

Imaging
black holes
in
GRAVITATIONAL WAVES

Acknowledgements

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1. Brief introduction
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The black holes are unique objects in which we can expect the manifestation of strong field, because they are sources of a strong gravitational field in the General Relativity. The future discovery of a black hole is not only the confirmation of General Relativity but it is one of the ways to the new physics - the physics of strong gravitational field, the physics of superhigh energy.

There are some strong arguments that there exist few black holes in our Galaxy. These arguments came from x-ray and simultaneous x-ray, optical observations. Unfortunately, since the x-ray emission released far from the event horizon, it is difficult to claim definitely that central object of a x-ray source is a black hole

The gravitational waves emitted by quasi-normal modes of a black holes are released near the event horizon and there exists an opportunity to identify the source of gravitational radiation as a black hole with 100% confidence.

Therefore, one of the most interesting aims of new born gravitational wave astronomy is to identify the source as a black holes and to measure its properties.

I would like to talk to you about the complete scenario leading to the formation of a black holes from coalescing neutron stars; and about the expected gravitational wave signal.

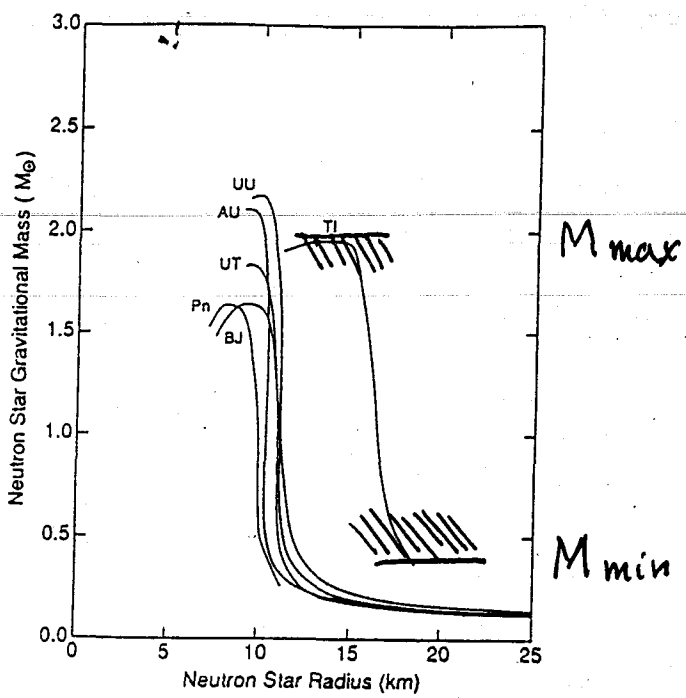


Fig. 3.5—Gravitational mass as a function of stellar radius for representative equations of state. The labeling is the same as in Figure 3.4.

From F.K. Lamb. *Neutron Stars and Black Holes*
 in: *Frontiers of Stellar Evolution*, ed. by D.L. Lambert
 Astron. Soc. of Pacif., p. 299 (1991)

$$M_{BH} = 2.5 - 4.5 M_{\odot}$$

Interval of allowed mass is narrow

One can divide the gravitational wave signal into 3 stages.

The first is the signal produced as the binary loses orbital angular momentum due to gravitational radiation.

The gravitational wave signal has nearly sinusoidal form with increasing amplitude and frequency.

$$f(t) = f_0 \left[1 - \left(\frac{100 \text{ kHz}}{\pi f_0} \right)^{8/3} \frac{768}{15} \frac{q}{(1+q)^2} 100 \text{ kHz} \cdot t(\text{sec}) \right]^{-3/8}$$

$$h(t) = 4 \pi^{2/3} 10^{-21} \left(\frac{M}{2M_\odot} \right)^{5/3} \frac{200 \text{ Mpc}}{R} \left[\left(\frac{100 \text{ kHz}}{\pi f_0} \right)^{8/3} - \frac{768}{15} \frac{q}{(1+q)^2} 100 \text{ kHz} \cdot t \right]^{3/8}$$

The total duration before coalescence is

$$t = 428 \text{ sec} \left(\frac{2M_\odot}{M} \right)^{8/3} \frac{(1+q)^2}{q}$$

and number of cycles is

$$N_{\text{cyc}} = 67622 \left(\frac{10 \text{ Hz}}{f} \right)^{5/3}$$

Signal-to-noise ratio is

$$\frac{\text{Signal}}{\text{Noise}} = h \sqrt{N_{\text{cyc}}} = 9 \cdot 10^{-22} 9^{1/2} \left(\frac{M}{2M_{\odot}}\right)^{5/3} \left(\frac{10\text{Hz}}{f}\right)^{1/6} \frac{2000}{R}$$

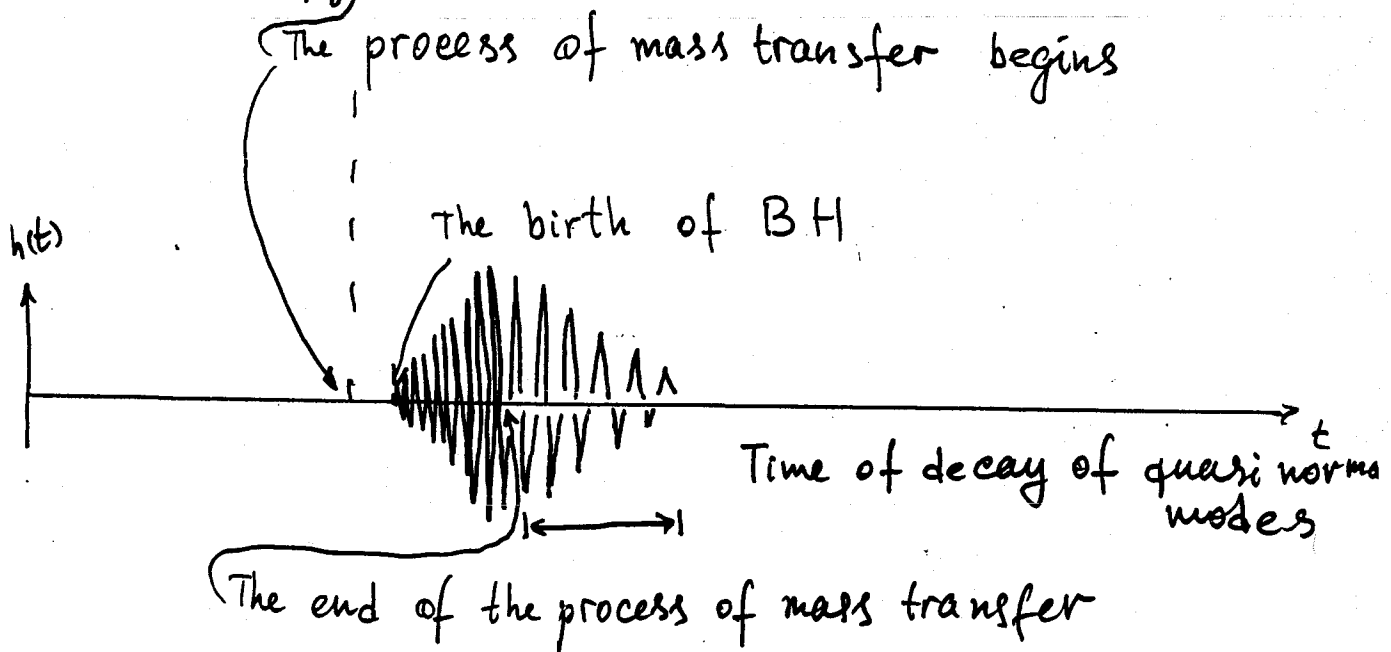
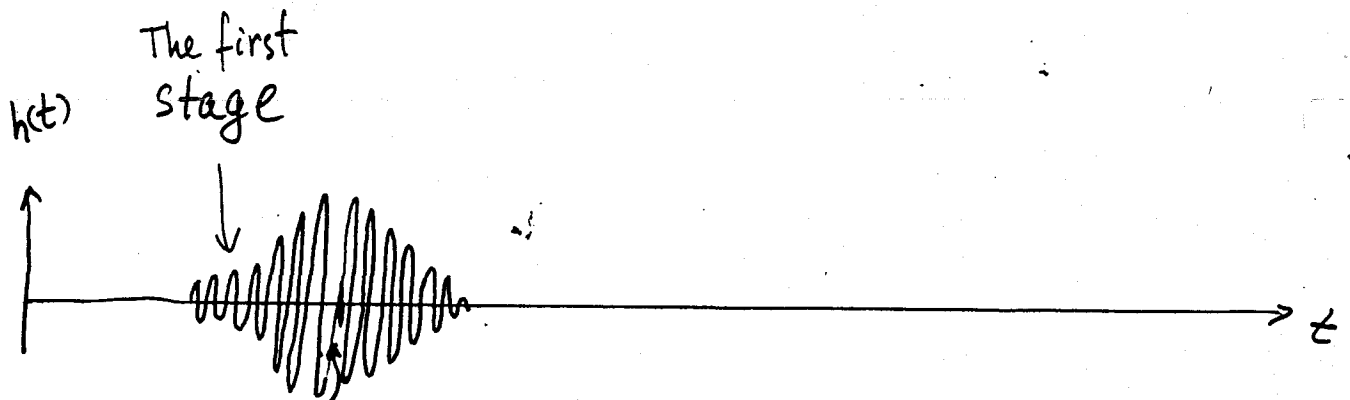
The second stage occurs when the neutron matter is transferred from the lighter neutron star to the heavier one, forming a single object.

The frequency of GW signal is constant approximately during the mass transfer and the amplitude of GW is decreasing with time.

The process begins when the separation between components is $\sim 40 - 50$ km.

The total number of cycles is 300 - 900.

The total number of cycles and the frequency of GW signal are determined by the properties of dense neutron matter.



$$\frac{J_{\text{orbital motion}}}{J_{\text{Black Hole}}} \sim \left(\frac{a_{\text{stab}}}{\frac{2GMt}{c^2}} \right)^{1/2} \sim 1$$

Potential energy of Black Hole \equiv kinetic energy of rotation
 (So called limiting Kerr Black Hole)

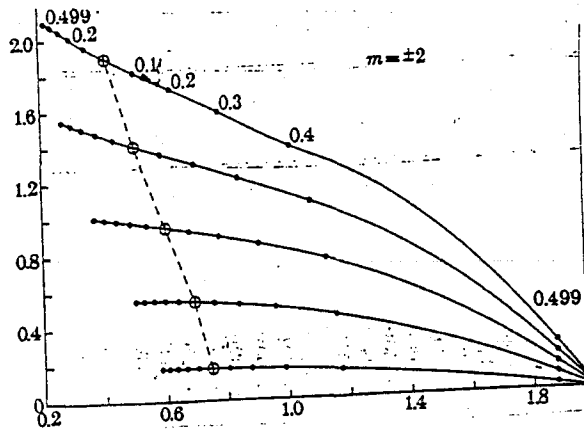


$$f_{\text{GW}} = \frac{2}{P_{\text{binary}}}$$



infinity
 r

f_{GW} = set of quasi-normal modes
of black holes

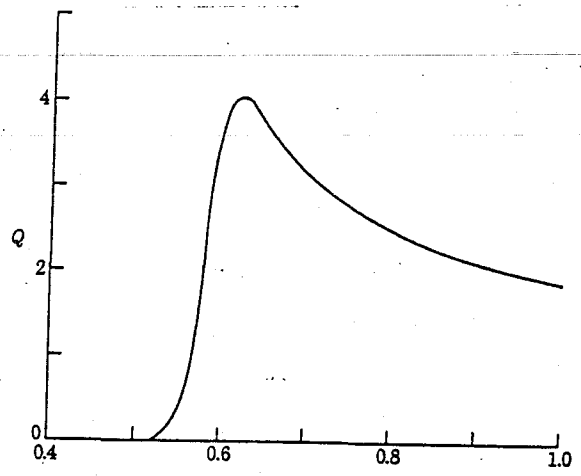


796 Hz

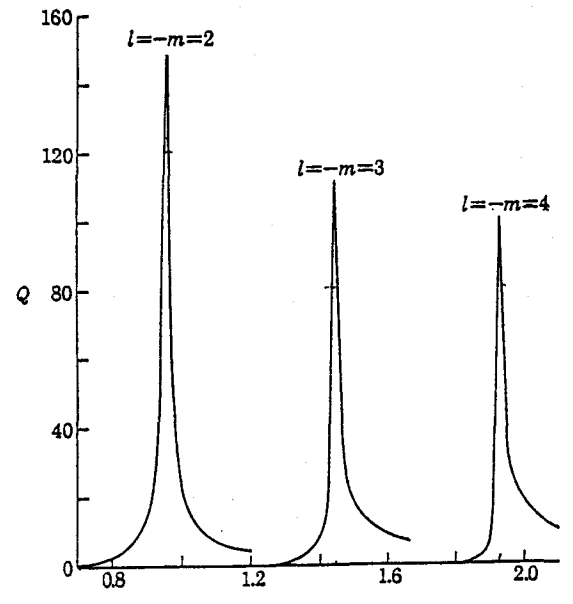
7,961 Hz ($M = 4 M_{\odot}$)

a	A_{lm}	ω_1
0.0000	(4.00000, 0.00000)	(0.747343, -0.177925)
0.1000	(3.89315, 0.02520)	(0.776500, -0.176977)
0.2000	(3.76757, 0.05324)	(0.815958, -0.174514)
0.3000	(3.61247, 0.08347)	(0.871937, -0.169128)
0.4000	(3.40228, 0.11217)	(0.960461, -0.155910)
0.4500	(3.25345, 0.11951)	(1.032583, -0.139609)
0.4900	(3.07966, 0.10216)	(1.128310, -0.103285)
0.4999	(3.02431, 0.07903)	(1.162546, -0.076881)

$l = 2 \quad m = 1$



$a = 0.4999 \quad f = 2,388 \text{ Hz} \quad \omega$



$a = 0.4999 \quad \omega$

Orbit

