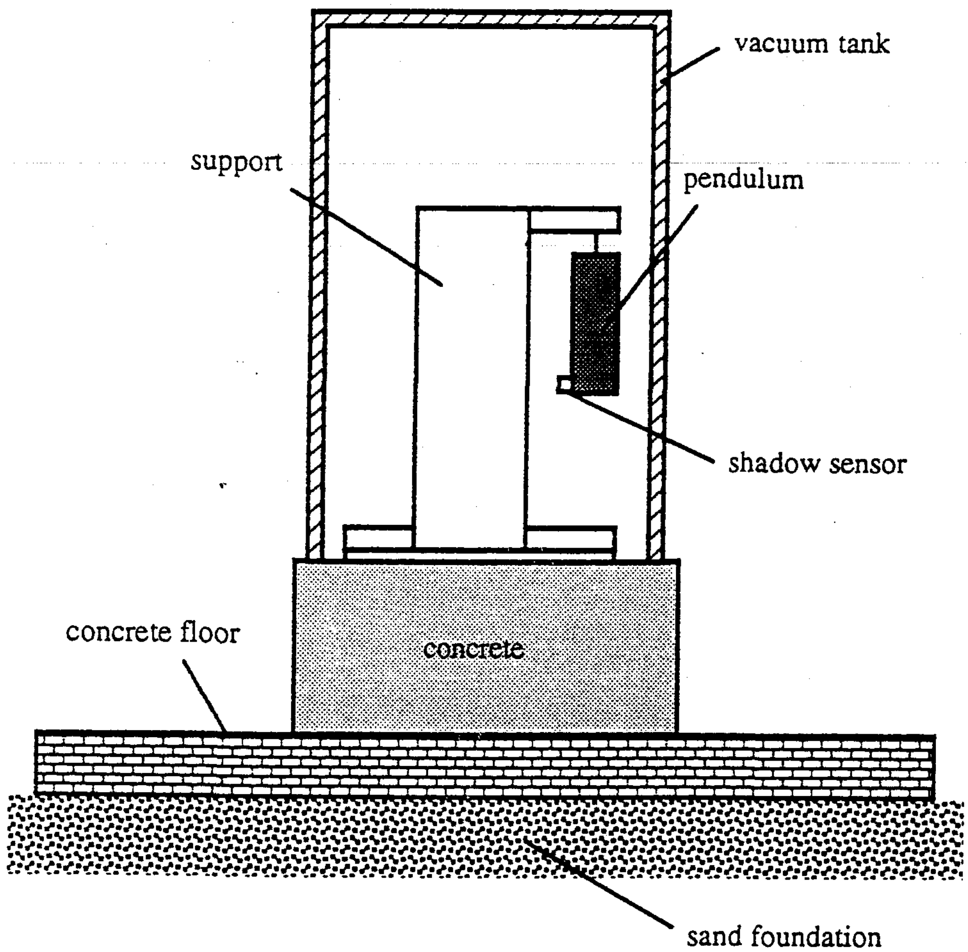

vacuum tank
support
pendulum
shadow sensor
concrete floor
concrete
sand foundation

Fig 12