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The Effect of Noise in the Modulation Reference Source to the Gravitational Wave Signal

Title

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INTRODUCTION

The noise in the RF modulation reference source can affect the measurement of the gravitational wave (GW) signal. This can happen through the gain fluctuations in the feedback loop controlling the length of the interferometer arms. If the gain fluctuates at the frequencies in the bandwidth of the gravitational wave the noise in the reference source will be added to the gravitational wave signal. The bandwidth of the gravitational wave signal is from 100 Hz to 1 KHz.

THE GAIN

Let $x(t)$ be the differential motion of the arm cavities and R be the interferometer response factor, which is a function of mirror reflectivities, recycling cavity length, asymmetry and etc. The fact that R is a function of frequency is not important for the present analysis. Assume that R is constant. Then the photocurrent at the differential mode photodiode is

$$I(t) = I_0 + E_0^2 J_0(\Gamma) J_1(\Gamma) R\, x(t) \sin \omega t,$$

where ω is the RF modulation frequency. At the mixer the photocurrent is multiplied by the demodulation current

$$D(t) = \sin(\omega t + \phi) + \frac{1}{3} \sin 3(\omega t + \phi) + \ldots,$$

which is a square wave signal. The constant ϕ is the phase that can be adjusted at the phase shifter. Since the signal part of the photocurrent does not have higher order harmonics we can neglect them in the demodulation current and assume

$$D(t) = \sin(\omega t + \phi).$$

To obtain the gravitational wave signal we have to low pass the mixer output, i.e. to average it

$$V(t) = \overline{I(t) D(t)}.$$

Here we average over the time period long compared to the modulation cycle but short compared to the period of the gravitational wave. The result is

$$\begin{aligned} V(t) &= E_0^2 J_0 J_1 R\, x(t) \overline{\sin \omega t \, \sin(\omega t + \phi)} \\ &= \frac{1}{2} E_0^2 J_0 J_1 R\, x(t) \cos \phi. \end{aligned}$$

The optimal demodulation is achieved when $\phi = 0$ modulo π.

Thus $V(t)$ is proportional to $x(t)$. The coefficient of proportionality is the optical gain of the interferometer. The noise in the reference source will cause the optical gain to fluctuate.

AMPLITUDE NOISE

To estimate the effect of the amplitude noise we approximate the Bessel functions by

$$J_0(\Gamma) \approx 1, \quad J_1(\Gamma) \approx \Gamma/2.$$

Then the GW signal is

$$V(t) \sim \Gamma x(t).$$

The amplitude noise of the reference source signal will cause the amplitude of the voltage on the Pockels Cell to fluctuate. These fluctuations appear in the optical sidebands as fluctuations of the modulation index $\delta\Gamma(t)$ and propagate further to the photodiode. The demodulation signal $D(t)$, however, is not affected by the amplitude noise. This is because the diodes in the mixer are saturated.

Thus instead of the idealized GW signal we will have

$$V(t) \sim (\Gamma + \delta\Gamma(t))x(t),$$

The restriction on the amplitude noise is that the high frequency fluctuations of the modulation index coupled to the low frequency motion should be much less than the estimated motion of the test mass in the bandwidth of the gravitational wave. To see how the noise is distributed over frequencies take Fourier transform

$$\tilde{V}(f) \sim \Gamma\tilde{x}(f) + \sum_{f'=0}^{f_1} \delta\tilde{\Gamma}(f - f')\tilde{x}(f')\Delta f.$$

The first term represents idealized signal while the sum is the effect of the amplitude fluctuations. The cut-off frequency in the sum is f_1=100 Hz. We neglected the contribution of terms at frequencies higher than f_1.

The maximum absolute value of the sum is

$$N \mid \delta\tilde{\Gamma}(f)\, \tilde{x}(0) \mid \Delta f,$$

where $N = f_1/\Delta f$ is the number of terms in the sum and $\tilde{x}(0)$ is the low frequency displacent.

We want the maximum absolute value of the sum to be much less than the absolute value of the first term. This gives us the restriction on the amplitude noise

$$\frac{|\delta\tilde{\Gamma}(f)|\Delta f}{\Gamma} \ll \frac{1}{N}\left|\frac{\tilde{x}(f)}{\tilde{x}(0)}\right|.$$

The amount of the amplitude noise that we can tolerate in the bandwidth $\Delta f = 1$ Hz is given in the following table.

f(Hz)	1	10	50	100	500
$\tilde{x}(f)(\mathrm{m}/\sqrt{\mathrm{Hz}})$	10^{-13}	10^{-14}	10^{-15}	10^{-17}	10^{-18}
Amp.Noise	10^{-2}	10^{-3}	10^{-4}	10^{-6}	10^{-7}

The approximations of the Bessel functions that we made are good for values of Γ up to 1.0 . However, our analysis is valid even for larger values of Γ. This is because the product $J_0(\Gamma)J_1(\Gamma)$ has maximum at $\Gamma = 1.076$. At this point there is a first order exclusion of the amplitude noise.

PHASE NOISE

The effect of the phase fluctuations can be analyzed in a similar way. Suppose $\delta\phi(t)$ is the phase noise in the reference source signal. This phase will be added to the optical sidebands, propagate into the recycling cavity and eventually appear as phase $\delta\phi(t-t_1)$ in the photocurrent $I(t)$. On the other hand, the phase will propagate through the phase shifter directly into the mixer and appear as $\delta\phi(t-t_2)$ in the demodulation signal $D(t)$. Due to the time delay $(t_1 - t_2)$ between the two paths the two phases are not the same. This time delay depends on the cable length, the length of the recycling cavity and other factors. We can approximate it as L/c, where L is of the order of 10 - 100 m for the 40-m inteferometer. The gravitational wave signal is then

$$\begin{aligned} V(t) &\sim x(t)\,\overline{\sin(\omega t + \delta\phi_{t-t_1})\,\sin(\omega t + \phi + \delta\phi_{t-t_2})} \\ &= \frac{1}{2}x(t)\cos(\phi + \delta\phi_{t-t_2} - \delta\phi_{t-t_1}) \\ &\approx \frac{1}{2}x(t)\left(1 - \phi\,\frac{L}{c}\frac{d\delta\phi}{dt}\right). \end{aligned}$$

As before take Fourier transform of the GW signal

$$\tilde{V}(f) \sim \tilde{x}(f) - \phi\frac{L}{c}\sum_{f'=0}^{f_1}\tilde{x}(f')\,\delta\tilde{\phi}(f-f')\,2\pi i\,(f'-f)\,\Delta f.$$

The maximum absolute value of the sum is

$$N\,\frac{L}{c}\,|\,\tilde{x}(0)\,\phi\,\delta\tilde{\phi}(f)\,2\pi f\,|\Delta f.$$

We want this expression to be much less than $\tilde{x}(f)$ for any frequencies f within the GW bandwidth. Then the restriction on the phase fluctuations is

$$|\phi\;\delta\tilde{\phi}(f)|\,\Delta f \ll \frac{1}{2\pi f}\left(\frac{c}{LN}\right)\left|\frac{\tilde{x}(f)}{\tilde{x}(0)}\right|.$$

By tuning the knob at the phase shifter and making the phase ϕ equal to zero we could in principle eliminate the phase noise in the first order. However, in practice we cannot adjust the phase to values better than $\phi \simeq 4^0$ (0.07 rad). This gives us the phase noise in the first order.

The phase noise for different frequencies in the bandwidth $\Delta f = 1$ Hz is given in the following table.

f(Hz)	1	10	50	100	500
$\tilde{x}(f)(\mathrm{m}/\sqrt{\mathrm{Hz}})$	10^{-13}	10^{-14}	10^{-15}	10^{-17}	10^{-18}
$\delta\tilde{\phi}(f)\Delta f$	-	-	0.1	10^{-2}	10^{-4}

CONCLUSION

We described a mechanism of how the noise in the modulation reference source can couple to the low frequency motion of the test masses. We estimated the upper limits of the amplitude and phase noise that can be tolerated by the interferometer without losing its sensitivity.