New Folder Name More on Vibration Isolation of Mechanical Equipment

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FROM: Hal Amick
DATE: 15 November 1995
SUBJECT: More on vibration isolation of mechanical equipment

NUMBER OF PAGES (including this page) 10

ORIGINAL TO FOLLOW: _ YES x NO

MESSAGE:

In my fax of a few days ago, I went to some length to encourage conservatism in the selection of spring isolators for rotating machinery. One of my colleagues suggested I should also point out the drawbacks of being too conservative with respect to spring selection.

A feature of springs that I did not discuss is what I call the "wavebearing" nature of springs, which occurs at frequencies much greater than the fundamental resonance frequency of the simple mass on the spring. Beranek calls them "standing waves". Norm Mason (of Mason Industries) calls it "surge". Pages 414-423 of Beranek's book *Noise and Vibration Control* discuss the basic physics of springs, including wavebearing. Figure 13.8 shows the measured performance of a Navy resilient mount (neoprene spring), which illustrates the degradation of isolator performance at wavebearing frequencies. The phenomenon exists for steel springs as well, but isn't as well documented as has been springs for Navy applications.

Also enclosed is a three-page letter from Norm Mason to Mason Berger Northeast in which he discusses specifics of surge in their steel springs. The last four lines of his table are of significance. We can see there is a potential problem if a 60 Hz device is supported on a 5" deflection spring. (It should be noted that this is not a well-documented phenomenon for steel springs, and this letter from Mason is one of the few things ever put in writing about the product.)

This phenomenon has been addressed in steel spring design by including a base of neoprene in series with the steel spring, which is intended to attenuate the higher frequency vibrations.

The lesson is: be cautious in applying conservatism for its own sake.

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MASON INDUSTRIES, Inc.
Manufacturers of Vibration Control Products

SPRING SURGE FREQUENCY

July 28, 1993

RECEIVED SEP 15 1993

MASON BERGER NORTHEAST
RICK NOVIA
214 ANDOVER STREET #5
WILMINGTON, MA 01887

Gentlemen:

Consultants are showing a renewed interest in spring surge frequencies.

Springs are normally selected to establish the natural frequency of the whole system. This primary frequency is the bouncing frequency or the rate at which an isolated foundation or machine bobs up and down when pushed or displaced. It is a function of the spring deflection.

Spring Surge Frequency is the frequency at which the coils themselves would vibrate if you strum the spring or displace just one coil and let it rebound quickly. The coils themselves surge or vibrate from top to bottom in a "slinky" like movement.

Engineers are often concerned that there will be vibration transmission if the operating speed of the equipment or some frequency generated by the equipment is close to this surge frequency. A 1" deflection spring system will have a primary of approximately 3 cycles per second (3 hz), and a surge frequency of 250 hz. This is 15,000 cycles per minute. Theoretically, a high speed compressor running at 15,000 rpm would excite this surge frequency and transmit noise. If the coils were heavy enough, there might be vibration transmission as well.

Another way to describe spring surge frequency is the resonant frequency at which the coils themselves vibrate top to bottom. The formula for arriving at this frequency is surprisingly simple. It is only a factor multiplied by the stress rate. The stress rate is the stress per inch of deflection.

Years ago we designed virtually all of our springs with a 100,000 psi solid stress. We allowed for 50% additional travel, so our springs were operating at 67% of the solid stress or about 67,000 psi. Since this was the stress at 1" deflection, the <u>stress rate</u> was 67,000 psi per inch as well.

Recently we re-designed our entire spring line to take advantage of the more exotic spring materials with higher stress limits. Our diameters divided by the compressed heights are still in the 0.8 range, but using these superior materials, the spring <u>package</u> became smaller. This more efficient reduced size helps in tight space situations and reduces cost when the springs are within structural housings like the Type SLR.

In response to an inquiry from Mason Berger East on behalf of a consulting firm, Tony Slota took the time to develop the enclosed reference, A-23334. Rather than listing every spring, we are providing the range of frequencies, both in cycles per minute and Hz. Should you be asked this question, you need only send A-23334 to give the consultant proper information. If you need a specific answer on any individual spring, please let us know.

It is interesting to note that the frequency of the spring is the constant times the stress rate rather than the actual stress in the spring at a rated deflection. This means that a spring designed for 4" deflection has the same surge frequency whether the spring is deflected 1", 3", 4" or overloaded at 5".

Should there be an application where 4" deflection is needed to develop the primary frequency but the surge frequency is close to the operating speed or some multiple of the operating speed, there would be two possible solutions.

The best answer would be to go to airsprings with a similar frequency. The airsprings have no surge problems.

The second choice would be a lower stressed spring, so the stress rate would go down. The surge frequency would drop in direct proportion to the reduction of the stress rate. However, cost must be considered. We normally buy our springs in large quantities. Four special springs might very well exceed the cost of an air system because the spring winders have expensive setup charges for short runs. A lower stress spring is more expensive to begin with as the spring becomes much heavier.

Should you have any questions about surge frequencies, please let us know.

Best regards,

MASON INDUSTRIES, INC.

Norm Mason

NM:fr

Enclosure: A-23334

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SURGE FREQUENCY OF SPRINGS

DATE JULY 23, 1993 A-23334

Derivation Reference: Mechanical Springs
By A.M. Wahl

McGraw Hill Book Company
Page 290

Stress Rate/Inch: $\frac{\text{Solid Spring Stress (psi)}}{\text{Solid Deflection (inches)}} = \frac{\text{psi}}{\text{inch}}$

Surge Frequency (C.P.M.): 0.2304 (Stress Rate) = Surge Frequency

Surge Frequency CPS (H_z): 0.00384 (Stress Rate) = Surge Frequency

SPRING GROUP	STRESS RATE KSI/INCH (NEAREST WHOLE NO)	SURGE FREQUENCY C.P.M. (NEAREST 0)	SURGE FREQUENCY H_z (NEAREST WHOLE NO)
"A" SPRINGS	60-90	13820-20740	230-346
"B" SPRINGS 20-280	41-52	9450-11980	157-200
"B" SPRINGS 450-1000	60-83	13820-19120	230-319
"C" SPRINGS	74-97	17050-22350	284-371
GROUP "30"	60-104	13820-23960	230-399
B-2 SPRINGS	48-58	11060-13360	184-223
C-2 SPRINGS	32-53	7370-12210	123-204
100 SERIES-2" DEFLECTION	33-47	7600-10830	128-182
100 SERIES-3" DEFLECTION	25-29	5760-6680	94-111
100 SERIES-4" DEFLECTION	17-21	3920-4840	64-80
100 SERIES-5" DEFLECTION	13-18	3000-4150	50-68

TSNM:fr

17140 3/8

Noise and Vibration Control

Edited by
LEO L. BERANEK
Chief Scientist, Bolt Beranek and Newman Inc.
Lecturer, Massachusetts Institute of Technology

To Hal Amick
Dear ~~Hal~~,
With my very best wishes,
Leo L Beranek

McGRAW-HILL BOOK COMPANY
New York St. Louis San Francisco Düsseldorf
Johannesburg Kuala Lumpur London Mexico
Montreal New Delhi Panama Rio de Janeiro
Singapore Sydney Toronto

datory that there be some damping present. As a rotating device is started or stopped, it will, in most cases, pass through a resonance range, i.e., a range of frequencies where the system response is magnified by a resilient mounting. If the machine can be accelerated rapidly, it may pass through this region so quickly that the amplitude of the transmitted force does not have time to build up to the steady-state levels indicated by Fig. 13.4. If, on the contrary, the machine accelerates slowly through the resonant range, then the transmitted force may become very large. In this case, a large amount of damping (say, $\zeta = 0.5$) may be required to prevent excessive vibration near resonance. An external damper can be installed to accomplish this. Fortunately, even with as large a damping as $\zeta = 0.5$, the amount of isolation achieved at higher frequencies is adequate in most cases.

Nonviscous Damping. The previous analysis considered only viscous damping, that is, systems in which the component of force due to damping in Eq. (13.12) is directly proportional to velocity. It has been shown that the effects at resonance due to other forms of damping can be represented in terms of an "equivalent viscous damping," using energy dissipation per cycle as the criterion of equivalence.[1] However, in such cases, the frequency effects of damping are generally quite different from those shown in Fig. 13.4.

For hysteresis or structural damping, the damping term depends on displacement instead of velocity. In this case the transmissibility at high frequencies is virtually independent of the damping. The equation for ϵ_f is

$$\epsilon_f = \left[\frac{1+\delta^2}{\left(1-\frac{\omega^2}{\omega_n^2}\right)^2+\delta^2}\right]^{1/2} \qquad (13.18)$$

where $\delta = 1/Q$

Q = amplification factor at resonance, equal to the ratio of the resonance frequency divided by the bandwidth at the half-power points; see Eq. (14.5)

Generally, δ is less than 0.2. A few calculations will show that Eq. (13.18) is closely approximated by Eq. (13.4) up to the frequencies where ω is very much larger than ω_n. At $\omega > 10\,\omega_n$, wave effects and other factors cause measured values of response to be significantly greater than those predicted by the approximations of Eqs. (13.14), (13.15), and (13.18).

Isolation at Acoustic Frequencies. The theory so far presented is that which is customarily found in books and papers dealing with vibration isolation.[2,3] With modifications to take care of the three-dimensional nature of the machine and the fact that several mounts are used,

it gives satisfactory results at relatively low frequencies. This infrasonic frequency range is where we are concerned about physical damage or fatigue failure. Unfortunately, results calculated by Eq. (13.15) predict attenuations for the audio frequency range which are apt to be very much higher than those achieved in practice.

Since there is nothing wrong with the mathematical development, we must look to the assumptions to explain this discrepancy. Figure 13.5 shows a rigid mass mounted on an isolator which in turn rests on an absolutely rigid foundation. If the foundation were actually rigid, the isolation problem would be of no interest since the foundation could not be moved. In actuality, almost any foundation and any reasonable machine will have many resonances in the audio frequency range.

Let us consider first the foundation. Since for small motions it is a linear system, when a force $F = F_0 \cos \omega t$ is applied to it, the vibratory velocity at the point of application is given by

$$v = UF_0 \cos(\omega t - \phi)$$

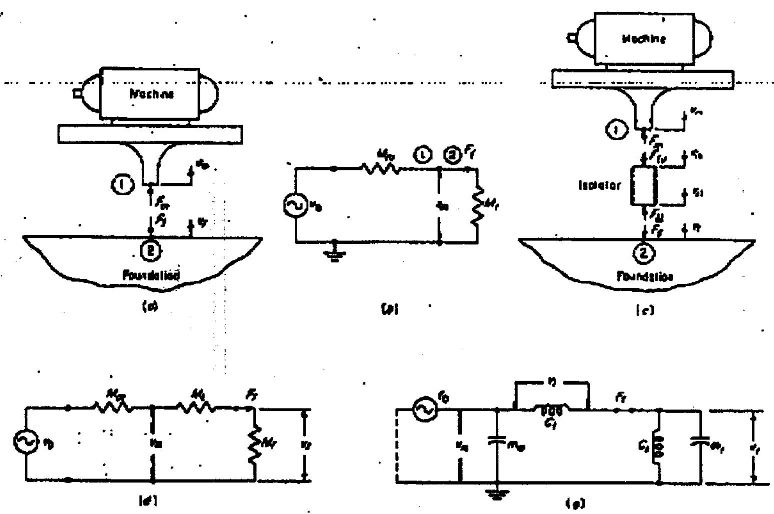

Fig. 13.5 Interconnection mobilities of a motor, isolator, and foundation: (a) mechanical diagram without isolator; (b) analogous schematic for this configuration with M_m = mobility of the machine, M_f = mobility of the frame, and v_0 = velocity the machine would have if it were freely suspended in space; (c) mechanical diagram with isolator; and (d) and (e) analogous schematic diagrams for this configuration with M_m, M_i, and M_f equal to the mobilities, in m/N-sec, for the machine, isolator, and foundation, respectively; m_M mass of machine in kilograms, C_i = compliance of the isolator in m/N, C_f = compliance of frame in m/N, and m_f = mass of frame in kilograms. Note also that $u_0 = M_m f_0$.

where U and ϕ are functions of frequency ω which we shall discuss below. This equation may also be written in the form

$$v = UF_0 \cos \omega t \cos \phi + UF_0 \sin \omega t \sin \phi$$

If we use the Euler equation

$$e^{j\theta} = \cos \theta + j \sin \theta$$

we may redefine the force F as the real part of

$$F = F_0 e^{j\omega t}$$

and the velocity at the point of application of the force as the real part of

$$\begin{aligned} v &= UF_0 e^{j(\omega t - \theta)} \\ &= Ue^{-j\theta} F_0 e^{j\omega t} \\ &= M_f F \end{aligned} \tag{13.19}$$

where M_f = *mobility*, defined as the complex number whose modulus $|M_f|$ is the rms magnitude of the velocity resulting from the application of a sinusoidal force whose rms magnitude is unity and whose argument θ is the phase angle between the velocity and the applied force

The internal mobility of the machine M_m can be defined in a similar manner. By superposition, the net motion of that point on the machine base where the machine is fastened to the foundation is the sum of the motion it would have due to internal forces only (that is, the point of attachment is free) and the motion due to the resisting force (Fig. 13.5*a* and *b*), or

$$v_m = v_0 + M_m F_m \tag{13.20}$$

It should be noted that v_0 is not necessarily a rigid-body motion of the machine and may be modified appreciably by internal resonances and antiresonances. It is not always easy to calculate but can be measured with reasonable accuracy if the machine is supported on mountings (isolators), which are very soft in comparison to M_m (that is, $M_m \ll M_i$, or in Fig. 13.5*b*, $M_f \gg M_m$).

Let us consider that point 1 on the machine is attached to point 2 on the foundation (Fig. 13.5*a*); then for motion of the combined system (which we denote by a tilde)

$$\tilde{v}_m = \tilde{v}_f$$
$$\tilde{F}_m = -\tilde{F}_f$$

From Eqs. (13.19) and (13.20)

$$\tilde{F}_f = \frac{v_0}{M_m + M_f} \tag{13.21}$$

By the analogous circuit of Fig. 13.5*b*, the generator velocity v_0 is equal to the force "flowing through" the elements times the total mobility of the circuit. If a massless isolator is interposed between the machine and its foundation (Fig. 13.5*c* and *d*), then for motion of the combined system

$$\begin{aligned} \tilde{v}_m &= \tilde{v}_{iu} \\ \tilde{v}_f &= \tilde{v}_{il} \\ \tilde{F}_m &= -\tilde{F}_{iu} = +\tilde{F}_{il} = -\tilde{F}_f \end{aligned} \tag{13.22}$$

We define the force in and relative velocity across the isolator by, respectively,

$$\tilde{F}_i = \tilde{F}_{iu} = -\tilde{F}_{il}$$
$$\tilde{v}_i = \tilde{v}_{iu} - \tilde{v}_{il}$$

which, with Eqs. (13.19), (13.20), and (13.22), becomes

$$\tilde{v}_i = M_i \tilde{F}_i = v_0 + M_m \tilde{F}_m - M_f \tilde{F}_f$$

Because

$$\tilde{F}_i = \tilde{F}_f = -\tilde{F}_m$$

we obtain (see Fig. 13.5*d*)

$$\tilde{F}_f = \frac{v_0}{M_m + M_i + M_f} \tag{13.23}$$

If we now redefine ϵ_f as the ratio of the force on the foundation with the isolator to that without

$$\epsilon_f = \left| \frac{F_{wi}}{F_{wo}} \right| = \left| \frac{M_m + M_f}{M_i + M_m + M_f} \right| \tag{13.24}$$ *

It will be found that this definition leads to the same result as the previous one if the previous assumptions are used.

It may also be shown that the displacement transmissibility ϵ_d will have the same value as the force transmissibility ϵ_f.

From Eq. (13.24) and Fig. 13.5*d* we find that an isolator does no good unless it is more flexible (has *greater* mobility, i.e., a greater value of M_i) than the sum of the mobilities of the machine and foundation. The mobility of a simple structure may be calculated, and that of any structure may be measured. At present, we have very few measured values, but those that are available indicate that M_m and M_f are very much larger

* The same result is obtained by Ungar and Dietrich[4] under more general assumptions.

than might be anticipated. It is rare to get more than 20 dB attenuation at acoustic frequencies ($\epsilon = 0.1$) with isolation mounts of reasonable stiffness, and it is not uncommon to get no attenuation at all. For this reason, very soft mounts ($f_n = 5$ to 6 Hz) are being offered, and special constructions, such as pneumatic mounts, are now marketed. In any case, we may see from Eq. (13.24) and Fig. 13.5*d* that if a mount is effective at all, a softer one (M_i larger) will be that much more effective.

Figure 13.5*d* can be expanded into Fig. 13.5*e* by noting that, if there is no dissipation, the mobilities are equal to

$$M_m = \frac{1}{j\omega m_{in}}$$

$$M_i = j\omega C_i$$

$$M_f = \frac{C_f/m_f}{j(\omega C_f - 1/\omega m_f)}$$

and

$$f_0 = v_0/M_{in}$$

where the symbols are defined in Fig. 13.1*f*. The mobility of the foundation M_f is represented here by a different equivalent compliance C_f and mass m_f for each resonance condition of the foundation.

In Fig. 13.6 a predicted curve and some measured values of ϵ are shown for an experimental setup.[5] In Fig. 13.7 a measured value for M_f of a typical baseplate is plotted.[6] The high-frequency effects displayed in these figures can be taken into account as due to wave effects

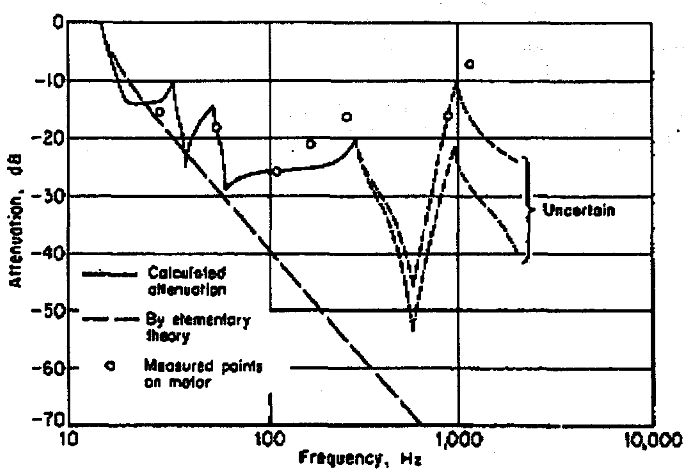

Fig. 13.6 Predicted and measured attenuation of an isolator.[5]

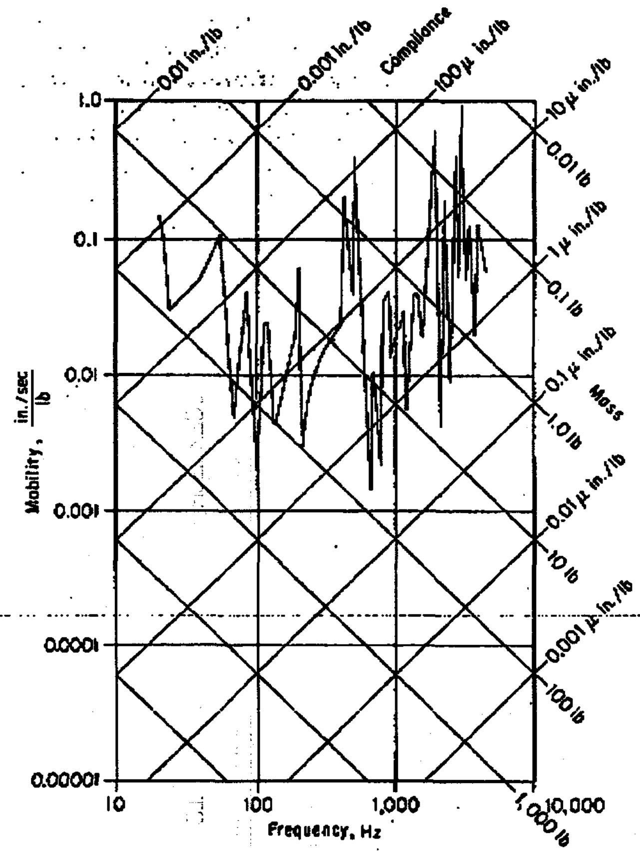

Fig. 13.7 The jagged line shows the measured mobility (in./sec-lb) as a function of frequency of a ribbed motor-generator baseplate 3/8 in. thick weighing 60 lb. Note that the lines running from upper left to lower right on the graph give the mobility as a function of frequency for the masses indicated. For example, 100 lb (a weight of 100 lb) at 100 Hz has a mobility of 0.0062 in./sec-lb. The lines running from upper right to lower left give the mobility as a function of frequency for the compliances (reciprocals of the spring constants) indicated. For example, a spring with a compliance of 10^{-5} in./lb has, at 1,000 Hz, a mobility of 0.062 in./sec-lb. This graph shows us that below 100 Hz this motor base behaves on the average like a mass of about 40 lb. Above 1,000 Hz it behaves on the average like a spring with a compliance of 3×10^{-6} in./lb. (*After Plunkett.*[6])

in the resilient mounting in which the mass on the mounting and/or the mass of the mounting are significant contributors. In Fig. 13.8, the effects are shown of such waves for two values of structural loss factor ($\eta = 0, 0.06$) and three values of the ratio of the mass on the mounting to the mass of the resilient element of the mounting ($\mu = 10, 100, 1{,}000$).[4] Superposed on these computed curves is a plot of experimental data taken from a transmissibility test of a U.S. Navy resilient mounting.[7]

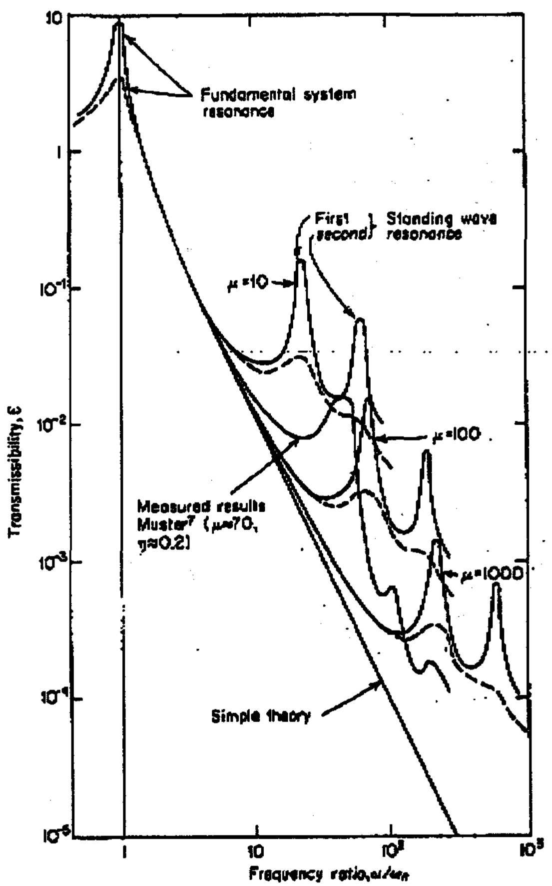

Fig. 13.8 Effect of the ratio μ of mass on an isolator to mass of the resilient element and of structural loss factor η on high-frequency transmissibility. The three solid calculated lines are for $\eta \approx 0$. The three dashed calculated lines are for $\eta \approx 0.06$.[4,7]

The computed and measured results are not directly comparable but the similarity is obvious.

Ungar and Dietrich[4] and Snowdon[8] have published detailed results of both analytical and experimental studies of the effects of isolator mass and damping on high-frequency performance of vibration isolators. The net effect of these phenomena is to increase significantly the transmission of forces or displacements through a resilient mounting over the levels that are predicted by the relatively simple classical theory. The first effects may occur at forcing frequencies as low as 10 to 30 times the natural frequency of the isolator mass system. It is clear that resilient mountings with good audio frequency isolation properties must be used in situations where both η and μ are large.

13.2 Vibration-isolation Systems

For systems in which more than a single degree of freedom must be considered in order to represent adequately the physical situation, there are no simple approaches. Fortunately, some multi-degree-of-freedom isolation problem procedures have been developed. For example, the SAE G-5 Committee on Aerospace Shock and Vibration has published a complete, stepwise procedure for the design of linear, center-of-gravity type vibration-isolation systems whose function is to protect fragile equipment.[9] A monograph devoted to the theory and practice of cushioning systems has been published recently by The Shock and Vibration Information Center (DoD).[10] However, the latter is not a design handbook. It contains no design procedures, but is intended to be a critical review of the current art in selecting, designing, analyzing, and using cushions.

A matrix approach to the fundamentals underlying vibration isolation for multi-degree-of-freedom systems has been developed by Smollen[11] which complements earlier analyses of Himelblau and Rubin,[12] who derived expressions for the effective stiffness and damping characteristics of a general isolation system and the associated equations of rigid-body motion using conventional techniques. They treat (see Fig. 13.9) the problem of a rigid body supported in a general way by isolators consisting of spring and damping elements. The solutions of specific examples are given by Smollen.[11]

13.3 Vibration Isolators and Isolator Materials

Four resilient materials are most commonly used in vibration isolators: metals in the form of various types of springs are used as well as mesh pads (or other shapes) of rubber, cork, and felt. Other materials, such as steel-mesh pads, pneumatic sacks, and even a gelatinous material similar to hectograph pad gel, have been used, but the majority of vibration isolators use one or more of the first four materials as the resilient element.

Metal Springs. Metal springs are by far the most commonly used, because the field of their use is as broad as that of machine design itself. They are used to isolate the most delicate scientific instruments from foundation vibrations, and yet masses up to 450 tons have been satisfactorily isolated with them. In theory, at least, the complete spectrum of frequencies can be isolated by metal springs. This is due, in part, to the large range of deflections which can be obtained by changing the dimensions and materials used in the design of the springs. This has been indicated in Fig. 13.10, which is a plot of Eq. (13.11*b*) with the nominal maximum permissible deflections of some elastic media superimposed upon it.

Metal springs have the advantages of ready interchangeability and several beneficial chemical characteristics (they resist corrosion by oil and water and are not affected by extremes of temperature). An advantage of industrial importance is that they can be produced in large quantities with only small variation in their individual characteristics. Inherently, metal springs have very little damping; the damping is about 0.001 of critical ($\zeta \approx 0.001$). However, external damping can be added to a system, if it is required by a particular application. A dashpot can be inserted in parallel with the metal spring (Fig. 13.3). Recoil mechanisms are sometimes built employing this principle—a metal spring for elasticity, a separate oil dashpot for a large amount of viscous damping.

Some resilient mountings are fabricated with a metal spring inside a rubber sack with a calibrated orifice that regulates the flow of air in and out of the sack and, thus, furnishes viscous damping. Coulomb (fric-

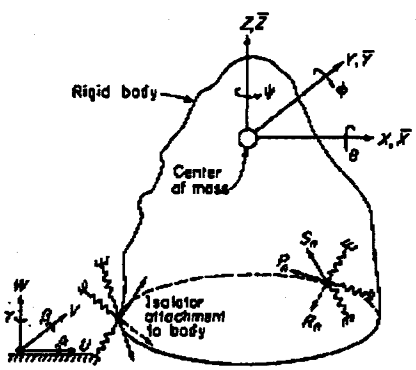

Fig. 13.8 General isolator support of rigid body (damping elements not shown).

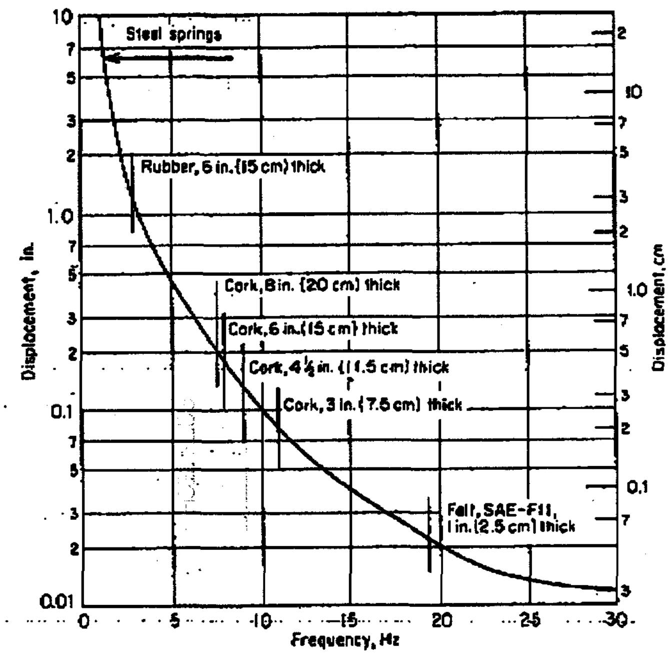

Fig. 13.10 Relation between static deflection and natural frequency. Typical natural frequencies of some practical isolators are shown.

tion) damping can also be supplied by external means. In leaf springs damping is caused by the rubbing action between individual leaves; in certain resilient mountings with steel mesh inserts it is caused by the rubbing of the wires on each other. Although it is unplanned in many cases, this form of damping occurs also in nonrigid mechanical joints where one part can slip relative to another.

For purposes of analysis, it is convenient to reduce coulomb and other nonviscous forms of damping to an equivalent viscous damping which uses energy dissipated per cycle as the basis of equivalence.[1]

Metal springs have the practical disadvantage of transmitting high frequencies very readily. For example, although the low natural frequency of an internal combustion engine (say, 15 Hz) can be isolated easily, the higher frequencies present are transmitted through the metal of the spring to the foundation. These higher frequencies in engines, for example, may range from two hundred to several thousand hertz and are due to detonation local resonances at the mounts and other sources. Transmission of these frequencies is minimized by ensuring that there