LIGO-T960088-00-D

96/05/15

FACSIMILE TRANSMISSION

The Institute of Space and AstronauticalScience(ISAS)
3-1-1 Yoshinodai, Sagamihara, Kanagawa 229, Japan
TEL:+81 427 (51) 3957 FAX:+81 427 (59) 4237
e-mail: knobuki@pleiades.sci.isas.ac.jp

TO: Dr. Seiji Kawamura
California Institute of Technology

FAX No.: 001-1-818-304-9834

Number of pages: 5
From: Nobuki Kawashima

MESSAGE:

Dear Dr. Kawamura:
川村静児様、
　よろしくご検討ください。

宇宙科学研究所
河島信樹

Conversion of the Fringe Function to the Displacement

ABSTRACT

The procedure to convert the fringe function of the laser interferometer with a suspended mirror system to the actual dispacement function is shown. Using the first and second derivatives of the fringe fuction, it is shown that the discrimation of the bright fringe and the turn-over of the mirror motion in a practically allowable accuracy.

1. Introduction

In the laser interferometer gravitational wave detector, the interferomete is locked at a dark fringe. In order to get a high sensitivity, the gain of the feedback system should be as high as possible, however, the hgher the gain, the narrower the dynamic range of the feedback and the system becomes very weak to external disturbances. The reason why is that in the locking, the fringe function is used. It has an dynamic range of less than one wave length. Above it, the function becomes non-linear.
In the laser interferometer gravitational wave detector, the mirror is suspended so that the mirror motion itself is slow, say 1 or 2 Hz. The fringe fucntion itself is, however, upconverted from the displacement function and it varies in the frequency range around 1 kHz. This makes the dynamic range very narrow especially when a large gain in feedback system is employed.
This paper is to establish a procedure to convert the fringe fuction to the displacement.

2. Characteristics of the Fringe Function and the Displacement

The fringe function can be expressed as

$$F(t) = \sin(A \cos 2\pi ft) \quad (1)$$

where

A = 100 - 1,000 : Amplitude of the displacement

$f = 1 - 5$: Frequency of the displacement

First derivative

$$F'(t) = A\, 2\pi f \cos(A \cos 2\pi ft) \sin 2\pi ft \tag{2}$$

Second derivative

$$F''(t) = A(2\pi f)^2[-A \sin(A \cos 2\pi ft) \sin^2 2\pi ft + \cos(A \cos 2\pi ft) \cos 2\pi ft]] \tag{3}$$

Let's assume that $t = t_0 + \Delta t$ and each function is expanded around t_0 ;

$$F(t) = \sin(A \cos 2\pi ft)$$

$$= \sin(A \cos 2\pi ft_0) + [A\, 2\pi f \cos(A \cos 2\pi ft_0) \sin 2\pi ft_0]\Delta t + [A(2\pi f)^2[- A \sin(A \cos 2\pi ft_0) \sin^2 2\pi ft_0 + \cos(A \cos 2\pi ft_0) \cos 2\pi ft_0]](\Delta t)^2 \tag{1a}$$

$$F'(t) = [A\, 2\pi f \cos(A \cos 2\pi ft_0) \sin 2\pi ft_0] + 2[A(2\pi f)^2[- A \sin(A \cos 2\pi ft_0) \sin^2 2\pi ft_0 + \cos(A \cos 2\pi ft_0) \cos 2\pi ft_0]](\Delta t) \tag{2a}$$

$$F''(t) = 2[A(2\pi f)^2[- A \sin(A \cos 2\pi ft_0) \sin^2 2\pi ft_0 + \cos(A \cos 2\pi ft_0) \cos 2\pi ft_0]] \tag{3a}$$

i) $\sin(A \cos 2\pi ft_0) = +/- 1$ $\cos(A \cos 2\pi ft_0) = 0$

This corresponds to the bright fringe.

$$F(t) = +/- \sin(A \cos 2\pi ft)$$
$$= +/- 1 -/+ [(A)^2(2\pi f)^2[\sin^2 2\pi ft_0](\Delta t)^2 \tag{1b}$$
$$F'(t) = -/+ 2[A^2(2\pi f)^2 \sin^2 2\pi ft_0](\Delta t) \tag{2b}$$
$$F''(t) = -/+ 2[A^2(2\pi f)^2 \sin^2 2\pi ft_0$$
$$= -/+ 2[A(2\pi f)^2 \; A \sin^2 2\pi ft_0 \tag{3b}$$

ii) $\sin 2\pi ft_0 = 0$ $\cos 2\pi ft_0 = +/- 1$

This corresponds to the turn-over of the displacement :

$$F(t) = \sin(A \cos 2\pi ft)$$
$$= +/- \sin(A) +/- [\cos(A)](\Delta t)^2 \quad (1c)$$
$$F'(t) = +/- 2[A(2\pi f)^2[\cos(A)](\Delta t) \quad (2c)$$
$$F''(t) = +/- 2A(2\pi f)^2 \cos(A) \quad (3c)$$

3. Conversion of the Fringe Function to the Displacement

To transform the fringe function to the displacement, a counter correspo nding to the the bright fringe is set and each time when the bright frin ge is detected from $F'(t) = 0$, the counter is count up in one direction (either add or subtract) until the turn-over of the displacement is dete cted. When the turn-over of the displacement is detected, the count-up s hould be reduced.

Since the turn-over is also detected from $F'(t) = 0$, it is necessary to discriminate them.

One possibility is to use the difference of $F(t)$. For the bright fringe, $F(t) = +/- 1$, while at the turn-over, $|F(t)| = |\sin(A)| < 1$. But it is not perfect, because sin (A) can be +/- unity.

Another and more clever is to use $F''(t)$. Comparing Eq.(3b) and (3c), g enerally, $F''(t)$ at the bright fringe is much larger than the one at the turn-over point, because in Eq.(3b), more A is multiplied.

However, near the turn-over point, it is not true. $F''(t)$ can be zero. W e need more accurately compare them.

We assume that at t_o, the displacement has turned over and at the same time, the bright fringe occurs. This would be the most severe case to be considered.

Let t_1 be the bright fringe neghboring to the turn-over point.

$F''(t)$ value at this neighboring bright fringe to the one at the turning point is estimated :

At the bright fringe, the first derivative

$$A\, 2\pi f \cos(A \cos 2\pi ft) \sin 2\pi ft = 0 \text{ so that}$$
$$A \cos 2\pi ft = N\pi \quad (4)$$

$$A \cos 2\pi ft^1 = N\pi \quad (5)$$
$$A \cos 2\pi ft_0 = (N+1)\pi \quad (6)$$
$$A \cos 2\pi ft_0 - A \cos 2\pi ft^1 = \pi \quad (7)$$

$$\delta t_1 = t_o - t_1$$
$$A - A(1 - (2\pi f \delta t_1)^2/2) = \pi \quad (8)$$
$$A/2\, (2\pi f \delta t_1)^2 = \pi$$

$$(2\pi f \delta t_1)^2 = 2\pi / A \tag{9}$$

$$\begin{aligned} F''(t_1) &= -2[A(2\pi f)^2 \; A \sin^2 2\pi f(t_0 + \delta t_1) \\ &\quad - 2[A(2\pi f)^2 \; A (2\pi f \delta t_1)^2 \\ &\quad - 2[A(2\pi f)^2 \; 2\pi \end{aligned} \tag{10}$$

Comparing Eqs.(3c) and (10), it is clear that F''(t) at the bright fring e neighboring the turn-over point is more than 2π larger than F''(t) at the turn-over point, when the turn-over point and the bright fringe coi ncides.

The procedure of conversion then is as follows :

1. When F'(t) = 0 is detected and unless the absolute value of $|F''(t)/[2[A(2\pi f)^2]|$ is not less than π, the counter should be count up/down in one direction.

2. When $|F''(t)/[2[A(2\pi f)^2]| < \pi$ is detected, the counter is closed b y a gate pulse until $|F''(t)/[2[A(2\pi f)^2]| > \pi$ is detected.

3. When $F''(t)/[2[A(2\pi f)^2] > \pi$ is detected, the counter is opened and the counter should be count down/up(in reverse direction) each time when F'(t) = 0.

This procedure is not perfect, because when the bright fringe is very ne ar to the turn-over point of the displacement, F''(t) bocomes small enou gh so that it is not recognized as the bright fringe. In most cases, how ever, this miscounting is for both of the bright fringes before and aft er the turn-over so that the effect will be only to decrease by a little bit the peak of the displacement function. Only in very rare cases, eit her one of the bright fringes neighboring the turn-over point is not cou nted, but still the effect is very small, usually cause an offset of 1 % or less.

4. Discussion

The procedure to convert the finge function of the interferometer to the displacement function is shown using the second derivative of the fring e function.