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For Lisa

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8/5/96 (1)

Modulation / Synch detection

From A.B. Carlson, pg 226.

$$x_c(t) = A_c J_0(\beta) \cos \omega_c t + \sum_{n\,odd}^{\infty} A_c J_n(\beta) \left[\cos(\omega_c + n\omega_m)t - \cos(\omega_c - n\omega_m)t \right] + \sum_{n\,even}^{\infty} A_c J_n(\beta) \left[\cos(\omega_c + n\omega_m)t + \cos(\omega_c - n\omega_m)t \right]$$

In phasor form:

$$x_c(t) = \left[A_c J_0(\beta) + \sum_{n\,odd}^{\infty} A_c J_n(\beta) \left(e^{jn\omega_m t} - e^{-jn\omega_m t} \right) + \sum_{n\,even}^{\infty} A_c J_n(\beta) \left(e^{jn\omega_m t} + e^{-jn\omega_m t} \right) \right] e^{j\omega_c t}$$

In terms of IFO variables:

$$E_{src} = E e^{j\omega_b t}\left[J_0(\Gamma) + \sum_{n\,odd}^{\infty} J_n(\Gamma) e^{nj\omega_m t} - \sum_{n\,odd}^{\infty} J_n(\Gamma) e^{-nj\omega_m t} + \sum_{n\,even}^{\infty} J_n(\Gamma) e^{nj\omega_m t} + \sum_{n\,even}^{\infty} J_n(\Gamma) e^{-nj\omega_m t}\right]$$

$$E_{src} = \left[E_0 + E_1 e^{j\omega_m t} - E_{-1} e^{-j\omega_m t} + E_2 e^{2j\omega_m t} + E_{-2} e^{-2j\omega_m t} + E_3 e^{3j\omega_m t} - E_{-3} e^{-3j\omega_m t} + \ldots \right]_{src} e^{j\omega_b t}$$

$$E_0 = E J_0(\Gamma)$$

$$E_1 = E_{-1} = E J_1(\Gamma)$$

$$E_2 = E_{-2} = E J_2(\Gamma)$$

$$E_3 = E_{-3} = E J_3(\Gamma)$$

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Detection for SBs to J_3:

$$E_{det} = \left[E_0 + E_1 e^{j\omega_m t} + E_{-1} e^{-j\omega_m t} + E_2 e^{2j\omega_m t} + E_{-2} e^{-2j\omega_m t} + E_3 e^{3j\omega_m t} + E_{-3} e^{-3j\omega_m t} \right] e^{j\omega_0 t}$$

$$E_{det}^* = \left[E_0^* + E_1^* e^{-j\omega_m t} + E_{-1}^* e^{j\omega_m t} + E_2^* e^{-2j\omega_m t} + E_{-2}^* e^{2j\omega_m t} + E_3^* e^{-3j\omega_m t} + E_{-3}^* e^{3j\omega_m t} \right] e^{-j\omega_0 t}$$

$P = E_{det} E_{det}^*$ (averaged @ ω_0)

$$= E_0 E_0^* + E_0 E_1^* e^{-j\omega_m t} + E_0 E_{-1}^* e^{j\omega_m t}$$
$$+ E_1 E_0^* e^{j\omega_m t} + E_1 E_1^* + E_1 E_{-2}^* e^{-j\omega_m t} + \cdots$$
$$+ E_{-1} E_0^* e^{-j\omega_m t} + E_{-1} E_{-1}^* + E_{-1} E_{-2}^* e^{j\omega_m t} + \cdots$$
$$+ E_2 E_1^* e^{j\omega_m t} + E_2 E_2^* + E_2 E_3^* e^{-j\omega_m t} + \cdots$$
$$+ E_{-2} E_{-1}^* e^{-j\omega_m t} + E_{-2} E_{-2}^* + E_{-2} E_{-3}^* e^{j\omega_m t} + \cdots$$
$$+ E_3 E_2^* e^{j\omega_m t} + E_3 E_3^* + \cdots$$
$$+ E_{-3} E_{-2}^* e^{-j\omega_m t} + E_{-3} E_{-3}^* + \cdots$$

$$p = [E_0 E_{-1}^* + E_1 E_0^* + E_{-1} E_{-2}^* + E_2 E_1^* + E_{-2} E_{-3}^* + E_3 E_{-2}^*] e^{j\omega_m t} + [E_0 E_1^* + E_1 E_2^* + E_{-1} E_0^* + E_2 E_3^* + E_{-2} E_{-1}^* + E_{-3} E_{-2}^*] e^{-j\omega_m t} + \text{DC terms} + \text{Higher f terms}$$

$$e^{j\omega_m t} = \cos(\omega_m t) + j \sin(\omega_m t) \equiv c + js$$

$$e^{-j\omega_m t} = \cos(\omega_m t) + j \sin(\omega_m t) = c - js$$

$$p = [E_0 E_{-1}^* + E_1 E_0^* + E_{-1} E_{-2}^* - E_2 E_1^* + E_{-2} E_{-3}^* + E_3 E_2^* + E_0 E_1^* + E_1 E_2^* + E_{-1} E_0^* + E_2 E_3^* + E_{-2} E_{-1}^* + E_{-3} E_{-2}^*] c + j [E_0 E_{-1}^* + E_1 E_0^* + E_{-1} E_{-2}^* + E_2 E_1^* + E_{-2} E_{-3}^* + E_3 E_2^* - E_0 E_1^* - E_1 E_2^* - E_{-1} E_0^* - E_2 E_3^* - E_{-2} E_{-1}^* - E_{-3} E_{-2}^*] s$$

$$E_1 E_0^* + E_0 E_1^* = 2 \mathrm{Re}(E_1 E_0^*)$$

$$E_1 E_0^* - E_0 E_1^* = -2 \mathrm{Im}(E_1 E_0^*)$$

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$$P = 2\,Re\left[E_0 E_{-1}^* + E_1 E_0^* + E_{-1} E_{-2}^* + E_2 E_1^* + E_{-2} E_{-3}^* + E_3 E_2^*\right] C - 2j\,Im\left[E_0 E_{-1}^* + E_1 E_0^* + E_{-1} E_{-2}^* + E_2 E_1^* + E_{-2} E_{-3}^* + E_3 E_2^*\right] S$$

$$Re\left[E_0 E_1^*\right] = Re\left[E_1 E_0^*\right]$$

$$Im\left[E_0 E_1^*\right] = -Im\left[E_1 E_0^*\right]$$

$$P = 2\,Re\left[E_0 E_{-1}^* + E_0 E_1^* + E_{-1} E_{-2}^* + E_1 E_2^* + E_{-2} E_{-3}^* + E_2 E_3^*\right] C - 2j\,Im\left[E_0 E_{-1}^* - E_0 E_1^* + E_{-1} E_{-2}^* - E_1 E_2^* + E_{-2} E_{-3}^* - E_2 E_3^*\right] S$$

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For sine modulation:

$$V_{demod} = 2\,Im\left[E_0 E_{-1}^* - E_0 E_1^* + E_{-1} E_{-2}^* - E_1 E_2^* + E_{-2} E_{-3}^* - E_2 E_3^*\right]$$

$$V_{quad} = 2\,Re\left[E_0 E_{-1}^* + E_0 E_1^* + E_{-1} E_{-2}^* + E_1 E_2^* + E_{-2} E_{-3}^* + E_2 E_3^*\right]$$

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For sine modulation:

$$V_{demod} = 2\,\mathrm{Im}\left[E_0 E_{-1}^* - E_0 E_1^* + E_{-1} E_{-2}^* - E_1 E_2^* + E_{-2} E_{-3}^* - E_2 E_3^*\right]$$

$$V_{quad} = 2\,\mathrm{Re}\left[E_0 E_{-1}^* + E_0 E_1^* + E_{-1} E_{-2}^* + E_1 E_2^* + E_{-2} E_{-3}^* + E_2 E_3^*\right]$$