LIGO-T960160-00-D

NIST

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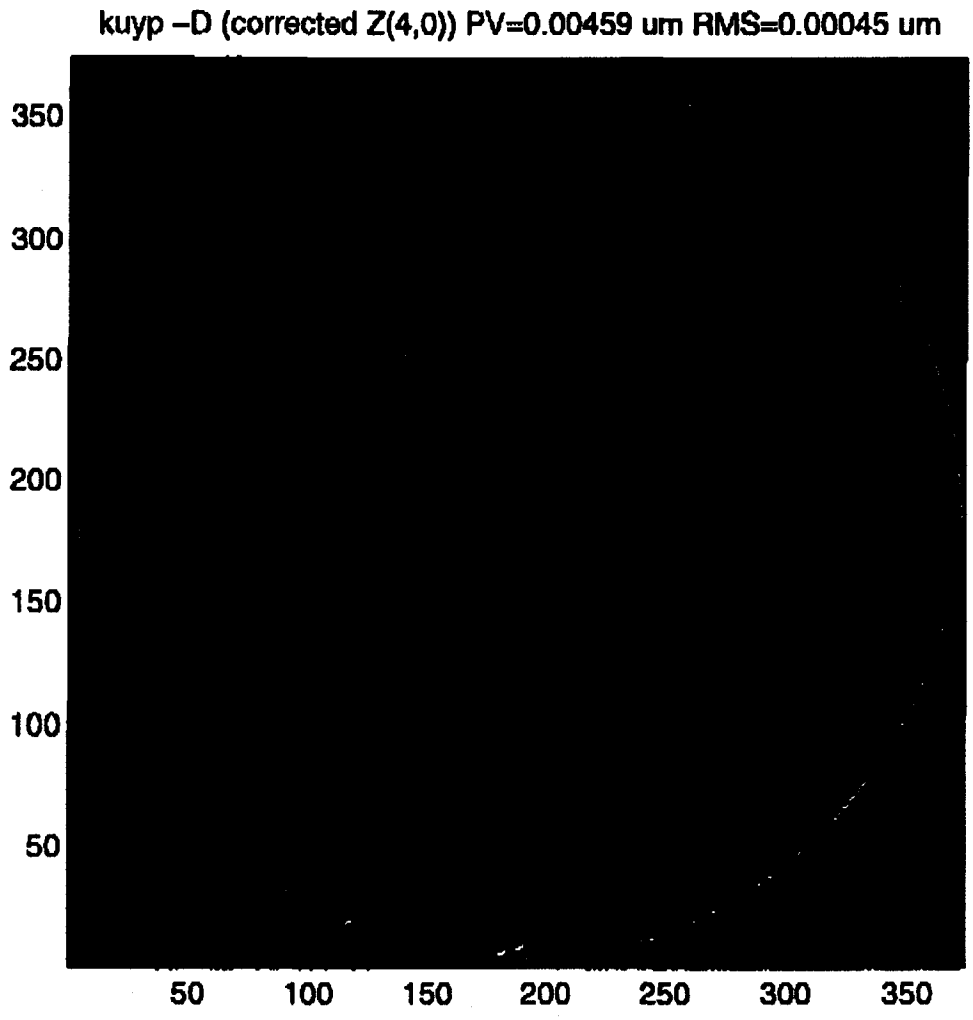
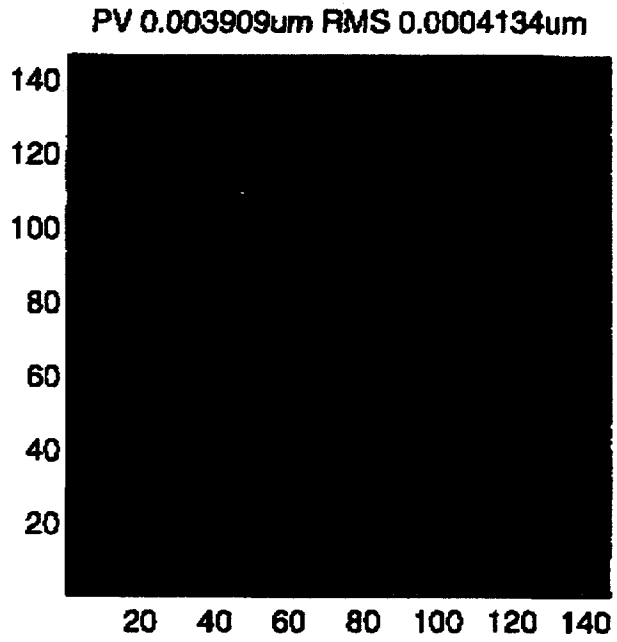
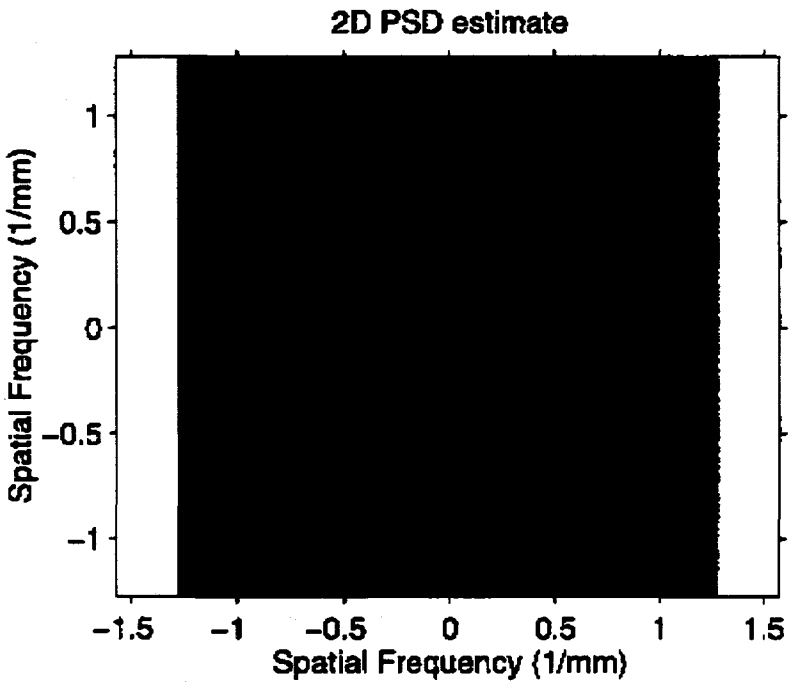
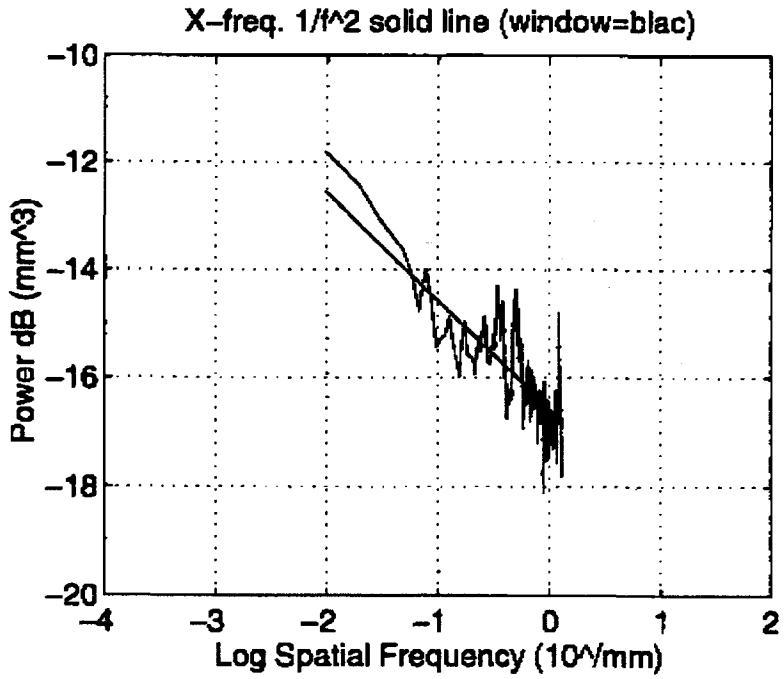
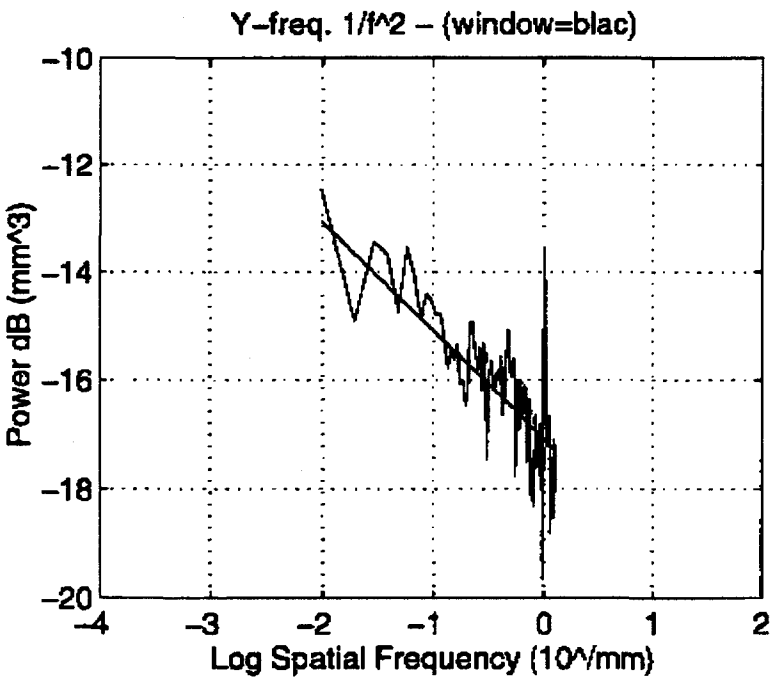
MESSAGE:


kuyp –D (corrected Z(4,0)) PV=0.00459 um RMS=0.00045 um
350
300
250
200
150
100
50
50
100
150
200
250
300
350


PV 0.003909um RMS 0.0004134um
140
120
100
80
60
40
20
20 40 60 80 100 120 140


2D PSD estimate
Spatial Frequency (1/mm)
1
0.5
0
−0.5
−1
−1.5 −1 −0.5 0 0.5 1 1.5
Spatial Frequency (1/mm)


X–freq. 1/f^2 solid line (window=blac)
Power dB (mm^3)
−10
−12
−14
−16
−18
−20
−4 −3 −2 −1 0 1 2
Log Spatial Frequency (10^/mm)


Y–freq. 1/f^2 – (window=blac)
Power dB (mm^3)
−10
−12
−14
−16
−18
−20
−4 −3 −2 −1 0 1 2
Log Spatial Frequency (10^/mm)

LIGO Pathfinder Measurements:

Systematic error arising from measuring a long radius spherical surface with respect to a flat

Non-zero fringe densities result in systematic errors in the measured wavefront by many commercially available phase measuring interferometers. Previous work[1] has shown that this error can be reduced through a numerical procedure. It appears, however, that in the special case of measuring a long radius spherical surface using a flat reference surface, the correction can be derived analytically. For the LIGO Pathfinder optics:

Radius of curvature = 6000 m
Test aperture = 0.15 m

Using the conventional sag equation, to a very good approximation:

$$\text{Sag} = (0.15)^2/(8*6000) = 0.46875 \times 10^{-6} \text{ m}$$

Coefficient of a(2,0) is half the sag, i.e. in this case 0.2344 μm (approx. 0.37 waves, in good agreement with measurements).

From previous work we know that, when we tilt a flat in front of the NIST owned WYKO 6000, the dominant aberration is coma. Experimentally, we know that the coma (a3,1) is related to tilt (a1,1) by:

$$a(3,1) = -1.3 \times 10^{-5} \times a(1,1)^2 + 0.006 \times a(1,1) - 0.0005$$

For small tilts, the linear term dominates, i.e. the coefficient of coma is k times the tilt (k = 0.006 for the NIST WYKO 6000)

We also know, from experiment, that for any small aperture within the full aperture, the slope of the aberration observed is the slope that would be observed at that position if the local slope were the slope across the entire aperture.

Now consider measurement of a long radius sphere. The problem is rotationally symmetric so, for simplicity, we will consider only a diameter. Using the Cartesian form of the Zernikes, (y=0), power takes the form:

$$z = 2x^2 - 1$$

and

$$dz/dx = 4x$$

Now the slope of coma has the form:

$$9x^2 - 2$$

So the systematic spurious slope resulting from the local slope of the radiussed surface is:

$$dz(\text{error})/dx = k.4x.(9x^2 - 2) = 36kx^3 - 8kx^2$$

which we can integrate to get the profile of the aberration:

$$A(x) = 9kx^4 - 4kx^2 + C$$

where C is a constant of integration. Now, the form of a(4,0) is:

$$a(4,0) = 6x^4 - 6x^2 + 1.$$

Assume that A(x) may be written as:

$$A(x) = \alpha(a(4,0)) + \beta\,(a(2,0)) + \chi$$

i.e.

$$A(x) = \alpha(6x^4 - 6x^2 + 1) + \beta\,(2x^2 - 1) + \chi$$

Then we can see, from the x^4 term that:

$$6\alpha = 9kx$$

i.e.

$$\alpha = 1.5\ k$$

and for the x^2 term:

$$-6\alpha + 2\beta = -4$$

i.e.

$$\beta = 2.5\ k$$

Explicitly, this analysis indicates that for unit (micrometer, wave) power, the systematic error in measured power and spherical aberration Zernike coefficients using the NIST WYKO 6000 (k = 0.006) are 0.015 and 0.009 (micrometer, wave) respectively.

For the LIGO Pathfinder Optics this converts to coefficients for:

spherical aberration:	2.1 nm
power:	3.5 nm

Now the systematic power term error leads to an error in the estimated radius of curvature of 38 m.

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[1] Evans C. "Compensation for errors introduced ...." CIRP Annals

Estimated ROC:

g	5.81 km
h	5.74
j	5.78
k	5.91
l	5.45
m	5.69