LASER INTERFEROMETER GRAVITATIONAL WAVE OBSERVATORY
- LIGO -
CALIFORNIA INSTITUTE OF TECHNOLOGY
MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Document Type	**LIGO-T960187-01 -**	**D**	2/20/97

Effect of Microseismic Noise on a LIGO Interferometer

F. Raab and D. Coyne

Distribution of this draft:

Detector Group

This is an internal working note
of the LIGO Project.

California Institute of Technology
LIGO Project - MS 51-33
Pasadena CA 91125
Phone (818) 395-2129
Fax (818) 304-9834
E-mail: info@ligo.caltech.edu

Massachusetts Institute of Technology
LIGO Project - MS 20B-145
Cambridge, MA 01239
Phone (617) 253-4824
Fax (617) 253-7014
E-mail: info@ligo.mit.edu

WWW: http://www.ligo.caltech.edu/

1 ABSTRACT

We estimate the range requirements for interferometer actuators arising due to the microseismic peak in the earth's seismic activity, using typical data from measurements taken at the Livingston, Louisiana LIGO site. We derive a seismic spectrum for design purposes that is a composite of measurements taken at the Hanford, Washington and Livingston, Louisiana LIGO sites and published data from laser strain measurements. The microseismic motion is found to be the main contributor to the estimated RMS motion. Using a statistical model for steady-state fluctuations of the microseismic noise, we derive specifications for the actuator ranges. However, the non-stationary nature of the microseism, particularly its connection to wave heights and storm activity in oceans, introduces significant uncertainty into this estimate. Using data from several seismographs that are part of the IRIS system of the USGS, we found that a "typical" variability of 10-12.5 dB in the microseism was characteristic of several stations in the Central and Western United States. Unfortunately, we could not definitively identify how the Livingston data fit within the yearly cycle of microseismic variability, which causes significant uncertainty in our estimated range specifications. We found that to satisfy the requirement that typical locked periods for an interferometer extend for 40 or more hours will require a range of between 14μm and 56 μm P-P. The upper bound is a worst case, reflecting our current uncertainty in the magnitude of microseism variability at the Livingston, Louisiana site. A reasonably conservative estimate, based on data from the nearest IRIS site at Crystal Cave, Missouri, indicates that a range of 40 μm P-P should be sufficient to achieve the required duration of locked periods. The goal of maintaining typical interferometer locking times of 3 months would require a sufficiently large range to withstand microseisms associated with large ocean storms that can be expected during any 3-month period. We estimate that achieving this goal would require a range on the order of 130 μm.

2 KEYWORDS

microseism, vibration, seismic, isolation, range

3 OVERVIEW

The seismic spectrum of the earth generally shows increasing power towards low frequencies. There are two characteristic features that cause large motions over small frequency bands. These are the earth tides near 10^{-5} Hz and a feature near 0.15 Hz, known as the microseismic peak. The earth tides are a coherent background driven by the motions of the sun and moon with diurnal and semidiurnal periods. The microseismic motion is more appropriately described as noise, resulting in a peak in the power spectrum whose width and frequency are comparable. The microseismic motion occurs at a frequency below the resonant frequencies of most laboratory apparatus. Thus for any apparatus whose physical size is much less than a kilometer, the relative motion resulting from microseismic excitation is largely common mode and hence often goes unnoticed. The 40-meter interferometer, for example, is relatively immune to the effects of microseismic excitation and also earth tides.

For interferometers with baselines of order several kilometers, the largest random contribution to the relative motions of mirrors in the arms can arise from the microseismic excitation. A considerable simplification occurs if we choose a non-inertial frame for this situation. Since most of the interferometer optics (with the exception of the end mirrors) are contained in a corner building that is of order 100 m on a side, this building and its optics respond to the microseism like a rigid body. Relative to a coordinate frame fixed within the corner building, the end mirrors (and the buildings which house them) are moving in response to the microseism. Throughout this document we will refer to this end-mass motion in the corner-building frame as *relative motion*.

4 A STATIONARY GROUND NOISE MODEL

We have generated a spectrum for the relative motions of mirrors separated by a 4-km distance from 10^{-4} to 10^{2} Hz, using a combination of LIGO data and published earth-strain data. The LIGO data is based on measurements of the amplitude spectral density of motion relative to an inertial frame done at the Livingston, LA site by A. Rohay of Batelle[1]. These measurements are valid at frequencies above 0.1 Hz. Below 0.1 Hz, we used data from laser strainmeters operated in the Western United States[6].

4.1. Livingston Seismic Data

Ground noise was measured at the LIGO Livingston by A. Rohay from October 26-November 3, 1995. Specifics of these measurements can be found reference [1]. We have used a power spectrum estimated from the data set[2] from October 30. We believe this data set is “typical” of the data taken during this period as it is neither the noisiest nor the quietest data in this set. In the analysis that follows, we suppose that this data is representative of the microseismic noise at this time of year. Variability of the microseismic noise is discussed further in section 6. The data, resampled from the plot in reference [1], are shown below in Figure (1). The seismometers used for these measurements give accurate ground noise readings from 0.1-50 Hz and give an upper bound on seismic motion above 50 Hz. The data exhibit a large peak between 0.1-0.3 Hz, which is the so-called microseismic peak caused by ocean-wave activity[3].

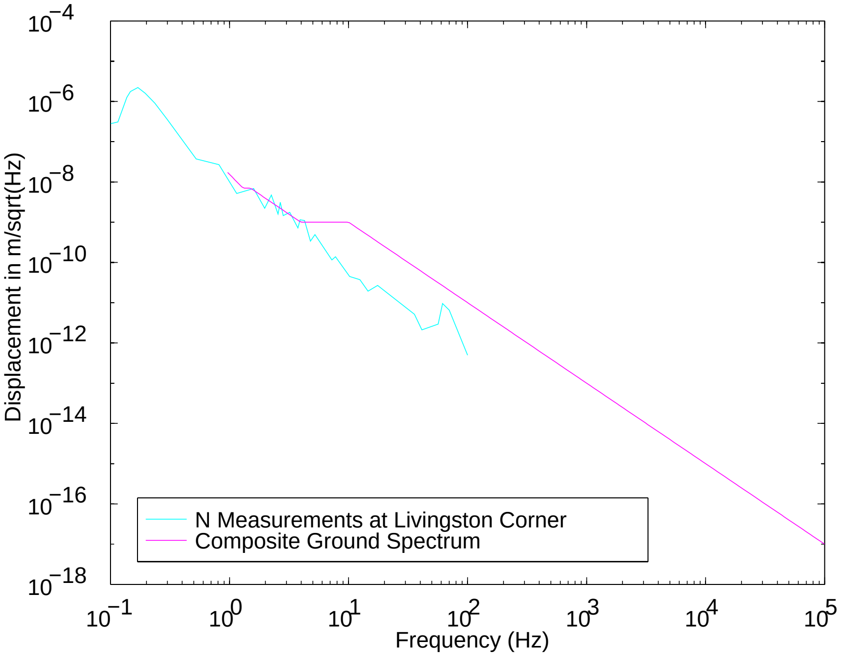


Figure (1) Ground noise spectrum measured at the Livingston, Louisiana site. (livingst.epi)

The data in Figure (1) specify ground motion relative to an inertial frame, which is the quantity measured by the seismometer. For LIGO the quantity of interest is the relative motion between the test masses at a separation of 4 km for full-length interferometers or 2 km for the Hanford half-length interferometer. Correlation measurements performed at the Livingston site[4] indicate that the microseismic peak is somewhat anticorrelated between the Southeast end station and the corner station, so that the relative axial motion between these two stations is approximately 60% larger than the motion of either station relative to an inertial frame. Note that the relative motion would be a 40% larger, even if the noise were totally uncorrelated between the two stations, so this anticorrelation is relatively weak.

4.2. Composite Ground Noise

The ground noise at frequencies above 1 Hz can exhibit significant fluctuations due to anthropogenic noise, also known as “cultural noise.” A ground noise spectrum was derived from seismic measurements made at the Hanford[5] and Livingston[1] sites by A. Rohay. The curve labelled “Composite Ground Noise” in Figure (1) represents an estimated envelope at each frequency of the typical noise found at either site at noisy times. We have assumed that ground noise at these frequencies is completely uncorrelated between the corner and end stations. The relative motion at these frequencies is then obtained by multiplying the “Composite Ground Noise” model by 1.4 to account for this lack of correlation.

4.3. Laser Strainmeter Data

Berger and Levine[6] have reported measurements of the earth strain in the frequency region between 10^{-4} Hz and 10^{-1} Hz, monitored by laser strainmeters at Pinon Flat and at the Poorman mine, both in the Western United States. These data sets agree among themselves and show a generally smooth spectrum $\tilde{h}^2(f) \approx f^{-1.8}$ to the seismic motion over this range of frequencies. We have assumed that this data is representative of low-frequency strain at the LIGO observatory sites and have multiplied the resultant strains by 4 km to estimate the relative ground motion between end and corner stations at Livingston at frequencies below 0.04 Hz.

4.4. Estimated Relative Motion Spectrum

The relative motion spectrum, shown below in Figure (2), was estimated by combining the microseismic data, the composite ground-noise model and the laser-strain data according to the criteria given above. The data were splined into a logarithmically-equally-spaced grid of data

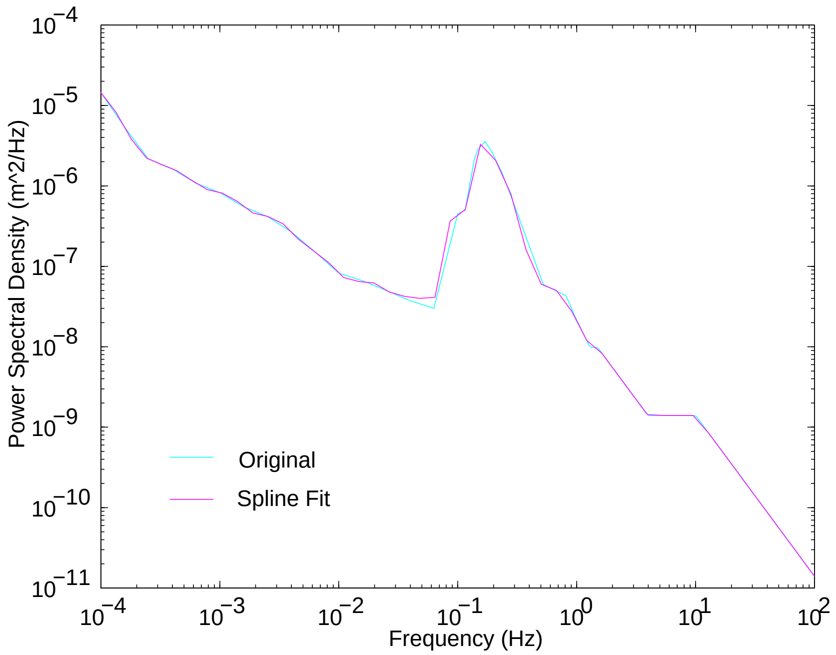


Figure (2) Estimated relative motion of end and corner station buildings at LIGO site in Livingston, Louisiana (compspec.epi).

pairs, spanning six decades shown in Figure (3) below, with a density of 8 points per decade.

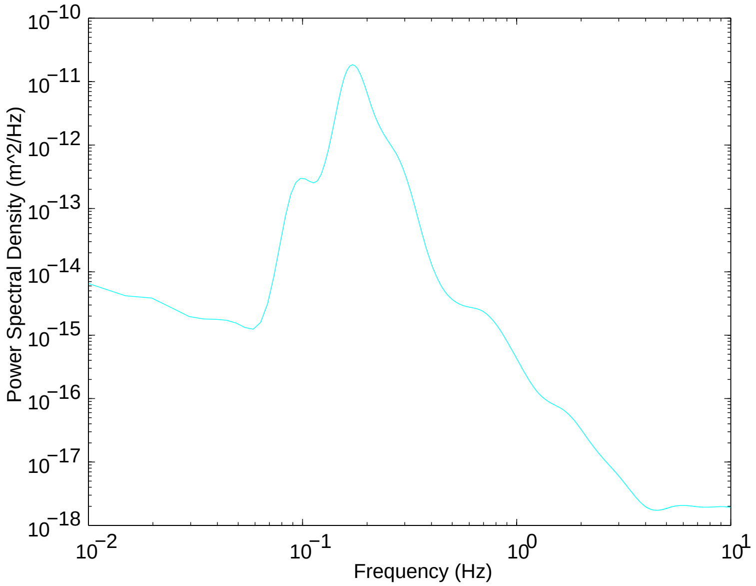


Figure (3) Composite ground-noise spectrum for the LIGO site at Livingston, Louisiana, splined to obtain 2048 frequency channels spaced at 0.0049-Hz intervals.(totlrelmot)

5 STATISTICS IN THE STATIONARY NOISE MODEL

The chance of exceeding some predetermined level of relative ground motion can be found for a process such as that described by the composite ground-noise spectrum of Figure (3), once the underlying statistic of the noise process is known. Strobach[7] has analyzed the process underlying the microseism as due to a superposition of sources having a characteristic frequency but random phases. The resulting distribution of amplitudes is described by a Rayleigh distribution of the form:

$$P(x) = \frac{x}{\sigma^2} \cdot e^{x^2/2\sigma^2} \tag{1}$$

Strobach has shown that this approximation successfully represents the statistics of long-period seismograms measured in the midwestern United States. We adopt this form of distribution for the analysis that follows, correcting later (Section 6) for variability in the intensity of the microseism.

5.1. Contributions to RMS Relative Ground Motion

The root-mean-square (RMS) relative ground motion was evaluated for each decade from 10^{-4}

Hz to 10^2 Hz. The contribution by frequency band is given in Table 1. From the table it is evident

Table 1. Relative Ground Motion by Band for Composite Motion Spectrum.

Band	**Frequency Range (Hz)**	**RMS Motion (μ m)**
1	$10^{-4} < f \leq 10^{-3}$	0.113
2	$10^{-3} < f \leq 10^{-2}$	0.032
3	$10^{-2} < f \leq 10^{-1}$	0.064
4	$10^{-1} < f \leq 1$	0.976
5	$1 < f \leq 10$	0.012
6	$10 < f \leq 100$	0.002
Total	$10^{-4} < f \leq 10^2$	0.985

that the relative ground motion is dominated by the microseismic peak, which appears in band 4.

5.2. Mean Correlation Time for Microseismic Motion

Since the relative ground motion over the range of 10^{-4} to 10^2 Hz is dominated by the microseismic peak, it seems reasonable that excursions from the mean amplitude of seismic motion should be describable by a single correlation time. To estimate this more precisely we have evaluated the autocorrelation associated with the spectrum given in Figure (3). The autocorrelation is defined as the cosine Fourier transform of the ground-motion power spectrum. If this power spectrum is defined by $\tilde{x}^2(f)$, then the resulting autocorrelation $\psi(\tau)$ is given by

$$\psi(\tau) = \int_0^\infty \tilde{x}^2(f) \cdot \cos(2\pi f\tau) d\tau . \qquad (2)$$

A plot of $\psi(\tau)$ is given in Figure 4. The autocorrelation exhibits oscillatory behavior with a period of 5.7 s and decays to 1/e in a time $\tau_e = 9.0$s. We adopt τ_e as the mean correlation time for the seismic spectrum over the frequency range 10^{-4} to 10^2 Hz.

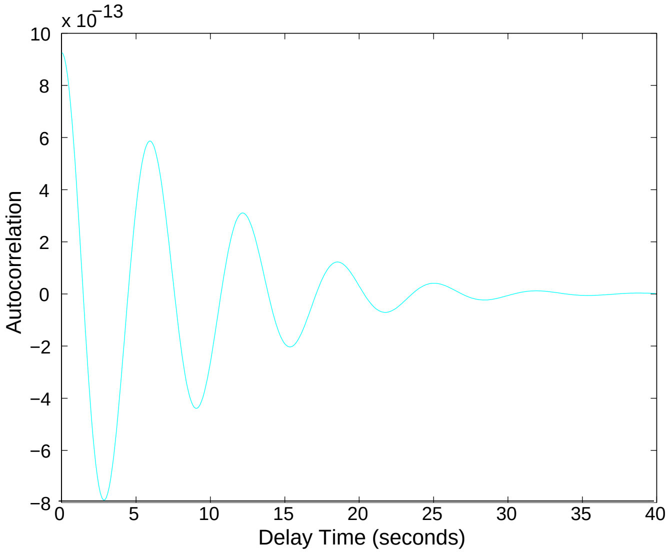


Figure (4) Autocorrelation for Livingston relative displacement spectrum. (la_corr.epi)

5.3. Expected Range in Stationary Model

The actuator range required to counteract the seismic perturbations on the test masses is determined in part by the transmissibility of the seismic-isolation stacks and the suspension. However, we see from Table 1 that the RMS ground motion is dominated by the microseismic peak. Since this occurs at frequencies below 0.5 Hz, it is below the resonances of the stacks and the suspension. Thus we expect a major fraction of the total RMS motion of the test masses can be accounted for by neglecting the stack and suspension resonances. The effect of stack resonances can then be taken into account later. We adopt this approach because the microseism is subject to a large variability from day to day and season to season, whereas the portion of the seismic spectrum near stack resonances (Bands 5 and 6 of Table 1) do not share the same fluctuations.

The duration of locked stretches that we require also influences the range requirement. We will evaluate the range needed for two specifications. We adopt the *requirement* that the interferometer can maintain lock for typical intervals of 40 hours[8]. We further adopt a *goal* that loss of lock should not typically occur during a 3-month interval of operation, to facilitate pulsar searches.

The probability of some relative displacement x exceeding some value a, due to noise characterized by a Rayleigh distribution with variance σ^2 and having a mean correlation time τ_e, is given by integrating equation (1) to obtain

$$P(x > a) = e^{-a^2/2\sigma^2} \qquad (3)$$

This can be solved to obtain the range required for a given probability to obtain

$$a = \sigma \cdot \sqrt{-2 \cdot \ln[P(z < a)]}\,. \tag{4}$$

For a 10% probability of exceeding the range in an interval T we set

$$P(z > a) = \frac{1}{10} \cdot \frac{\tau}{T}\,. \tag{5}$$

This results in the minimum range values shown in Table 2 below.

Table 2. Minimum Range Specifications

Specification	Typical Lock Duration	Minimum Range
Requirement	40 hours	4.9σ
Goal	3 months	5.7σ

If the microseism were truly a steady-state process over the course of a year, we could estimate the range requirement by taking $\sigma \cong 0.99\mu\text{m}$ from the total RMS relative motion in Table 1 and applying the minimum range requirement in Table 2. This would result in a range requirement of $4.8\mu\text{m}$ RMS or $14\mu\text{m}$ peak-to-peak (P-P). However we need to look at the variability of the microseism before establishing the range specifications.

6 NON-STATIONARITY OF THE MICROSEISM

The microseism is known to have significant seasonal variability. The measurements taken at the Livingston site represented too small an interval of time to deduce these statistical properties, which would have required monitoring over a period of at least one year. To estimate the typical fluctuations, we looked at other sources of data for various sites and made the assumption that the microseism exhibits similar variability at any site in the United States.

6.1. Survey of USGS Data on Microseism Variability

Data from a world-wide network of seismic observatories has been compiled by the United States Geological Service (USGS) and can be accessed on the World-Wide Web[9]. We looked at data from four seismological observatories that were closest to the line connecting the two LIGO observatories. These observatories were near Albuquerque, New Mexico, Corvalis, Oregon, Crystal Cave, Missouri and Tucson, Arizona. The data from these sites is reproduced in Appendix 1. The Albuquerque, Crystal Cave and Tucson sites are more than 100 miles inland, similar to the Hanford, Washington site. The distance from the Corvalis site to the Pacific Ocean is comparable to the distance of the Livingston, Louisiana site to the Gulf of Mexico.

The seasonal data on the website give the "typical" seismic acceleration spectral density by quarter over a year. All four observatory sites showed a seasonal variation in the microseism of 6.6-12.2 dB as indicated in Table 3. Crystal Cave and Tucson showed higher typical microseismic activity during the October-November time frame, the time of year when the Livingston data of reference [1] was obtained. However the Albuquerque and Corvalis observatories showed higher microseismic peaks at other times of year. The inland sites all showed comparable levels of microseismic activity whereas the activity at the Corvalis site was about 10 dB higher.

Table 3. Seasonal Variation of the Microseism at IRIS Stations

	Direction of Motion		
IRIS Station[a]	**East (dB)**	**North (dB)**	**Peak Season**
ANMO	10.6	9.5	Jan-Jun
TUC	12.2	11.1	Oct-Mar
CCM	8.9	9.2	Oct-Mar
COR	6.6	8.1	Jan-Jun

[a] ANMO = Albuquerque, New Mexico; TUC = Tucson, Arizona; CCM = Crystal Cave, Missouri; COR = Corvalis, Oregon.

6.2. Ocean-Storm Activity and Microseism Variations

Storms at sea can cause much larger increases in the microseism than the seasonal variations of "typical" microseismic levels would indicate. The Sources of Ambient Microseismic Ocean Noise (SAMSON) experiment measured the direct correlation between wind, ocean-wave activity and ocean-bottom seismic noise during the October-November 1990 time frame, off the coast of North Carolina[10]. The 27-day record showed two large storms that increased ocean-bottom seismic activity by 20-30 dB and two smaller storms that increased ocean-bottom activity by about 10 dB. The two large storms generated peak microseismic activity at the ocean bottom seismograph of about 10 μm RMS, which lasted for about 12 hours. If this level of noise occurred in Livingston, a range of 45 μm RMS or 130 μm P-P would be needed to have a reasonable chance of maintaining interferometer lock throughout the duration of the storm.

Curiously, there was also a single incidence when ocean-bottom activity increased by about 10 dB when the local winds and wave activity were calm. This was believed to due to microseismic waves that originated from a very large but distant storm. The RMS microseismic motion during periods away from the large storms appears to be comparable to the Corvalis seismological observatory data during quarters of large "typical" activity.

6.3. Microseismic Variability and the Livingston Data

The Livingston seismic data may well have been taken at a time of the year typically characterized by large microseismic motion. The nearest seismological station in the IRIS group, Crystal Cave, is approximately 500 miles from the Livingston site and has its seasonal extremes of microseismic activity during autumn and winter. Since autumn is the peak hurricane season in the Gulf of Mexico, we would expect to have larger wave activity in the Gulf at this time of year which should enhance the microseism. If the seismic spectra taken at Livingston represent seasonally quiet spectra, then we must allow additional margin, accounting for the seasonal variation exhibited by the IRIS stations. Crystal Cave is the nearest IRIS station to Livingston and Corvalis is, like Livingston, very near the sea. The microseismic variability at these station could reasonably be expected to be most like Livingston, indicating that an additional factor of 2.8 margin, based on a 9 dB seasonal variability should be sufficient. This would give a range requirement of 40 μm P-P The most conservative margin would use the largest variation in Table 3 as representative of Livingston and would allow for the unlikely possibility that the Livingston data were taken at the most quiet time of year for the microseism. This would require an additional factor of 4 margin (12.2 dB), for a range requirement of 56 μm P-P.

6.4. Effect of Microseism Variability on Estimated Range

If we assume that the Livingston composite spectrum is based on measurements at a time of year when the seasonal variation is near a high value, it seems reasonable to view the minimum range that satisfies the *requirement* (i.e., loss of lock being unlikely in 40 hours) as adequate. Large ocean-storm activity could greatly increase the probability of losing lock within any particular 40-hour interval, but it is improbable that an arbitrary 40-hour interval would overlap such storm activity. Taking $\sigma \cong 0.99\mu\mathrm{m}$ from the total RMS relative motion in Table 1 and applying the minimum range requirement in Table 2, we obtain a required range of $4.8\mu\mathrm{m}$ RMS or $14\mu\mathrm{m}$ peak-to-peak (P-P).

Similarly, the minimum-range *goal* of maintaining an interferometer in resonance for 3 months would correspond to $16\mu\mathrm{m}$ P-P, except that it is unlikely that large ocean-storm activity would not typically occur in a 3-month interval. To survive a large storm would require nearly an order of magnitude increase in range. One or two large storms per month can be expected in the Atlantic Ocean, and similar or higher rates of large storms seem reasonable during hurricane season in the Gulf of Mexico.

If we make the worst case assumption, that the Livingston spectrum we have analyzed corresponds to a seasonal minimum typical level, then we should allow at least another factor of 4.2 margin (i. e. 12.5 dB), based on the typical variations exhibited by data from the IRIS stations. A range of 56 μm P-P would then be needed to satisfy the *requirement*.

7 CONCLUSIONS

We have attempted to estimate the range requirements as driven by the microseismic peak in the earth’s seismic activity, using typical data from measurements taken at the Livingston, Louisiana LIGO site. If the microseism were describable by a stationary random process this data would be sufficient. However the non-stationary nature of the microseism, particularly its connection to wave heights and storm activity in oceans, requires other data. The best data would be measurements of the microseism in Livingston over a minimum period of one year. We do not have access to such data and acquiring new data of sufficient duration will not be possible prior to the final design of the initial LIGO seismic isolation. Using published data from several seismographs that are part of the IRIS system of the USGS, we were able to find that “typical” variability in the microseism was similar at several stations in the Central and Western United States. Unfortunately, we could not definitively identify how the Livingston data fit within the yearly cycle of microseismic variability. Clearly, more data from a seismograph nearby the Livingston site would sharpen our estimates considerably. Satisfying the *requirement* that typical locked periods for an interferometer extend for 40 or more hours will require a range of between 14μm and 40 μm with reasonable certainty. A very conservative case might argue for extending this range up to 56 μm P-P. The *goal* of maintaining typical interferometer locking times of 3 months would require a sufficiently large range to withstand microseisms associated with large ocean storms that can be expected during any 3-month period. This goal would require a range on the order of 130 μm.

The key source of uncertainty in these range estimates is our lack of knowledge about the variation of the microseism at the Livingston site. We have not yet been able to identify a better source of data on the Livingston site, either with regard to seismic, oceanographic or meteorologic data, which could be used to constrain the uncertainties in our estimates. A search for other published data should continue as time allows. It will not be possible to compile a sufficiently thorough set of new seismic data at Livingston before the actuator design for the LIGO seismic isolation and suspension is frozen. However further data from the Livingston site could be beneficial if measurements were started soon.

8 REFERENCES

[1] A. Rohay, Ambient Ground Vibration Measurements at the Livingston, Louisiana LIGO Site, LIGO-C961022-A-D, (1996).

[2] The data set used was the median amplitude spectrum for a one-hour period at the corner station on 10/30/95 at 7:00 am, which appears in Figure 6-2 of reference [1].

[3] J. M. Babcock, B. A. Kirkendall and J. A. Orcutt, *Bull. Seismol. Soc. Am.*, **84**, 1991, (1994) gives a brief description and references to papers describing the physics contributing to seismic noise in the vicinity of the microseismic peak.

[4] A. Rohay, Analysis of Differential Motions at the Hanford, Washington and Livingston, Louisiana LIGO Sites, (1996).

[5] A. Rohay, Ambient Ground Vibration Measurements at the Hanford, Washington LIGO Site, LIGO 950571-02-D.

[6] J. Berger and J. Levine, *J. Geophys. Res*., **79**, 1210, (1974).

[7] K. Strobach, *Bull. Seismol. Soc. Am*., **55**, 365, (1965).

[8] D. Shoemaker, Detector Subsystem Requirements Document, LIGO-E960112-05-D, (1996).

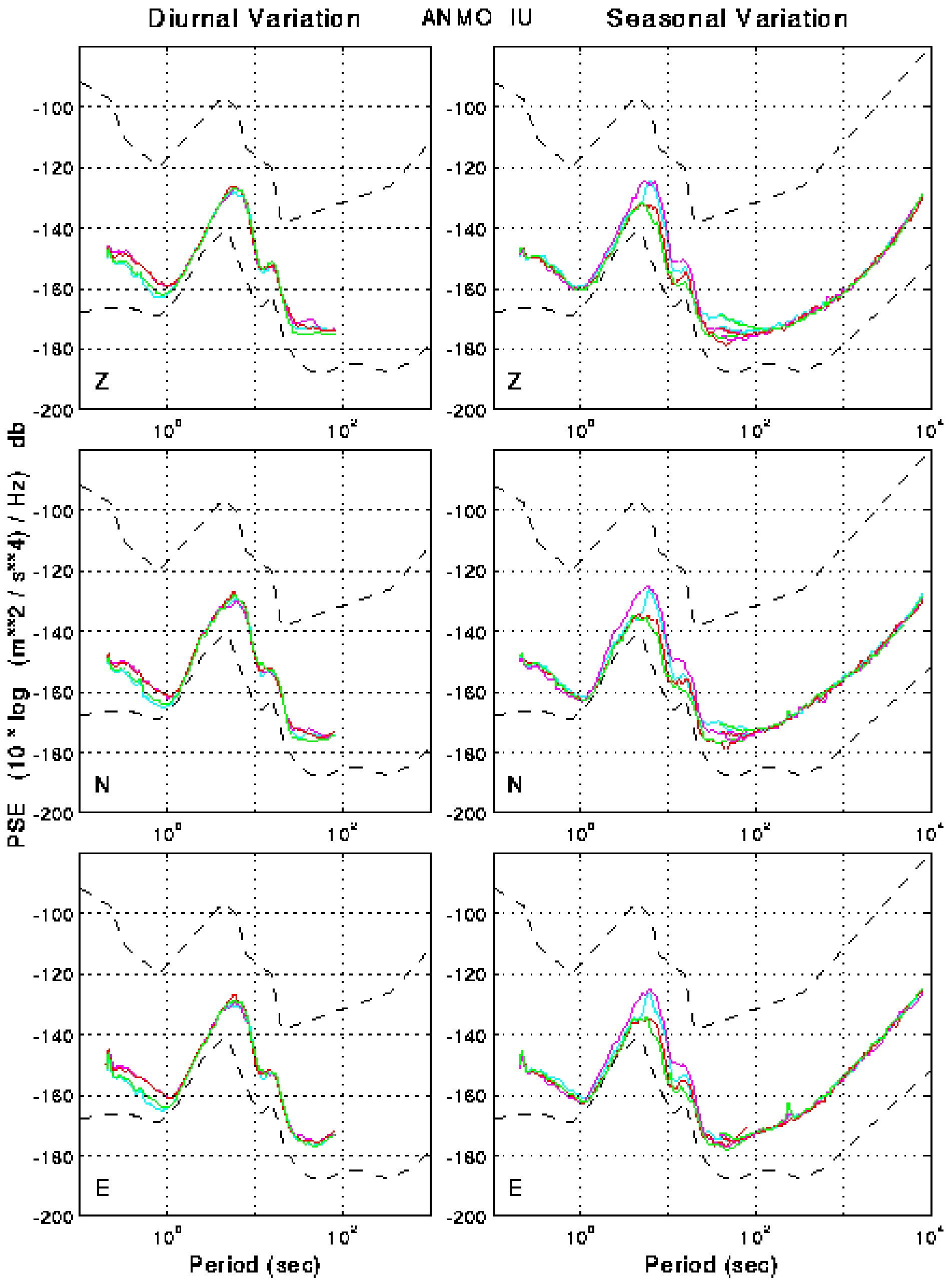
[9] Data from the IRIS network of the USGS can be found on the World-Wide Web at http://asl-www.cr.usgs.gov/web_gsn/gsnmnt/network/network.htm.

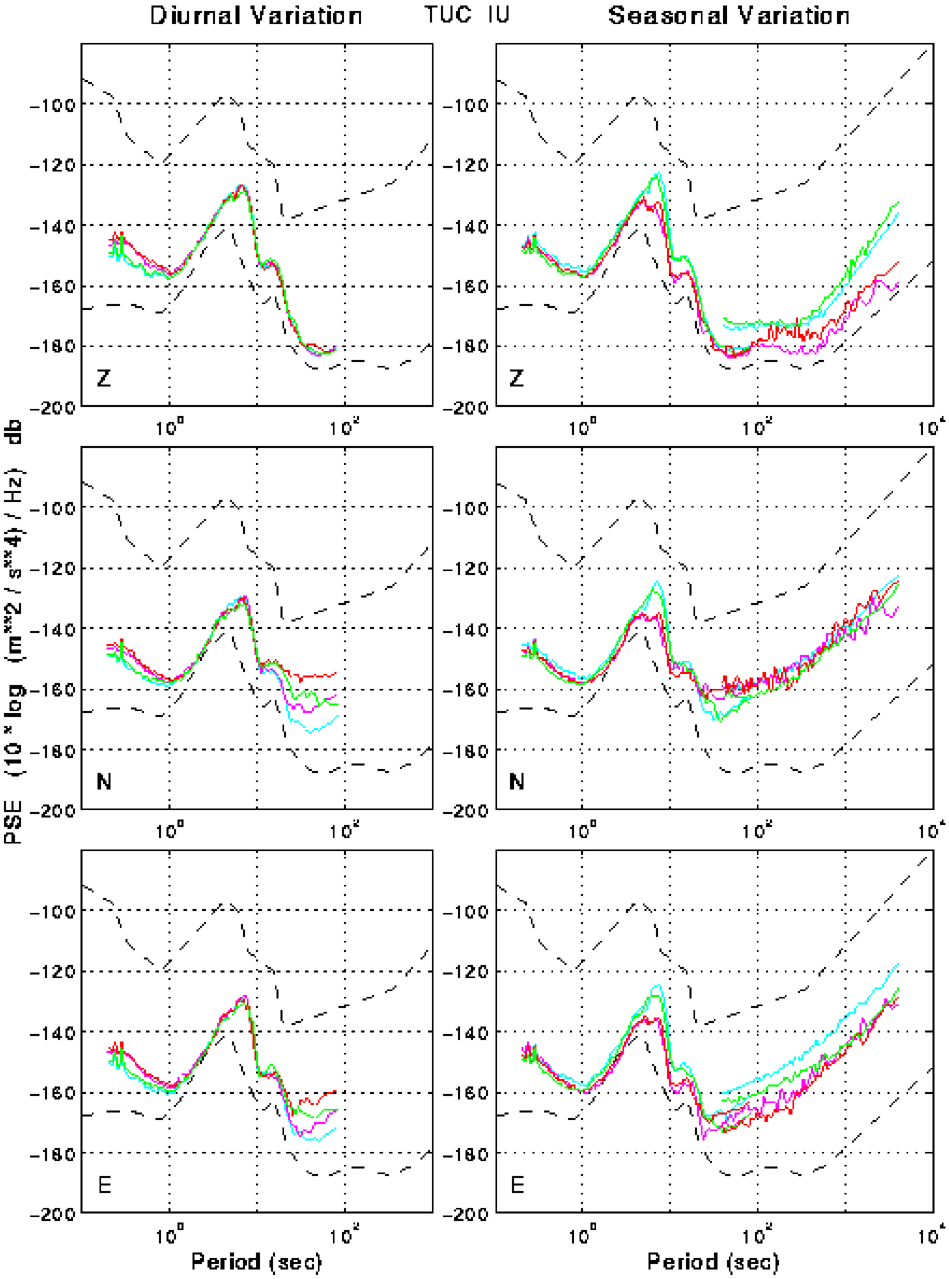
[10] J. M. Babcock, B. A. Kirkendall and J. A. Orcutt, *Bull. Seismol. Soc. Am.*, **84**, 1991, (1994).

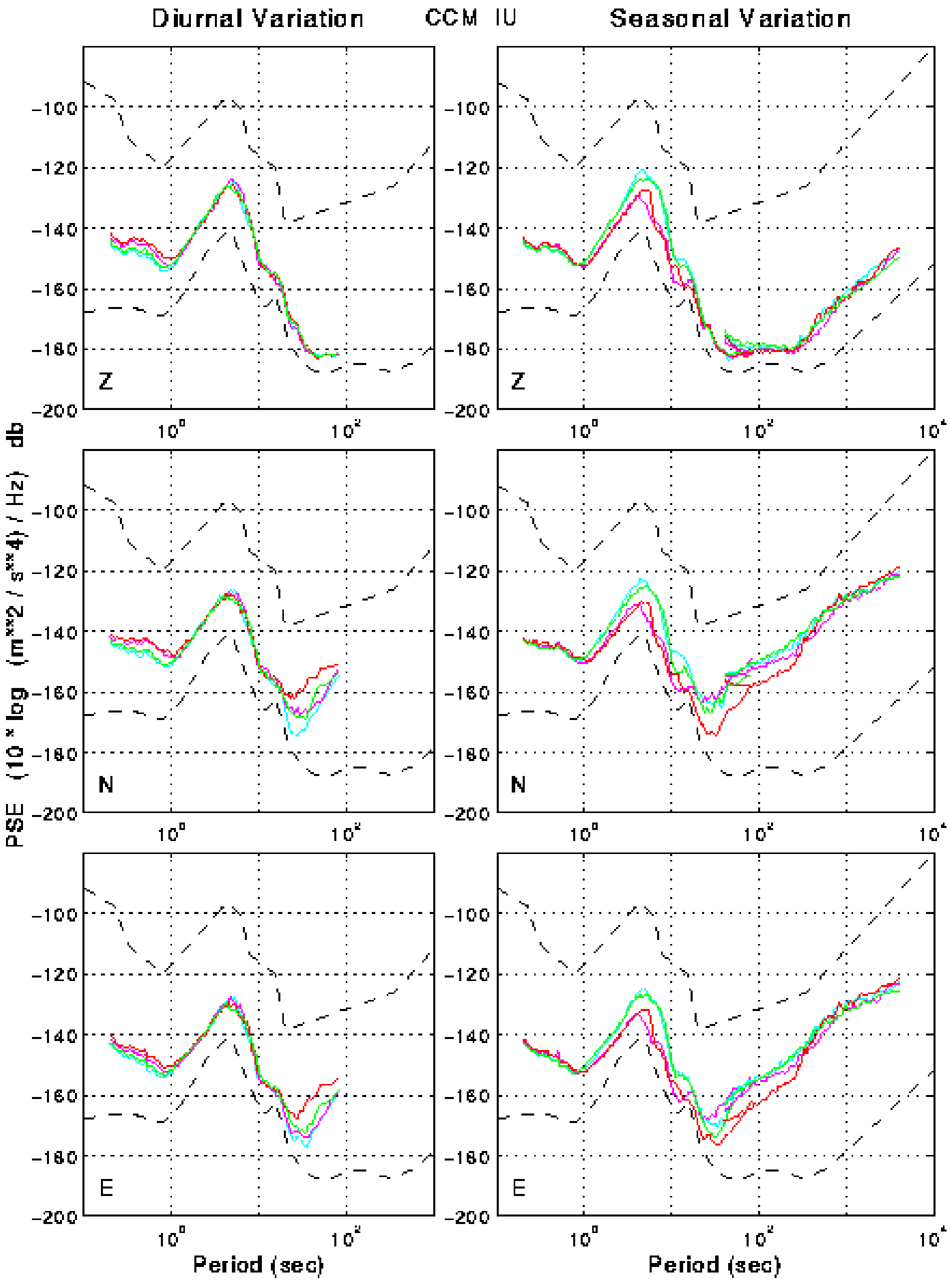
APPENDIX 1 USGS DATA ON MICROSEISMIC VARIABILITY

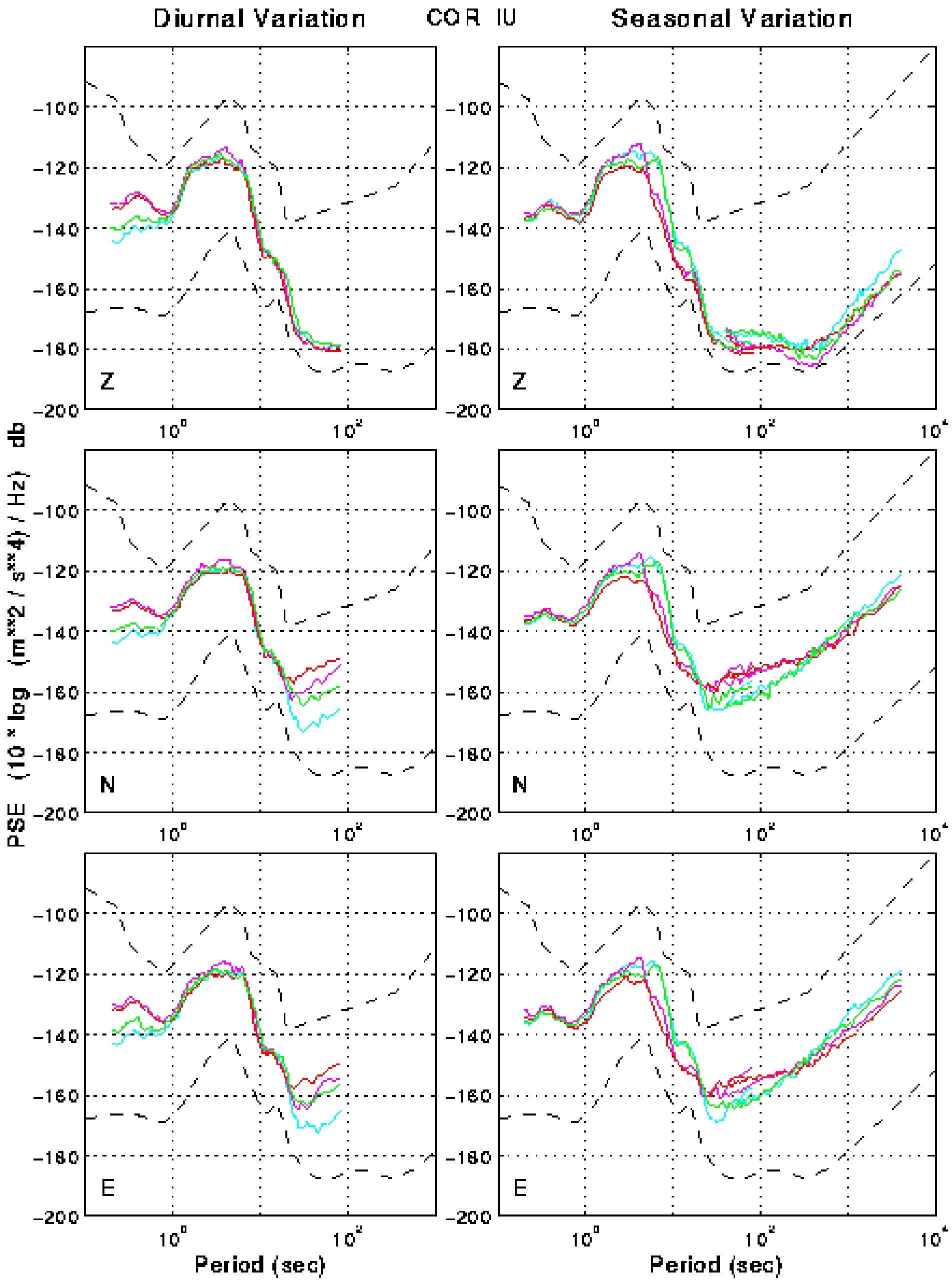
The following data summarizes data from seismological stations that are part of the IRIS system deployed by the USGN. The acronyms indicate the station names (ANMO = Albuquerque, New Mexico; TUC = Tucson, Arizona; CCM = Crystal Cave, Missouri; COR = Corvalis, Oregon). In the graphs in the right column, the colors represent the season during which the data were taken (blue = Jan-Mar; pink = Apr-Jun; red = Jul-Sep; and green = Oct-Dec). In the graphs on the left the colors represent the time day that the data were taken (blue = 00:00-06:00; pink = 06:00-

12:00; red = 12:00-18:00; green = 18:00-24:00).




Diurnal Variation
TUC IU
Seasonal Variation
PSE (10 * log (m**2 / s**4) / Hz) db
-100
-120
-140
-160
-180
-200
Z
N
E
10^0
10^2
10^4
Period (sec)


Diurnal Variation
CCM IU
Seasonal Variation
PSE (10 * log (m**2 / s**4) / Hz) db
-100
-120
-140
-160
-180
-200
Z
N
E
10⁰
10²
10⁴
Period (sec)


Diurnal Variation
COR IU
Seasonal Variation
PSE (10 * log (m**2 / s**4) / Hz) db
−100
−120
−140
−160
−180
−200
Z
N
E
10^0
10^2
10^4
Period (sec)
Period (sec)