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Optical Pathlength Noise in Sensitive Interferometers Due to Residual Gas

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Abstract

An analysis of the noise in the apparent optical pathlength in an interferometer due to the presence of statistical fluctuations in the passage of gas molecules through a laser beam is presented. An experimental test using a 40 m long prototype interferometer confirms the main predictions of the model regarding the dependences on frequency, pressure, molecular polarizability and molecular speed. The results are used to derive the vacuum requirements for the planned Laser Interferometer Gravitational-wave Observatory (LIGO).

1. Introduction

A new generation of large gravitational wave detectors based on precision laser interferometry is currently under development, including the Laser Interferometer Gravitational-wave Observatory (LIGO) Project¹ in the U.S. and the VIRGO Project² in Europe. These detec-

tors are designed to sense gravitational waves by measuring very small changes in separation of test masses suspended at the vertex and ends of an L-shaped vacuum system. A gravitational wave of the appropriate polarization will cause the test masses along one arm to move toward each other while causing those on the other arm move apart, with a reversal of the motion one-half cycle later. A laser interferometer using mirrors on the test masses monitors changes in the separations of the test masses. These interferometers will operate in the frequency range from about 10 Hz to 10 kHz, a range set at low frequencies by the spectrum of ground motion and at high frequencies by the expected absence of strong sources. The wave strength is characterized by dimensionless strain $h = \Delta L/L$, where L is the length of the arms and ΔL is the difference in arm length induced by the gravitational wave. The initial LIGO detectors are designed to have a sensitivity of $\approx 10^{-21}$; subsequent detectors may push this sensitivity to 10^{-23} or below. LIGO will have arm lengths of 4 km; thus the required precision of the measurement of ΔL is $\approx 10^{-18}$ m. Because of their extreme sensitivity, these interferometers are ^{affected by} sensitive to noise sources which are unimportant in most other applications.

One such noise source is the effect of residual gas in the laser beam paths. Even though these detectors are installed in high vacuum systems, residual gas in the laser path is a ^{even a good U.S.} significant potential noise source. ^{return can be significant} Statistical fluctuations of the number of gas molecules in the laser beam change the optical pathlength between the test masses in an interferometer. These changes will be sensed by the interferometer as apparent changes in length, ^{making identical} ~~creating~~ ^{myself} a source of noise. Heuristically, these variations can be thought of as fluctuations in the effective index of refraction. Because the vacuum system is one of the most costly parts of these installations it is important to have an accurate assessment of the vacuum required. In this letter, we derive the expected noise as a function of gas pressure and species, and describe an experiment to verify the effect.

2. Calculation of the effect

To compute the expected noise level for a given gas species, we first calculate the effect on the average phase of the light in the laser beam due to the presence of a single molecule and express this as a contribution to the apparent optical pathlength. This contribution depends on the molecular species and on the position of the molecule as it traverses the beam. In the limit where gas molecules cross the laser beam without collision (an approximation valid for the operating regimes of all proposed gravitational wave interferometers), the noise due to a population of residual gas molecules may then be calculated by summing their contributions using a Boltzmann distribution to give the expected noise expressed as a power spectral density.

The contribution of any given molecule i to the optical pathlength measured by the laser beam depends on its position in the beam. We will assume that the laser beam propagates along the z axis in a TEM00 mode from z_1 to z_2 with a an electric field in cylindrical coordinates

$$E(r, z) = E_0 \frac{\omega_0}{\omega(z)} \exp \left\{ -i[kz - \eta(z)] - r^2 \left(\frac{1}{\omega^2(z)} + \frac{ik}{2R(z)} \right) \right\} \quad (1)$$

where $w(z)$ is the beam spot size, assumed to vary slowly with z . , and $R(z) =$

The electric field of the incident light induces an electric dipole in the molecule which produces a reradiated field

$$E_i(\mathbf{r}) = \alpha E(\mathbf{r}_i) \frac{e^{ik|\mathbf{r}_i - \mathbf{r}|}}{|\mathbf{r} - \mathbf{r}_i|} \quad (2)$$

The molecular polarizability α can be obtained from the index of refraction of the gas n_o by $\alpha = (n_o - 1)/(2\pi\rho_o)$ where ρ_o is the number density of gas molecules at which n_o is measured.

In the pressure regime which is relevant for gravitational wave interferometers ($P \leq 10^{-6}$ torr), the mean free path of the gas is much larger than the laser beam diameter, and the molecules cross the beam without collisions. A particular molecule i passing through the

beam will have a radial coordinate (measured from the beam center) $r_i(t) = (b^2 + (v_t(t - t_i))^2)^{1/2}$ where b is the impact parameter (distance of closest approach to the beam axis), v_t is the speed of the molecule in the plane perpendicular to the beam, and t_i is the time of closest approach to the optic axis. The resulting optical pathlength, as measured by the phase of the light, is the sum of contributions from a large number of individual molecules

$$L(t) = L + \sum l_i(t) \quad (3)$$

where the contribution from the i th molecule, $l_i(t)$, depends on b , t_i and v_t . If we assume that the molecules have a Boltzmann velocity distribution, then a simple extension of the formula for the spectral noise density of a series of pulses^{Yariv} gives

$$S_L(f) df = 2 \int \int \int \frac{dN(b, v_t, T)}{dt} db dv_t dz \quad (4)$$

where the rate of molecules making their closest approach to the optic axis (per unit length along the axis) with impact parameter b and transverse velocity v_t is

$$\frac{dN(b, v_t, T)}{dt} db dv_t dz = \frac{4\rho v_t^2}{v_0^2} \exp[-\frac{v_t^2}{v_0^2}] db dv_t dz \quad (5)$$

where v_0 is the mean speed of the gas molecules,

$$v_0 = (2kT/M)^{1/2}.$$

Sensitivity of gravitational wave interferometers is conventionally expressed in terms of the amplitude spectral noise density of either displacement ($\tilde{x}(f)$) or strain ($\tilde{h}(f)$), defined as the square root of the power spectral density of the corresponding quantity⁴. A simple calculation gives the predicted power spectral density of the noise $S_x(f)$

$$S_x(f) = \frac{4\rho(2\pi\alpha)^2}{v_0} \int_{z_1}^{z_2} \frac{\exp[-2\pi f w(z)/v_0]}{w(z)} dz. \quad (6)$$

Current plans for gravitational wave interferometers call for cavity geometries with g values in the range x - x ; the resulting beam radius $w(z)$ varies by less than a factor of 2 along the length of the arms. Taking $w(z) \approx w_0$, we may estimate $h(f)$ as

$$h(f) = \frac{\sqrt{2S_x(f)}}{L} \approx 4\pi\alpha \left(\frac{2\rho}{v_0 w_0 L} \right)^{\frac{1}{2}} \quad (2\pi f \ll v_0/w_0) \quad (7)$$

This latter form for $h(f)$ shows the noise due to residual gas falling as $L^{-1/2}$ for fixed beam radius. For diffraction-limited optics, however, the beam radius w_0 increases as $L^{1/2}$, so that in practice the strain noise $h(f)$ actually falls as $L^{-3/4}$ (assuming fixed gas properties).

3. Experimental tests on the LIGO 40-meter interferometer

The predictions of Eq. 6 were tested using a prototype gravitational-wave interferometer⁵. A simplified schematic of this interferometer is shown in Fig. 1. The two 40 m long arms of the interferometer are defined by four test masses, which are suspended from vibration-isolated platforms. Mirrors on the test masses form two resonant optical cavities. Optoelectronic control systems maintain the resonance condition in each cavity by monitoring the relative phase between incident laser field and the returning leakage of the internal cavity field, using a radiofrequency phase modulation technique⁶. The laser wavelength is stabilized to a resonance of one cavity (the “primary”), and the other cavity (the “secondary”) is forced into resonance with the laser by applying a magnetic force to permanent magnets attached to one of its mirrors. The force required to attain resonance in the secondary is thus proportional to the difference between the optical paths of the two arms (modulo half an optical wavelength), and is recorded and analyzed. This readout is calibrated by applying a test force of known magnitude to one mirror of the primary cavity, using a similar magnet arrangement. Natural disturbances of the actual arm length, such as seismic excitation, cause significant motions of the test masses below a few hundred Hertz, but above 800 Hz the motion of the test masses was less than $3 \times 10^{-18} \frac{\text{m}}{\sqrt{\text{Hz}}}$ at the time of these measurements.

Each cavity of the 40 m interferometer consists of one plane mirror and one concave mirror with a radius of curvature of 62 m, giving a beam radius $w(z)$ which varies between 2.2 and 3.7 mm. The condition that the mean free path for the gas must exceed the beam diameter thus limits the validity of Equation 6 to pressures below approximately 30 mT.

A similar pressure limit was imposed purely for technical reasons, due to the danger of high voltage discharges within the vacuum system. At these pressures, the light species (predominantly hydrogen and water) which are expected to dominate the LIGO residual gas would ^{have} produce too small an effect in ~~the 40-meter instrument~~ to be accurately measured against other noise sources. To enhance the effect, three heavier gas species, with larger values of α and smaller values of v_o , were chosen: Xe, CO₂, and N₂.

The interferometer's vacuum vessel was pumped by a combination of turbomolecular and cryogenic pumps, and in normal operation attained a pressure of approximately 3×10^{-6} T. A small metering volume was instrumented with a wide-range capacitance manometer vacuum gauge⁷ whose calibration is independent of gas species. The ratio R of this metering volume to the volume of the interferometer vessel (approximately 3600 liters) was measured by opening an isolation valve separating the two at a variety of initial pressures and noting the final pressures after equilibration. For Xe and N₂ measurements, the metering volume was evacuated and filled to R times the desired final pressure of the interferometer vessel. After measuring and recording the interferometer background noise level, the pumps were sealed off and the isolation valve opened to admit the gas. This procedure resulted in an estimated pressure uncertainty of ± 0.2 mT at the lowest ^{sample} pressures studied. For the CO₂ series, a change to the metering apparatus required direct readout of the manometer at the final pressure, limiting the precision to ± 0.5 mT.

Noise spectral densities measured at several different pressures for Xe are shown in Figure 2. Note that the noise due to the gas is approximately independent of frequency, in agreement with Equation 6, over the frequency range from approximately 800 to 2000 Hz (where the gas noise dominated the baseline interferometer noise); for the relatively small beams in this interferometer, the exponential cut-off in the noise is above 10 kHz. Narrow peaks in the background and all the measurements are due to mechanical resonances of the interferometer structures or powerline interference. At the highest pressures (above about 10 mT) some small peaks appeared as well; these may be caused by acoustic coupling to the test masses through the sample gas. At the lowest pressures, the observed noise was only slightly higher

than the baseline noise. The background noise level was subtracted from each measurement in quadrature to determine the contribution due to the gas alone.

The observed noise contribution from each gas is compared with the prediction in Figure 3. The points show the measured noise in the region around 1.5 kHz, while the curves show the predictions of Eq. 6. The observation that the measured noise appears to fall slightly below the prediction at the highest pressure for each of the gases may be attributable to the breakdown of the theory as the mean free path of the gas particles becomes comparable with the beam diameter. The fractional fluctuations in column density are expected to decrease as collisions begin to correlate particle trajectories.

4. Pressure requirements for large gravitational wave interferometers

The results of this analysis can be used to set the pressure requirements for the vacuum systems of large gravitational wave observatories such as LIGO. The residual gas in clean, baked stainless steel vacuum systems is predominantly hydrogen, frequently higher than 99%. Other potentially significant species from the standpoint of index fluctuations include water vapor, nitrogen, carbon monoxide, and carbon dioxide, in approximate order of their expected contribution. Table 1 shows the requirements derived for these gases to allow both the initial LIGO target and advanced LIGO goal sensitivities to be reached without significant degradation ($\tilde{h}(f) \lesssim 2 \times 10^{-23}/\sqrt{\text{Hz}}$ at 200 Hz and $\tilde{h}(f) \lesssim 2 \times 10^{-24}/\sqrt{\text{Hz}}$ at 100 Hz, respectively). Higher-mass components, while slower-moving and generally more polarizable, are expected to constitute negligible fractions of the total residual gas in LIGO.

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FIGURES

Fig. 1. Semi-schematic diagram of the LIGO 40-meter interferometer.

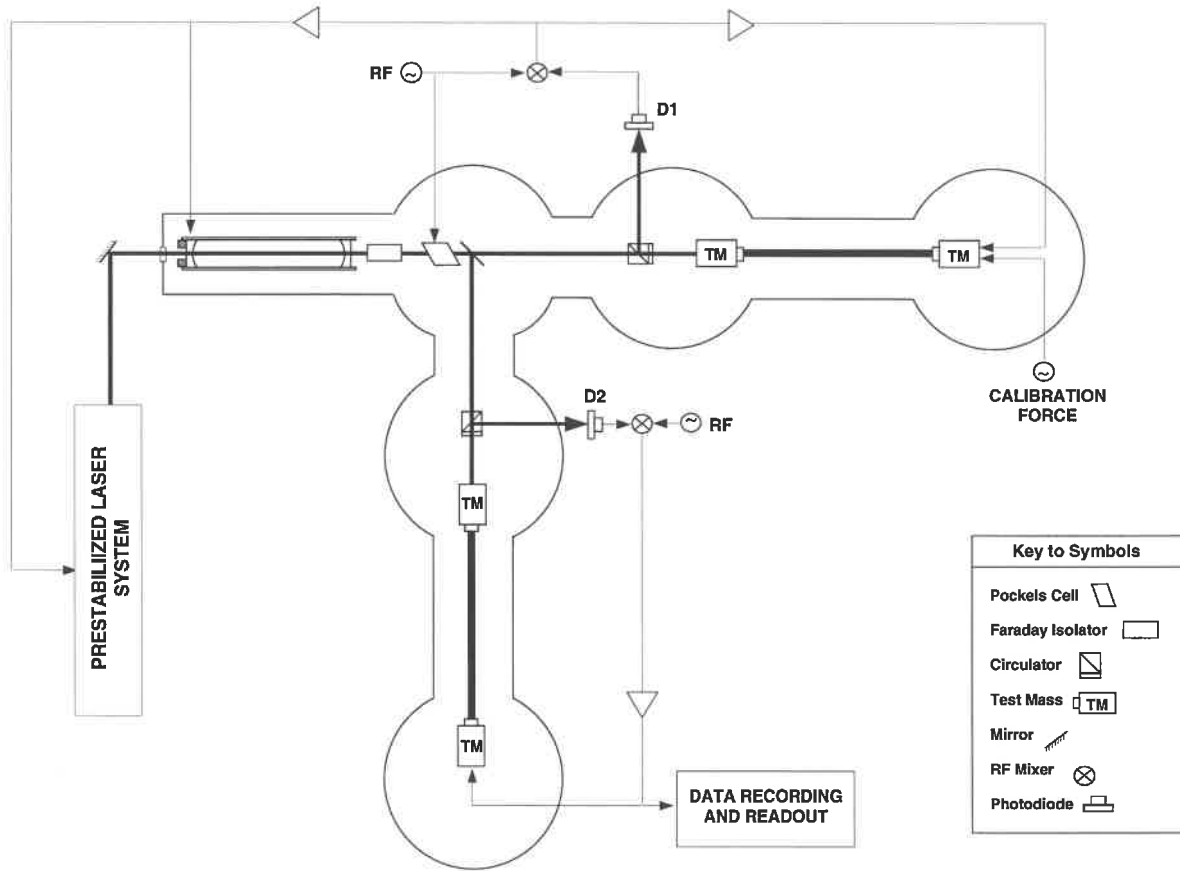


Fig. 2. Interferometer displacement noise spectral density measured at low pressure (background, lowest curve) and with Xenon gas at four pressure levels.

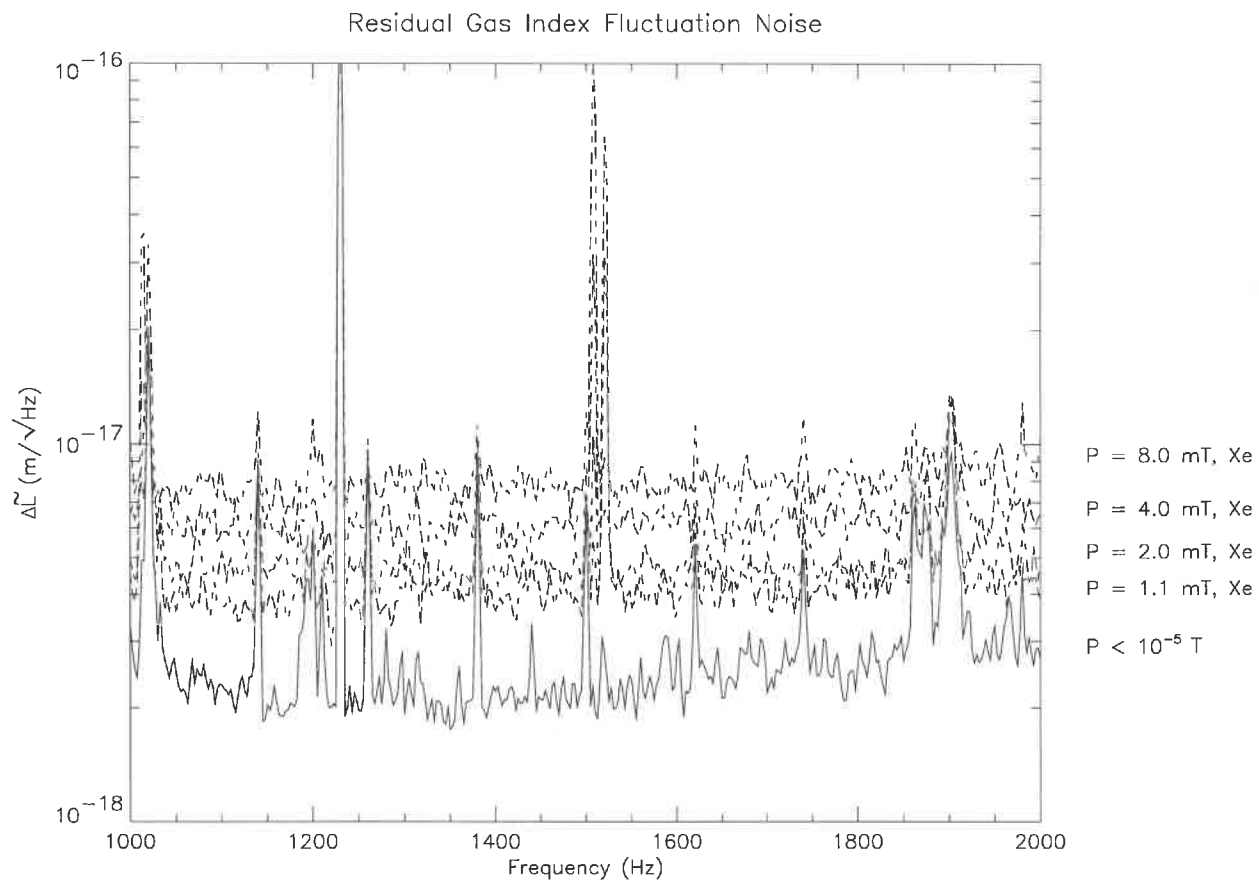
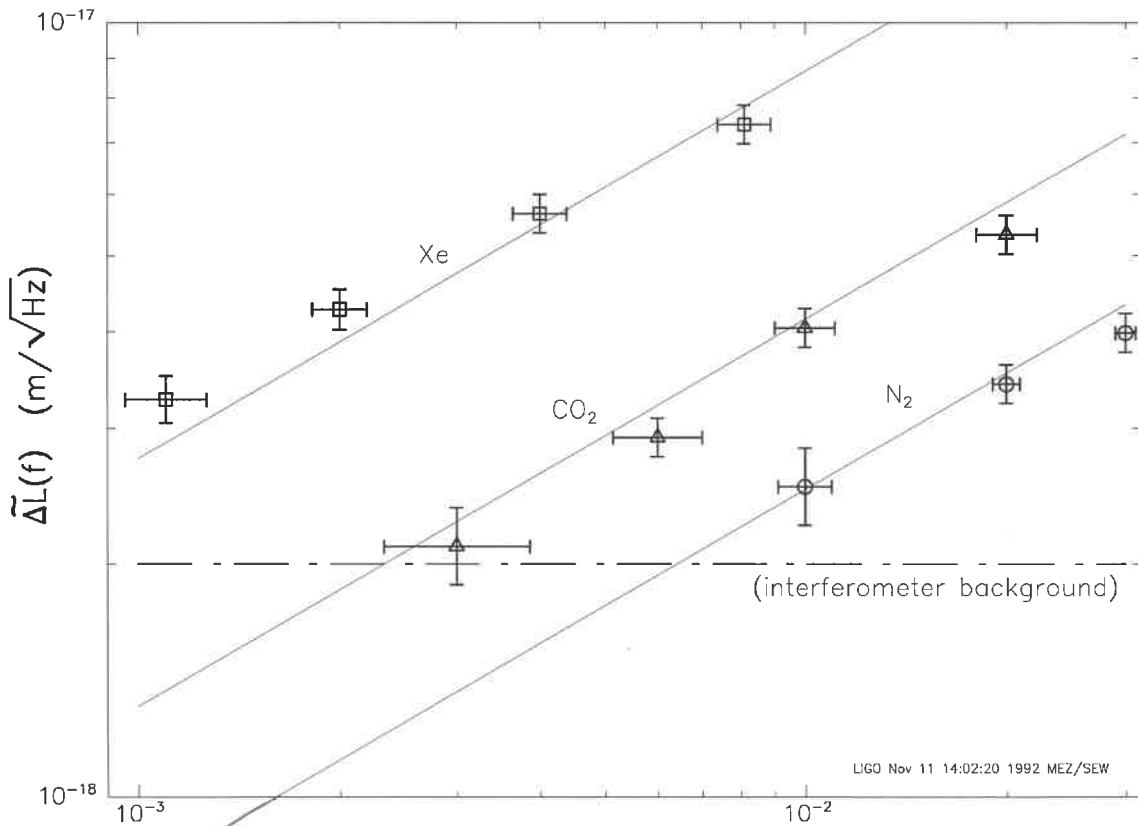


Fig. 3. Interferometer displacement noise spectral density predicted by Equation 6 (lines) and directly measured (data points) as a function of pressure for Xe, CO₂, and N₂. The interferometer noise background (dashed line), measured before admitting and after pumping away each test gas, was subtracted in quadrature from the noise measured at each pressure. The points represent local averages in the spectral neighborhood of 1300 Hz, where the interferometer background was lowest. Vertical error bars include estimated uncertainties in the background subtraction and in the absolute calibration of the interferometer readout. Horizontal error bars include specified scale factor error, offset, and readout precision of the capacitance manometer.



N_2 : $8 \times 10^{-19} \frac{m}{\sqrt{Hz}}$
 @ 10^{-3} torr
 H_2O : $8 \times 10^{-19} \frac{m}{\sqrt{Hz}}$
 at 1.6×10^{-3} torr
 @ 10^{-7} torr, $8 \times 10^{-21} \frac{m}{\sqrt{Hz}}$
 from ~~not~~ N_2

TABLES

Table 1. Allowable beam tube residual gas partial pressures for initial and advanced LIGO interferometers. Equation 6 is employed using the planned LIGO parameters $L = 4$ km and $w_0 \approx 22$ mm. With all gas species present at the specified limits, the respective interferometer strain sensitivities would be degraded by no more than 5 %. Other species are considered unlikely to occur at partial pressures which could contribute significantly.

Gas	Allowable Partial Pressure (Torr)	
	Initial LIGO	Advanced LIGO
H ₂	1×10^{-6}	1×10^{-9}
H ₂ O	1×10^{-7}	1×10^{-10}
N ₂	6×10^{-8}	6×10^{-11}
CO	5×10^{-8}	5×10^{-11}
CO ₂	2×10^{-8}	2×10^{-11}