

**New Folder Name External Phase Modulation**

---

"External Phase Modulation Interferometry," by Malcolm B. Gray, Andrew J. Stevenson,  
Charles C. Harb, Hans-A. Bachor, David E. McClelland, July 12, 1994

---

Distributed by S. Whitcomb 7/22/94

PLEASE PASS ALONG PROMPTLY

COPY 1

A. Abramovici  
J. Camp  
J. Carri  
D. Durance  
A. Gillespie  
S. Kawamura  
A. Kuhnert  
T. Lyons

COPY 2

J. Mason  
R. Savage  
V. Schmidt  
L. Sievers  
R. Spero  
M. Zucker

cc: B. Barish  
F. J. Raab  
M. Regehr  
R. Vogt  
S. Whitcomb -original  
MIT Science Group  
Project File  
Science File



July 12, 1994

# External Phase Modulation Interferometry

Malcolm B Gray, Andrew J Stevenson, Charles C Harb  
Hans-A Bachor, David E McClelland

Department of Physics and Theoretical Physics  
Faculty of Science  
The Australian National University  
Canberra ACT 0200  
Australia

## 1. Introduction

In this report, we investigate the performance and operational requirements of a generic external modulation scheme, in which the output of a Michelson interferometer is coherently combined with a reference beam tapped off the input beam. This constitutes a Michelson interferometer within a Mach-Zehnder interferometer.

In Section 2 we analyse our generic model and derive the expected signal-to-noise performance, accounting for non-ideal features one might expect in practice. We find that both electronic noise in the detectors and imperfect fringe contrast in the Michelson interferometer increase the required optical power tap-off fraction, and reduce the achievable sensitivity. By contrast, we find that the optimum phase modulation depth in the local oscillator arm and the required DC optical phase settings for both the Michelson and local oscillator arms are fixed, regardless of interferometric fringe visibility or detection system quality.

We find that the ultimate achievable sensitivity in an ideal external modulation system is comparable with that of internal modulation, and that both schemes (for different reasons) suffer about a 33% sensitivity penalty compared to ideal shot-noise-limited direct detection, when simple modulation and demodulation waveforms are used.

Section 3 describes our optical and electronic configuration, including technical considerations affecting the design, such as the need to achieve reasonable Gaussian beam matching throughout the interferometer, to adjust the optical power tap-off fraction into the local oscillator beam, and to compensate vibrational disturbances in the interferometer with piezo-controlled mirrors. Our

layout is designed to match the theoretical model as far as possible, to confirm the analytic predictions.

Section 4 describes our experiments, performed to measure the achievable interferometric sensitivity, to compare the results to theoretical predictions, and to investigate aspects of the technical design of practical interferometers which do not fall into the realm of an analytic model. Our Michelson interferometer has a high contrast ratio ( $\sim 600:1$  in some instances) while the equivalent ratio in the Mach Zehnder is more like 100:1 when optimised. We find when using an input optical power of around 9mW, that electronic noise arising in our detectors and amplifiers is the dominant noise component dictating the optimal power splitting fraction and the ultimate sensitivity. In our measurements, electronic noise is greater by a factor of  $\sim 60$  than the shot noise associated directly with the dark fringe transmission from the Michelson. This indicates that the Michelson could have a fringe contrast as poor as 10:1, or the laser power could be as high as 500 mW, before the dark fringe shot noise figures as prominently as electronic noise.

The main feature dictating our suppression of laser technical noise is our ability to lock the Michelson interferometer to its dark fringe and reduce the residual RMS phase deviations. By means of an integrating servo loop with a 5 kHz response bandwidth, we are able to suppress acoustic vibrations in the Michelson interferometer sufficiently to eliminate laser intensity noise (including the strong relaxation oscillation peaks) and thus enable shot-noise-limited sensitivity down to  $\sim 100$  kHz, where laser frequency fluctuations remained the dominant noise contribution. These frequency fluctuations couple into the signal intensity spectrum due to the optical path length mismatch inside the Michelson interferometer.

Section 5 highlights our principal findings and conclusions.

## 2. External modulation interferometry : A generic model

### 2.1 Motivations for using external modulation

The optical power emerging from a Michelson interferometer fluctuates due to changes in the effective optical path difference between the two interferometer arms, and also due to input intensity fluctuations such as technical noise of the laser source. The desired signal from the interferometer may be completely masked by the technical laser noise, even if the detected optical power is relatively low.

Combining the Michelson output with a coherent phase-modulated beam at a beam combiner, as depicted in Fig. 1, will result in a homodyne photocurrent containing an additional intensity modulation at the applied phase modulation frequency, in general, as well as components at the sum and difference of the modulation and signal frequencies. These sum and difference frequency components will yield the required signal information after demodulation. Technical noise can be

nulled out in the same process by selecting the correct interferometer and local oscillator phases.  
 We now demonstrate analytically how this works.

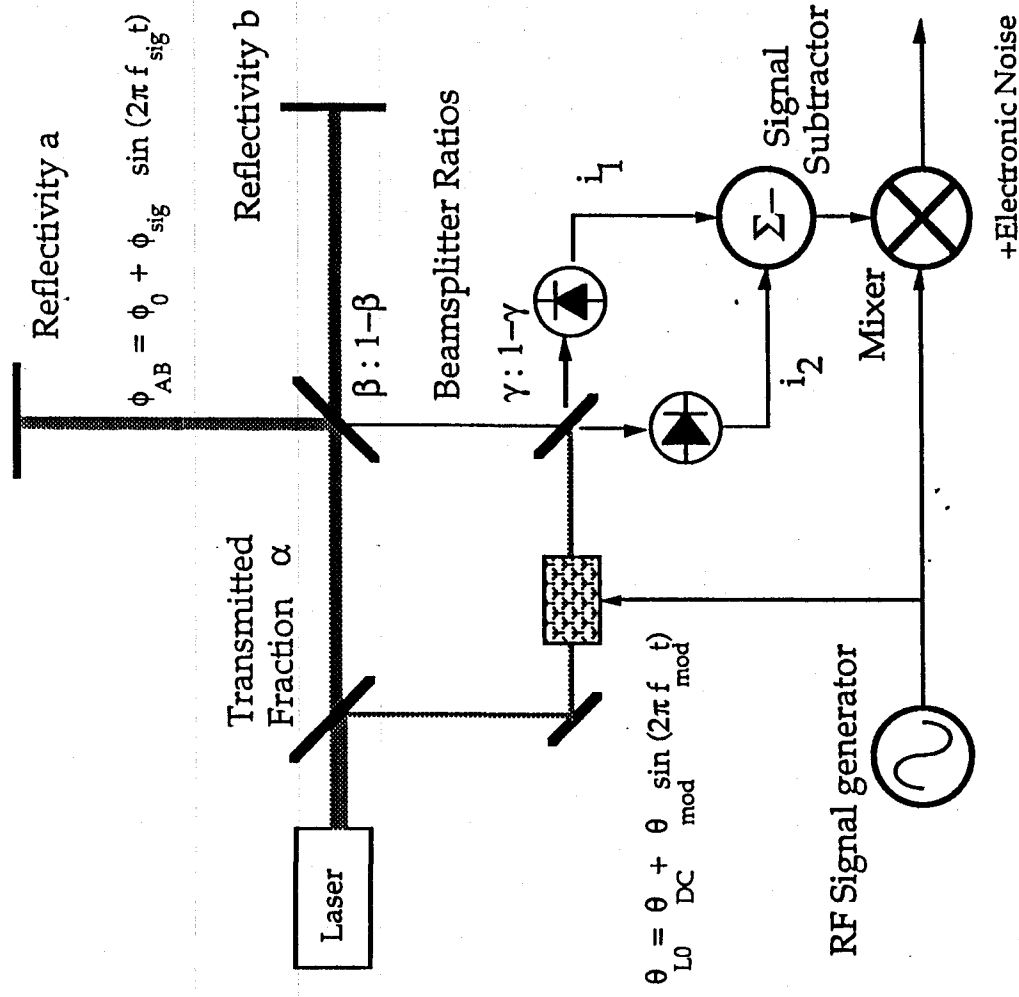


Figure 1: Schematic plan of a generic external modulation interferometer.

## 2.2 Optical fields in the interferometer

In this analysis, we will assume perfect matching of all beams in the interferometer, ie. all beams have the same wavefront curvature at points of interference, ie. at the detectors, and all transverse field distributions are identical at the photodetectors. We also assume that the entire optical field is detected at each photodetector. In this model, therefore, all fields are parametrised by just two quantities - *Amplitude* and *Phase*.

We model changes in the arm length difference in the interferometer by equal and opposite phase changes in the two arms  $\phi(t)/2$ . The input light is associated with an electric field amplitude  $E$ , and is split into two beams at a simple lossless beamsplitter (loss is modelled by assuming a smaller effective input field  $E$ ). The interferometer input field has amplitude:

$$E_{mi} = \alpha^{1/2} E \quad (1)$$

where  $\alpha$  is the (power) transmissivity of the tapoff beamsplitter. An ideal passive beamsplitter introduces a relative phase of  $\pi/2$  between the two output fields, so we may write the local oscillator field emerging from the beamsplitter in complex notation:

$$E_{l0} = i(1 - \alpha)^{1/2} E \quad (2)$$

We now trace both beams through the apparatus to determine what field amplitudes will add at the detectors after the final beam combiner.

The interferometer input field is split at a 50:50 beam-splitter (losses subsumed into the non-ideal, end mirror reflectivities), propagates along each arm of the Michelson to a non-ideal mirror, and back again, recombining at the beamsplitter to produce an output field

$$E_{mo} = i\alpha^{1/2} E (e^{i\phi/2} \sqrt{a} + e^{-i\phi/2} \sqrt{b}) / 2 \quad (3)$$

where  $a$  and  $b$  are the power reflectivities of the two end mirrors in arm A and arm B respectively. The factor of  $i$  signifies the  $\pi/2$  phase shift incurred by each component undergoing one transmission and one reflection at the beamsplitter to get from the interferometer input to its output.

The electric fields incident on the two photodetectors are then a coherent sum of the Michelson output field and the modulated local oscillator field, taking into account phase shifts incurred in the local oscillator arm ( $\theta = \theta(t)$ , due to the combined effects of the external arm phase modulator and the DC optical path difference) and in the final beam combiner ( $\pi/2$ ):

$$E_1 = (E_{mo} + iE_{l0}e^{i\theta}) / \sqrt{2} \\ = (E/\sqrt{2}) [i\alpha^{1/2} E (e^{i\phi/2} \sqrt{a} + e^{-i\phi/2} \sqrt{b}) / 2 - (1 - \alpha)^{1/2} e^{i\theta}] \quad (4a)$$

$$E_2 = (iE_{mo} + E_{l0}e^{i\theta}) / \sqrt{2} \\ = (E/\sqrt{2}) [-\alpha^{1/2} (e^{i\phi/2} \sqrt{a} + e^{-i\phi/2} \sqrt{b}) / 2 + i(1 - \alpha)^{1/2} e^{i\theta}] \quad (4b)$$

### 2.3 Photocurrents

These fields produce photocurrents proportional to the square of their magnitude. We define the detector responsivity  $\rho$  in terms of the induced photocurrent  $I(t)$  due to an incident electric field  $E(t)$ , assuming detector response much faster than the fastest intensity fluctuations:

$$I(t) = \rho E(t) E^*(t) \quad (5)$$

The photocurrents from the two detectors are then given by substituting (4a) and (4b) into (5):

$$I_1 = (E^2 \rho / 2) \left\{ (a+b) \alpha / 4 + (1-\alpha) + \cos \phi (ab)^{1/2} \alpha / 2 - (\alpha(1-\alpha))^{1/2} \left[ \sqrt{b} \sin(\phi/2 + \theta) - \sqrt{a} \sin(\phi/2 - \theta) \right] \right\} \quad (6a)$$

$$I_2 = (E^2 \rho / 2) \left\{ (a+b) \alpha / 4 + (1-\alpha) + \cos \phi (ab)^{1/2} \alpha / 2 + (\alpha(1-\alpha))^{1/2} \left[ \sqrt{b} \sin(\phi/2 + \theta) - \sqrt{a} \sin(\phi/2 - \theta) \right] \right\} \quad (6b)$$

The signal phase  $\phi$  and the modulation phase  $\theta$  are time dependent, and for simplicity, we will assume sinusoidal variations as follows:

$$\phi = \phi_0 + \phi_s \sin(\omega_s t) \quad (7a)$$

$$\theta = \theta_0 + \theta_m \sin(\omega_m t) \quad (7b)$$

Here,  $\phi_0$  and  $\theta_0$  are the DC phase offsets in the Michelson interferometer and the local oscillator beam respectively, while  $\phi_s$  and  $\theta_m$  are modulation depths due to the signal and external modulation at frequencies  $\omega_s$  and  $\omega_m$  respectively.

#### 2.4 Frequency components in the photocurrents

Substituting the phase variations (7a) and (7b) into (6a) and (6b) gives an expression for photocurrent which can be expanded using Bessel functions. The current exhibits DC and AC components, at frequencies corresponding to the baseband modulations at  $\omega_s$  and  $\omega_m$ , intermodulation components at  $\omega_m \pm \omega_s$ , as well as higher order harmonics which will not figure in the demodulation process to follow. We now give expressions for the components of interest.

The DC photocurrent in each detector is as follows:

$$I_{1,2}(\text{DC}) = (E^2 \rho / 2) \left\{ (1-\alpha) + \alpha \left[ (a+b)/4 + \cos \phi_0 (ab)^{1/2} J_0(\phi_s) / 2 \right] \pm (\alpha(1-\alpha))^{1/2} J_0(\phi_s/2) J_0(\theta_m) \times \left[ \sqrt{a} \sin(\phi_0/2 - \theta_0) - \sqrt{b} \sin(\phi_0/2 + \theta_0) \right] \right\} \quad (8)$$

where the  $\pm$  sign refers to current at detectors 1 (+) and 2(-). The first three terms represent the total DC power incident on the two beam combiner ports. The term  $(1-\alpha)$  represents power from the local oscillator arm, and the other two terms proportional to  $\alpha$  represent power emerging from the Michelson interferometer, complete with the standard phase dependence  $\cos \phi_0$ . When there is no signal in the interferometer,  $J_0(\phi_s)$  becomes unity and perfect fringe contrast can result if  $a = b$  (identical mirrors). The presence of signals in the interferometer destroys the perfect interference in this DC picture, because the phase variations allow light to be transmitted through the interferometer even when the DC phase  $\phi_0$  is set to 0.



The term following the  $\pm$  sign, proportional to  $\sqrt{[\alpha(1-\alpha)]}$ , represents power variation at the detectors due to phase differences between the local oscillator and the Michelson output.

Laser intensity noise at low frequencies is carried implicitly in the  $E^2$  factor. (It is this feature which prevents direct detection at the shot noise limit for low frequency signals.)

The baseband signal component (which can be lost in intensity noise associated with the DC component) is:

$$I_{1,2}(\omega_s) = E^2 \rho \left\{ -\alpha \sin\phi_0 (ab)^{1/2} J_1(\phi_s/2) \right. \\ \left. \pm (\alpha(1-\alpha))^{1/2} J_1(\phi_s/2) J_0(\theta_m) \left[ \sqrt{a} \cos(\phi_0/2 - \theta_0) - \sqrt{b} \cos(\phi_0/2 + \theta_0) \right] \right\} \\ \times \sin(\omega_s t) \quad (9)$$

where again the  $\pm$  sign refers to detectors 1 and 2 respectively. The first term represents the square law detector response to optical sidebands and carrier emerging from the Michelson. The term following the  $\pm$  sign represents interaction between optical sidebands emerging from the Michelson and the optical carrier of the local oscillator.

Similarly, a component appears at the external modulation frequency:

$$I_{1,2}(\omega_m) = \mp E^2 \rho (\alpha(1-\alpha))^{1/2} J_0(\phi_s/2) J_1(\theta_m) \\ \times \left[ \sqrt{a} \cos(\phi_0/2 - \theta_0) + \sqrt{b} \cos(\phi_0/2 + \theta_0) \right] \sin(\omega_m t) \quad (10)$$

This component is solely due to interaction at the detector between the optical carrier from the Michelson and the external modulation sidebands from the local oscillator. There is no self-interaction between the local oscillator sidebands and its own carrier, as these retain strict phase modulation symmetry. This component, like the DC component in Equation (8) carries its own complement of laser intensity noise showing up as a modulation of the  $E^2$  factor. This can be viewed in the frequency domain as a consequence of the convolution theorem - both signals and noise are shifted to higher frequencies centred around  $\omega_m$  by a modulation at  $\omega_m$ .

Finally, the component containing the shifted signal information is given by:

$$I_{1,2}(\omega_m \pm \omega_s) = \pm E^2 \rho (\alpha(1-\alpha))^{1/2} J_1(\phi_s/2) J_1(\theta_m) \\ \times \left[ \sqrt{a} \sin(\phi_0/2 - \theta_0) + \sqrt{b} \sin(\phi_0/2 + \theta_0) \right] \\ \times \left[ \cos(\omega_m - \omega_s)t - \cos(\omega_m + \omega_s)t \right] \quad (11)$$

These shifted components are due to the interaction at the detector between the optical modulation sidebands from the Michelson and Local Oscillator beams.

The photocurrent from detectors 1 and 2 are then combined in a R.F. splitter/combiner. The 180° output of the R.F. splitter/combiner is expressed as:

$$I_{\Sigma-} = (I_1 - I_2) / \sqrt{2} \quad (12)$$

Performing this subtraction operation on the frequency shifted signal sidebands results in:

$$I_{\Sigma-}(\omega_m \pm \omega_s) = \sqrt{2} E^2 \rho (\alpha (1-\alpha))^{1/2} J_1(\phi_s/2) J_1(\theta_m) \\ \times \left[ \sqrt{a} \sin(\phi_0/2 - \theta_0) + \sqrt{b} \sin(\phi_0/2 + \theta_0) \right] \\ \times \left[ \cos(\omega_m t - \omega_s t) - \cos(\omega_m t + \omega_s t) \right] \quad (13)$$

### 2.5 Demodulation of the frequency-shifted signals and noise

The operation of the mixer is to shift this signal back to baseband. This is achieved by multiplying the subtracted photocurrent output by a sinusoid at the modulation frequency:

$$D(t) = d \sin(\omega_m t + \chi) \quad (14)$$

This demodulation waveform is offset relative to the modulation waveform by an RF phase  $\chi$  and its amplitude  $d$  includes any gain or attenuation in the mixer. Multiplying (13) by (14) and disregarding all high frequency terms (ie. sum-frequency terms near  $2\omega_m$  in the spectrum) gives the recovered signal component at a receiver located at the mixer output. Sum-frequency terms are filtered out, or integrated out by the receiver, in practice. The variance due to demodulated signal alone in the mixer output is:

$$\sigma_{sig}^2 = 2 d^2 E^4 \rho^2 \alpha (1-\alpha) J_1^2(\phi_s/2) J_1^2(\theta_m) \cos^2 \chi \\ \times \left[ a \sin^2(\phi_0/2 - \theta_0) + b \sin^2(\phi_0/2 + \theta_0) \right] \\ + 2 (ab)^{1/2} \sin(\phi_0/2 - \theta_0) \sin(\phi_0/2 + \theta_0) \quad (15)$$

Clearly it is important to select the correct demodulation phase  $\chi$  (by careful control of time delays). From now on,  $\cos^2 \chi = 1$  will be assumed.

The total variance at the mixer output also contains a shot noise component, resulting from the subtraction of two uncorrelated shot noise contributions (half of the shot noise from each detector, due to the action of the passive splitter/combiner):

$$\sigma_{shof}^2 = 2 d^2 e B \times [ \langle I_1 \rangle + \langle I_2 \rangle ] / 2 \quad (16)$$

Here  $B$  is the resolution bandwidth of the receiver used, and  $e$  is the fundamental electronic charge. There is no dependence on the demodulation phase  $\chi$ , and only the long-term average photocurrents  $\langle I_1 \rangle$  and  $\langle I_2 \rangle$  are important in calculating shot noise. Substituting (8) into (16) gives:

$$\sigma_{\text{shot}}^2 = d^2 e B E^2 \rho \left[ (1-\alpha) + \alpha (a+b)/4 + \alpha / 2 (ab)^{1/2} J_0(\phi_s) \cos(\phi_0) \right] \quad (17)$$

Electronic noise due to detectors and amplifiers must also be considered when evaluating the total noise at the receiver, and this can be expressed as a noise variance after the mixer:

$$\sigma_{\text{elec}}^2 = d^2 e B E^2 \rho \epsilon \quad (18)$$

Here,  $\epsilon$  is the ratio of the measured electronic noise after the mixer to the shot noise which would be observed at the same place if all of the optical power entering the interferometer was incident on the detectors (eg. set  $\alpha = 0$  in (17)). We have chosen to scale electronic noise in this manner for experimental and mathematical convenience. Note,  $\epsilon$  depends not only on electronic noise, but scales inversely with input optical power.

There is a third noise component of interest in this scheme - laser intensity noise in the demodulated photocurrent spectrum. This noise component at the receiver is proportional to the subtracted photocurrent at frequency  $\omega_m$  (see Equation (10)) as discussed above and is therefore sensitive to the operating phases  $\theta_0$  and  $\phi_0$ . At the mixer output this  $\omega_m$  component appears as a DC component surrounded by its related technical noise. The DC component can be approximated (setting  $\sqrt{a} = \sqrt{b} = 1$ ) by:

$$I_{\text{DC}} = d \sqrt{2} E^2 \rho (\alpha (1-\alpha))^{1/2} J_1(\theta_m) \cos(\phi_0/2) \cos(\theta_0) \quad (19)$$

With the associated technical noise variance given by:

$$\sigma_{\text{tech}}^2 = 8 RIN d^2 B E^4 \rho^2 \alpha (1-\alpha) J_1^2(\theta_m) \cos^2(\phi_0/2) \cos^2(\theta_0) \quad (20)$$

where  $RIN$  is the relative intensity noise of the laser with respect to the DC component it is associated with. This noise will not be included in the general signal-to-noise expressions below - it turns out that at the optimum operating phases, this noise term vanishes.

## 2.6 Signal-to-Noise ratio in the demodulated output - general case

The signal to noise power ratio for the external modulation system is then given by  $\sigma_{\text{sig}}^2 / (\sigma_{\text{shot}}^2 + \sigma_{\text{elec}}^2 + \sigma_{\text{tech}}^2)$ :

$$S/N = 2 E^2 \rho \alpha (1-\alpha) J_1^2(\phi_s/2) J_1^2(\theta_m) \times$$

$$a \sin^2(\phi_0/2 - \theta_0) + b \sin^2(\phi_0/2 + \theta_0) + 2 (ab)^{1/2} \sin(\phi_0/2 - \theta_0) \sin(\phi_0/2 + \theta_0)$$


---


$$e B [ \alpha (a + b)/4 + (1-\alpha) + \alpha / 2 (ab)^{1/2} J_0(\phi_s) \cos(\phi_0) + \epsilon ]$$
(21)

The signal-to-noise ratio in (21) is general for our configuration. However, we may simplify this expression by noting that only a few particular situations are of interest. We are aiming to optimise the signal-to-noise ratio by optimising the phases  $\phi_0$  and  $\theta_0$  in the Michelson and local oscillator beams, respectively, the modulation depth  $\theta_m$  in the local oscillator arm, and the optical splitting ratio  $\alpha$ .

Shot noise is minimised when  $\phi_0 = \pi$  in the denominator, corresponding to a minimum in the Michelson output power. Putting this condition into (21) in turn demands that we select  $\theta_0 = 0$  in the numerator to maximise signal power. This implies that at the detectors, the local oscillator field is optically in quadrature with the resultant field from the Michelson. The S/N varies with local oscillator phase like  $\cos^2 \theta_0$ , when the Michelson phase is optimised. With the optimum optical phase settings throughout, equation (21) simplifies to :

$$S/N = \frac{4 E^2 \rho \alpha (1-\alpha) J_1^2(\phi_s/2) J_1^2(\theta_m) [1 + V] R}{e B [ 1 + \epsilon - \alpha + \alpha R(1 - V J_0(\phi_s))/2 ]}$$
(22)

where  $R = (a+b)/2$  is the mean power reflectivity of the Michelson interferometer mirrors and  $V = (a b)^{1/2}/R$  is the fringe visibility of the Michelson interferometer.

### 2.7 Deviations from the ideal interferometer: Signal-to-Noise penalties

Practical interferometers will never operate exactly at the desired phase all of the time, due to experimental conditions (eg. vibrations and thermal drifts) resulting in an RMS phase error. The effects of this phase error can be analysed by adding technical noise to (21) and making the simplifying assumption that  $a = b = 1$  resulting in:

$$S/N = \frac{8 E^2 \rho \alpha (1-\alpha) J_1^2(\phi_s/2) J_1^2(\theta_m) \sin^2(\phi_0/2) \cos^2(\theta_0)}{e B [ 1 - \alpha / 2 (1 - J_0(\phi_s) \cos(\phi_0)) + \epsilon + 8 RIN/e E^2 \rho \alpha (1-\alpha) J_1^2(\theta_m) \cos^2(\phi_0/2) \cos^2(\theta_0) ]}$$
(23)

We can then examine the sensitivity of (23) to phase errors in the Michelson by plotting the  $S/N$  as a function of phase  $\phi_0$ , for a small signal  $\phi_s \ll 1$  assuming that the local oscillator phase is optimally fixed  $\theta_0 = 0$ . In Fig 2, the effect of different levels of laser intensity noise is demonstrated - the higher the intensity noise, the smaller the tolerable RMS phase deviation. For signal frequencies around the laser relaxation oscillation frequency (~300 kHz), our laser restricts us to curve e in Fig 2, and hence requires an RMS phase error of less than 100  $\mu$ radians in order to achieve maximum signal-to-noise performance. Fortunately the relative intensity noise of our laser is strongly frequency dependant and falls off rapidly above and below the relaxation frequency. Curve c for instance, corresponds to a signal frequency of approximately 500 kHz while at 2 MHz, where all quantitative tests were performed, the maximum RMS phase error that can be tolerated is in excess of 10 mradian (curve b Fig(2)).

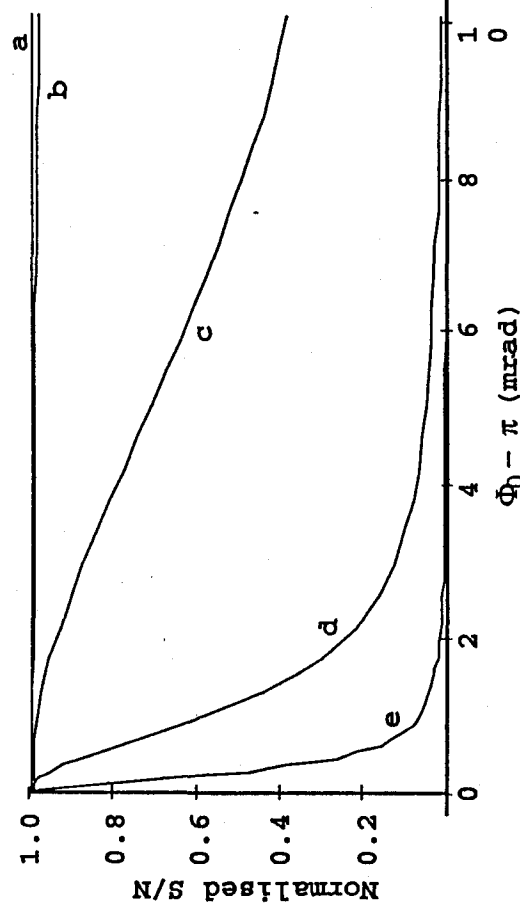


Figure 2: Normalised signal-to-noise power ratio versus dc Michelson phase  $\phi_0$ , from (23), assuming the lo phase  $\theta_0=0$  and  $R = V = 1$  (perfect optics) for various levels of laser relative intensity noise : a,  $RIN=0$  (perfect noise limited laser); b:  $RIN = 1 \times 10^{-14}$ ; c:  $RIN = 1 \times 10^{-12}$ ; d:  $RIN = 0.5 \times 10^{-10}$ ; e,  $RIN = 1 \times 10^{-9}$ .

A tradeoff occurs between power tapped off into the local oscillator (which boosts the frequency shifted signal strength) and power lost from the Michelson (which reduces the original signal). The frequency shifted signal strength is proportional to  $\alpha(1-\alpha)$ , and is optimal when  $\alpha = 0.5$ . However, tapping off light into the local oscillator leads to extra shot noise at the detectors. In an ideal configuration, this is the only noise term, and  $S/N$  is actually optimum when  $\alpha \rightarrow 1$ . When unrelated noise is present, e.g. electronic noise, the optimum  $\alpha$  falls somewhere between 0.5 and 1.

This trade-off between signal and local oscillator beam power is illustrated in Figure 3(a), which plots normalised signal to noise from (22) against the optical splitting ratio  $\alpha$ , for different amounts of electronic noise  $\epsilon$ , assuming all other parameters are optimised. In the ideal case of no electronic noise,  $\epsilon = 0$ , curve 3(a) appears to rise linearly and approach its optimum at  $\alpha = 1$ . (In fact, the signal to noise drops sharply to 0 in a very small range close to  $\alpha = 1$ , because there is a

non-zero shot noise due to signal-related optical transmission in the Michelson.) Figure 3(b) plots the optimum value of  $\alpha$  in an optically ideal interferometer as a function of electronic noise as well as the associated S/N ratio. The greater the electronic noise, the more power must be tapped off into the local oscillator arm to optimise sensitivity, but the worse that optimum becomes. Therefore it is experimentally prudent to use electronically quiet detectors/amplifiers or increase the input laser power (both operations reduce  $\epsilon$ ).

### 2.8 Signal-to-Noise ratio for small signals in the ideal limit

In general, we are interested in detecting extremely weak signals. In the limit  $\phi_s \ll 1$ , we may use Bessel function small argument approximations:  $J_0(\phi_s) \rightarrow 1$  and  $J_1^2(\phi_s/2) \rightarrow \phi_s^2/16$ . Substituting these approximations into (22) yields:

$$S/N = \frac{E^2 \rho \alpha (1-\alpha) \phi_s^2 J_1^2(\theta_m) [1+V] R}{4 e B [1+\epsilon - \alpha + \alpha R(1-V)/2]} \quad (24)$$

When the mirrors are both close to perfect R and V approach 1. Equation (24) is useful for comparison to measured signal-to-noise, because each of the independent physical parameters can be determined experimentally.

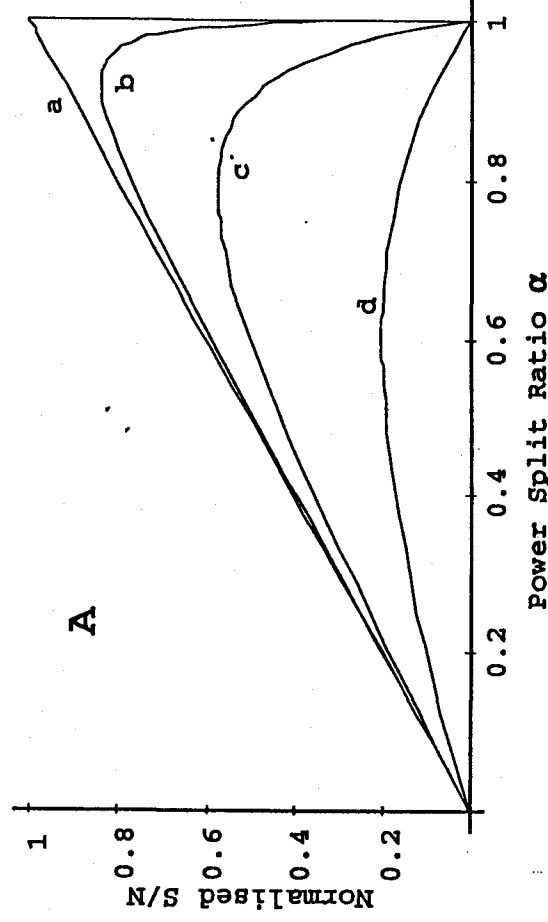


Figure 3 (a): Normalised signal-to-noise power ratio versus optical split ratio  $\alpha$ , for various levels of electronic noise  $\epsilon$  assuming perfect optics ( $R = V = 1$ ): a,  $\epsilon = 0$ ; b,  $\epsilon = 0.008$ ; c,  $\epsilon = 0.08$ ; d,  $\epsilon = 0.8$ .

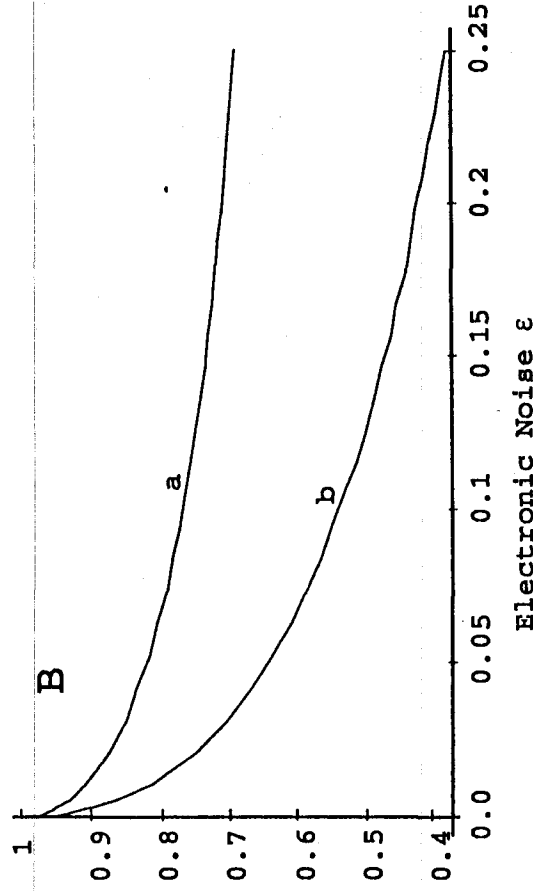


Figure 3 (b): Curve "a": Optimum optical power split ratio  $\alpha$  versus electronic noise assuming ideal optics ( $R = V = 1$ ). Curve "b": The resulting signal-to-noise ratio when the optimum value of  $\alpha$  (given by curve "a") is used, versus electronic noise  $\epsilon$ .

The modulation depth  $\theta_m$  in the external arm is an independent variable and can be set to maximise  $J_1^2(\theta_m)$  when  $\theta_m = 1.84$  radians. In this case  $J_1^2(\theta_m) = 0.338$ . Experimentally this is achievable using a resonant Pockels cell, but the required crystal is long, often producing significant wavefront distortions. Larger modulation depths lead to more power being transferred to higher order sidebands, not utilised by the demodulation process.

Assuming an ideal interferometer (ie, no electronic noise,  $\epsilon = 0$ , perfect Michelson interferometer fringes,  $R = 1$ ,  $V = 1$  and optimum modulation  $\theta_m = 1.84$  radians in the external arm) we have

$$(S/N)_{\text{ideal}} = 0.338 E^2 \rho \phi_s^2 \alpha / 2 e B \quad (25)$$

Here, it is apparent that to optimise the signal to noise ratio, we need to set  $\alpha$  as large as possible, ie.  $\alpha \rightarrow 1$ . In this ideal limit,

$$(S/N)_{\text{ideal}} \rightarrow 0.676 [E^2 \rho \phi_s^2 / 4 e B] \quad (26)$$

Equation (26) is in agreement with previous literature<sup>1,2</sup>. The factor in the brackets is the S/N limit which would be achieved if an ideal *Direct Detection* scheme had been used<sup>3</sup>.

$$(S/N)_{\text{direct}} \rightarrow E^2 \rho \phi_s^2 / 4 e B \quad (27)$$

The penalty (1.7dB on the log scale) relative to direct detection can be interpreted as the effect of inefficiencies inherent in the simple signal recovery process modelled here (single frequency modulation and demodulation). In principle, it is possible to construct a more complex modulation

scheme (involving higher harmonics) which can approach the direct detection limit, with 0.676 in (26) replaced by unity <sup>4</sup>.

It is also noteworthy that the signal to noise obtained by the *Internal Modulation* technique is very close to that in (26) above:

$$(S/N)_{\text{internal}} \rightarrow 2/3 [E^2 \rho \phi_s^2 / 4 e B] \quad (28)$$

where the penalty in that case is due to demodulation of non-stationary shot noise<sup>5</sup>. The decision to use external modulation rather than internal modulation must be based therefore on technical considerations rather than inherent sensitivity limits.

### 3. Experimental Configuration

A schematic of our experimental external modulation arrangement, detailing the principal electronic and optical components, is shown in Fig 4. This configuration closely matches that of the model analysed in the previous section.

#### 3.1 Optical configuration

The lightsource is a "MISER" Nd:YAG CW laser (Lightwave Electronics model 120) operating at 1064 nm. The output beam is directed through a Faraday isolator and then into a variable beam splitter consisting of a rotatable half wave plate and a polarising beam splitter. The vertically polarised output from the variable beam splitter traverses a lens and then enters the Michelson sensing interferometer.

A Pockels cell in one arm of the Michelson interferometer provides a small, calibrated, optical phase "signal", while a neutral density filter in the other arm provides optical amplitude compensation to obtain good fringe visibility at the Michelson output. By this technique, we are able to obtain a fringe contrast in the Michelson of around 600:1 (fringe visibility  $V = 0.9967$ ). The factor ultimately limiting contrast is wavefront distortion in the Pockels cell.

The local oscillator beam emerges with an orthogonal polarisation to that of the Michelson beam and so another half wave plate is used to rotate the polarisation state by 90°. The variable beam splitter allows experimental adjustment of the power splitting ratio  $\alpha$  by rotation of the initial half wave plate.

Under normal operating conditions, the Michelson interferometer is locked so that the output port is tuned to a dark fringe. The optical power injected into the Michelson interferometer is then reflected back toward the laser. It proved necessary to misalign the Michelson reflected beam slightly to prevent it re-entering the laser, as the 30dB Faraday isolator alone was insufficient to



prevent laser oscillations due to optical feedback. These were particularly observable when the Michelson phase modulation (signal) frequency corresponded to the relaxation oscillation frequency of the laser (~ 300 kHz).

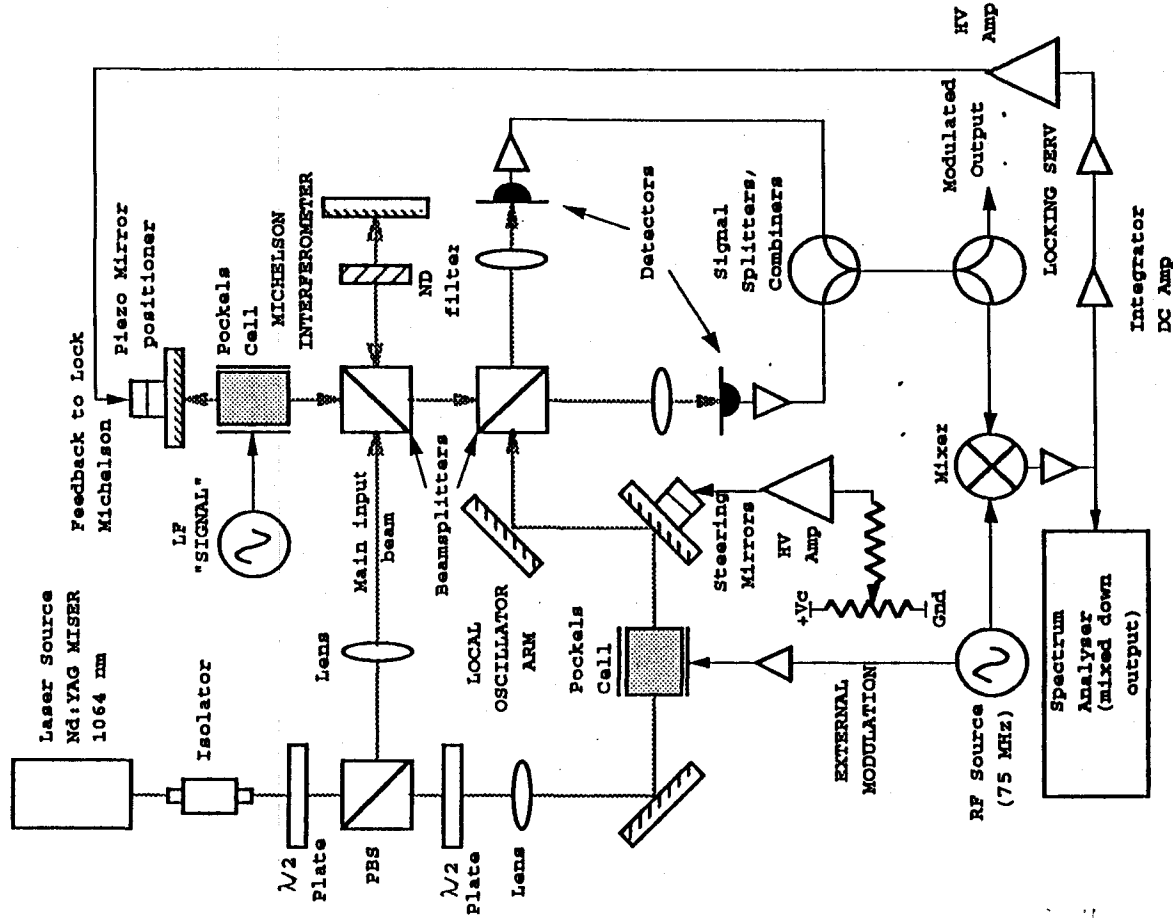


Figure 4: Detailed schematic of the external modulation interferometer experiment.

The local oscillator beam traverses a mode matching lens, a resonant Pockels cell (New Focus model 4003, resonant at 75 MHz) and is then directed into the final beam combiner via two steering mirrors. By adjusting the control voltage on the piezo actuator, the relative phase of the local oscillator beam can be tuned with respect to the Michelson output. The Michelson output beam and the local oscillator beam are coherently combined at a 50:50 beam splitter, and the two outputs are focussed onto a balanced pair of detectors. The best obtained fringe contrast for this Mach-Zehnder interferometer (measured by combining the local oscillator beam with an equal intensity beam from

just one arm of the Michelson interferometer) was around 150:1, limited by wavefront distortion in the resonant Pockels cell. This is more than adequate to ensure near optimal operation.

The Michelson end-mirrors in each arm are placed to ensure accurate matching of the two Gaussian beams at the output of the Michelson. It should be noted that matching the spot-sizes and wavefront curvatures of two beams does not necessarily correspond to matching their absolute optical path lengths. Geometric factors in one arm of the interferometer, such as refraction in a high-index element such as the Pockels cell, can mean that it is necessary to shorten the other arm (rather than lengthen it as intuition might suggest) to correctly match the two Gaussian output beams. Note: This will unavoidably introduce an optical path length mismatch, comparable to the length of the Pockels cell.

The primary purpose of the lens in the local oscillator beam is to create a beam-waist inside the resonant Pockels cell, but this necessitates an identical lens in the Michelson input beam, placed so that both beams arriving at the final beam combiner experience the same optical evolution. It proves more effective to match the beam evolutions as closely as possible in the first instance than to correct for different beam paths subsequently with extra lenses.

### 3.2 Electronic configuration

The light is focussed onto a pair of balanced InGaAs detectors (Epitaxx ETX500) AC coupled via a  $50\Omega$  linear amplifier, with a flat frequency response from about 6 MHz to 250 MHz. The output voltages are subtracted by a passive  $180^\circ$  signal combiner to coherently combine signals from both detectors, yielding a 3dB improvement in the attainable signal-to-noise ratio (since the shot noise from each detector is uncorrelated).

The output from the passive signal combiner is itself split at a passive signal splitter and one output is fed directly to a receiver (a Hewlett Packard HP8568B spectrum analyser) while the other is mixed down to recover the original signal spectrum using an RF mixer. The local oscillator for the mixer is derived from the external modulation signal source at 75 MHz, with an appropriate time delay to optimise the demodulation phase ( $\chi$  in Section 2). This RF electronic operation was modelled in Section 2 by multiplying equations (13) and (14). Experimentally the signal-to-noise is independent of the mixer gain  $d$ , in accordance with Equation (19). There is also a 3dB gain in S/N (when the correct demodulation phase is chosen) because frequency convolution combines coherent signal sidebands and incoherent noise sidebands. The demodulated signal spectrum is fed to the receiver (HP8568B), and also acts as an error signal for locking the Michelson interferometer.

Electronic noise arising in the detectors and amplifiers is characterised by  $\epsilon$ , as defined in Equation (18). This is determined experimentally by measuring the electronic noise directly at the mixer output and comparing it to the shot noise measured at the same point for a given optical power incident on the detectors. Since Equation (18) normalises electronic noise to the maximum possible shot noise level (which would be observed if all the light was incident on the detectors), it is

necessary to scale up the measured shot noise appropriately to account for this definition. In this manner we obtain values of  $\epsilon \sim 0.08$  for an effective total optical power of 9mW.

To reduce vibration-induced RMS phase excursions from the Michelson dark fringe (and hence to reduce the amount of optical transmission and associated technical noise from the laser), it is necessary to actively lock the Michelson to the dark fringe. Interferometric phase locking is achieved by integrating the error signal, and amplifying the result to drive a piezo-controlled mirror in one arm of the Michelson. The desired locking condition is complete nulling of the 75 MHz component in the subtracted photodetector output, which corresponds uniquely to a dark fringe in the Michelson interferometer, but good 75 MHz cancellation is only achieved when the local oscillator optical phase is approximately correct. This local oscillator phase is set by manually tuning the piezo mirror in the local oscillator arm. When the feedback loop is active, locking follows automatically as soon as the local oscillator is within  $\sim 45^\circ$  of the correct phase; subsequent fine tuning of the local oscillator phase optimises the demodulated signal at the receiver.

The response of the integrator, the following amplifiers and the piezo mirror mount combine to produce an open loop unity gain at around 5 kHz. This is sufficient to counteract differential acoustic and vibrational noise in the interferometer so that laser technical noise appearing in the demodulated output spectrum is suppressed by more than 50 dB under normal operating conditions.

## 4. Experimental Results

### 4.1 Motivation for these experiments:

The simple model of a non-ideal external modulation system analysed in Section 2 is only worthwhile if it can genuinely describe a realistic experimental interferometer. It is necessary to perform a series of experiments to verify the predictions of the theory (or otherwise), to determine whether or not we have included sufficient (or superfluous) non-ideal features in the model. Experimental measurements will also reveal technical issues which the model cannot predict, such as potential difficulties in operating a practical instrument, or the relative sensitivities of the device to various technical noise sources such as acoustic disturbances.

### 4.2 Locking the interferometer

To confirm the experimental predictions of our generic model of the external modulation scheme in Section 2, it is first necessary to lock both the Michelson interferometer and the local oscillator beam to their correct operating phases. Manual locking is sufficient in the local oscillator arm because the final sensitivity depends only weakly on the local oscillator phase ( $S/N$  varies like  $\cos^2\theta$  in Section 2). In the benchtop Michelson interferometer, on the other hand, manual locking to a dark fringe is completely ineffective, as acoustic and low frequency mechanical vibrations produce a significant RMS differential phase error. This leads to a certain amount of unwanted