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The Effect of Earth Tides on LIGO Interferometers

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Abstract

Earth tides cause a significant variation in the separations between test masses in the long arms of a LIGO interferometer. Earth-tide effects were estimated at the applicable site coordinates and the relevant arm bearings. The maximum possible contribution from alignment of the moon and sun was included to obtain the maximum possible peak tide. The appropriate angular factors were then used to derive common-mode and differential-mode arm-length changes for the two sites. The maximum common-mode drifts are approximately 370 microns peak-to-peak (P-P) at Hanford and 420 microns P-P at Livingston. This corresponds to a peak-to-peak strain of ~ 10^{-7}, which can be removed by a slight adjustment of the laser frequency (equivalent to changing the temperature of the reference cavity by 0.2 K). The maximum differential-mode drifts are approximately 210 microns P-P at Hanford and 200 microns P-P at Livingston. These require a differential change in arm lengths that exceeds the range of the suspension actuators. Providing the end- and mid-station BSC seismic isolation with actuators capable of 120 microns P-P motion in each arm should allow differential earth-tide motion to be handled with adequate margin.

Keywords: earth tides, actuator range

1 OVERVIEW

Because of their large size, the LIGO interferometers are affected by phenomena that normally do not affect precision measurements made with smaller scale apparatus. Examples of such phenomena are the microseisms and the earth tides. The microseism, earth vibrations driven by ocean-wave activity, occur with a period of about 6 seconds and exhibit random fluctuations. The effects of the microseism are treated elsewhere[1]. Here we treat the earth tides, deformations of the earth induced by the gravitational pull of the sun and moon. These are coherent effects that are, a priori, predictable in phase and amplitude. The earth tides produce a strain that is small over the dimensions of a single room or laboratory building but accumulates to a sizable value over the kilometer-scale separations of the mirrors in LIGO and VIRGO.

2 MAGNITUDE OF EARTH TIDES OVER LIGO BASELINE

We have used a derivation of the magnitude of the earth tides by Melchior[2], which we outline here for convenience.

2.1. Approximate Scale and Symmetry of the Tides

The potential of interest for deriving the tides on earth due to an external body of mass M at a distance d is given by

$$W(A) = \frac{3}{4}GM\frac{r^2}{d^3}\left\{\cos^2\theta\,\cos^2\delta\,\cos 2H + \sin 2\theta\,\sin 2\delta\,\cos H + 3\left(\sin^2\theta - \frac{1}{3}\right)\left(\sin^2\delta - \frac{1}{3}\right)\right\} \quad (1)$$

where $A(r, \phi, \lambda)$ is the point at which the potential is evaluated, θ is the polar angle (increasing toward South) and λ is longitude (increasing toward West), and δ is the declination for the external body. The quantity H is the local hour angle, given by

$$H(A) = H - \lambda A = \omega t' - \alpha - \lambda \tag{2}$$

where H is the hour angle of the external body, t' is the sidereal time at the origin of the longitudes, ω is the sidereal angular velocity of the earth's rotation and α is the right ascension for the external object.

The last term in equation (1) is called the zonal term, which is independent of time and therefore will be dropped from further discussion in this paper. The middle term gives rise to a diurnal variation of the tides known as the tesseral component. The first term gives rise to a semi-diurnal effect known as the sectorial component of the tides. If the external body were in the plane of the equator, then the amplitude of the tesseral wave would be zero and the sectorial wave would give rise to a quadrupolar distortion of earth with nodes at the poles. The sectorial wave would be zero for the case of the external body lining up with the polar axis, in which case the tesseral wave would give rise to a quadrupolar distortion of earth with nodes on the equator.

A quantity called Doodson's constant, defined by

$$D = \frac{3}{4} G M_{lunar} \frac{a^2}{d^3} \tag{3}$$

is used to define the size of tidal effects. Here $a = 6371.0236$ km is the radius of a sphere with the same volume as earth and d is the mean earth-moon distance.

The tidal displacement h of equipotential surfaces on the earth due to the presence of the moon is given by setting $gh = W$, where g is the local acceleration due to gravity. The maximum tidal displacement of equipotentials is then given by the ratio of Doodson's constant D to the acceleration due to gravity g at the earth's surface:

$$\frac{D}{g} = 26.76 \text{ cm.} \tag{4}$$

The amplitude of displacement of equipotential surfaces at an arbitrary point on the earth's surface can be obtained from equation (4) by multiplying by the appropriate trigonometric expression, which can vary from +1 to -1. Thus the moon can displace equipotential surfaces by as much as 53.52 cm peak-to-peak (P-P). The tides due to the sun are weaker, causing maximum displacements of equipotential surfaces by 24.61 cm P-P, or 0.4599 of the lunar tide. The quantity *D/g* will set the scale for the earth tides.

2.2. Elastic Deformations of the Earth

The shift in equipotential surfaces causes tidal forces in the earth that deform the earth's surface. Since the earth is a solid, it resists this force and the corresponding deformation is much less than the shift in equipotential surfaces. The elastic deformation can be parametrized in terms of the "Love numbers" h and l.

2.2.1. Tesseral-Wave Components

The diurnal component of the elastic deformation of the earth, known as the tesseral wave, is[3]

$$d_1 = \left[(h-4l)\cos^2\beta + (h-2l)\sin^2\beta - 2l\frac{\tan H}{\cos\theta}\cos\beta\sin\beta\right]\cdot \left[\frac{D}{ag}\left(\frac{r}{a}\right)^2\left(\frac{c}{d}\right)^3\sin 2\theta\sin 2\delta\cos H\right] \quad (5)$$

where the direction cosine $\cos\beta = \sin z\cos\psi$, with local zenith angle z and azimuth ψ, with $\psi = 0$ due North. It is convenient to separate the time dependence into quadratures as follows:

$$d_1 = \left\{[(h-4l)\cos^2\beta + (h-2l)\sin^2\beta]\cos H - \left[\frac{2l\cos\beta\sin\beta}{\cos\theta}\right]\sin H\right\}\cdot\frac{D}{ag}\left(\frac{r}{a}\right)^2\left(\frac{c}{d}\right)^3\sin 2\theta\sin 2\delta \quad (6)$$

2.2.2. Sectorial-Wave Amplitude

The semi-diurnal amplitude of deformation of the earth, known as the sectorial wave, is given[4] by

$$d_2 = \left\{\left[h + 2l\left(\frac{1-2\sin^2\theta}{\sin^2\theta}\right)\right]\cos^2\beta + \left[h - 2l\left(\frac{1+\sin^2\theta}{\sin^2\theta}\right)\right]\sin^2\beta + \left[4l\left(\frac{\cos\theta}{\sin^2\theta}\right)\tan 2H\right]\sin\beta\cos\beta\right\}\cdot\frac{D}{ag}\left(\frac{r}{a}\right)^2\left(\frac{c}{d}\right)^3\sin^2\theta\cos^2\delta\cos 2H \quad (7)$$

with the same definitions as given above. Rearranging terms to isolate the quadratures of the time dependence, we obtain

$$d_2 = [\{[h\sin^2\theta + 2l(1-2\sin^2\theta)]\cos^2\beta + [h\sin^2\theta - 2l(1+\sin^2\theta)]\sin^2\beta\}\cos 2H + \{(4l\cos\theta)\sin\beta\cos\beta\}\sin 2H]\cdot\frac{D}{ag}\left(\frac{r}{a}\right)^2\left(\frac{c}{d}\right)^3\cos^2\delta \quad (8)$$

3 ESTIMATION OF LARGEST TIDAL AMPLITUDES

We have estimated the worst-case tidal displacements for LIGO interferometers at Hanford and Livingston. This was done by using the geographical longitude, latitude and bearings of the corner station and arms and adding coherently the largest tesseral-wave and sectorial-wave amplitudes, obtained by maximizing the trigonometric functions that vary due to varying declination and zenith amplitude.

3.1. Approximate Tidal Changes in Interferometer Arm Lengths

We define common-mode and differential arm-length changes for a LIGO interferometer by ΔL_+ and $2\Delta L_-$, respectively, where

$$\Delta L_{\pm} = \frac{1}{2}(\Delta x_1 \pm \Delta x_2) \quad (9)$$

and Δx_i refers to the tidal length deformation in the *i*th arm. These tidal deformations are defined for the tesseral-wave component by

$$\begin{aligned}\Delta x_1(t) &= a_{1c}\cos H + a_{1s}\sin H\\ \Delta x_2(t) &= a_{2c}\cos H + a_{2s}\sin H\end{aligned} \quad (10)$$

where the a_{ic} and a_{is} differ between arms 1 and 2 due to the differences in bearing. Factoring these relations into quadratures of the time dependence, we obtain

$$\Delta L_{\pm}^{T} = \frac{1}{2}[(a_{1c} \pm a_{2c})\cos(H) + (a_{1s} \pm a_{2s})\sin(H)] \quad (11)$$

which is of the form

$$\Delta L_{\pm}^{T} = \left|\Delta L_{\pm}^{T}\right| e^{iH}. \quad (12)$$

Clearly the quantity $\left|\Delta L_{\pm}^{T}\right|$ represents the maximum amplitude of the tesseral tidal deformation due to a single external body. In the spirit of obtaining a worst-case estimate we have approximated the quantity $\cos\beta = \sin z\cos\psi$ in equations (5) through (8) as $\cos\beta \cong \cos\psi$. We then have for the tesseral component

$$\begin{pmatrix} a_{ic} \\ a_{is} \end{pmatrix} = \begin{pmatrix} [h-4l]\cos^2\psi + [h-2l]\sin^2\psi \\ -\dfrac{2l\cos\psi\sin\psi}{\cos\theta} \end{pmatrix} \cdot \frac{D}{ag}\left(\frac{r}{a}\right)^2 \sin 2\theta \sin 2\delta \,. \qquad (13)$$

Similarly, the time dependence of the sectorial component can be expressed as

$$\begin{aligned} \Delta x_1(t) &= b_{1c}\cos 2H + b_{1s}\sin 2H \\ \Delta x_2(t) &= b_{2c}\cos 2H + b_{2s}\sin 2H \end{aligned} \qquad (14)$$

with coefficients given by

$$\begin{pmatrix} b_{ic} \\ b_{is} \end{pmatrix} = \begin{pmatrix} [h\sin^2\theta + 2l(1-2\sin^2\theta)]\cos^2\psi + [h\sin^2\theta - 2l(1+\sin^2\theta)]\sin^2\psi \\ 4l\cos\theta\sin\psi\cos\psi \end{pmatrix} \cdot \frac{D}{ag}\left(\frac{r}{a}\right)^2 \cos^2\delta \,. \qquad (15)$$

Factoring these relations into quadratures of the time dependence, we obtain

$$\Delta L_{\pm}^{S} = \frac{1}{2}[(b_{1c} \pm b_{2c})\cos(2H) + (b_{1s} \pm b_{2s})\sin(2H)] \qquad (16)$$

which is of the form

$$\Delta L_{\pm}^{S} = \left|\Delta L_{\pm}^{S}\right| e^{i2H} \,. \qquad (17)$$

The quantities $2\Delta L_{-}^{S,T}$ are directly comparable with the displacement induced by gravitational waves.

4 RESULTS

The tidal effects due to the sun and the moon can interfere constructively in what are known as the Spring Tides. The strength of this largest tide can vary depending on the relative phases of the solar and lunar motions. We have taken the worst case, assuming that the tidal effects add coherently, by setting

$$\frac{D}{ag} = \frac{0.39\text{meters}}{6370\text{kilometers}} = 6.1\times 10^{-8} \qquad (18)$$

where 0.39 meters is the scalar sum of tidal effects (motion of equipotentials) due to the sun and the moon. We have used $\sin 2\delta \cong 0.76$ for the lunar declination, but we have set $\cos^2\delta \cong 1$. The resulting estimates for the tidal deformations of the interferometer arms can therefore be expected to be conservative. The Love numbers, $h = 0.62$ and $l = 0.062$ were obtained from geophysical data[5]. Local earth radius was computed separately for each site. The resulting estimates of maximal tidal effects are listed in Tables 14 and 15, below.

Table 1: Earth-Tide Effects for Hanford[a]

Quantity	*Value*	*Units*
Site Properties:		
Site Longitude	119 24 27.1 W	deg, min, sec, E/W
Site Latitude	46 27 18.5 N	deg, min, sec, N/S
Bearing of Arm 1	N 36.8 W	N/S, deg, E/W
Bearing of Arm 2	S 53.2 W	N/S, deg, E/W
Diurnal (Tesseral) Wave:		
Common-Mode Amplitude	113.6	microns peak-max
Differential-Mode Amplitude	22.0	microns peak-max
Semi-Diurnal (Sectorial) Wave		
Common-Mode Amplitude	71.0	microns peak-max
Differential-Mode Amplitude	31.1	microns peak-max

a. Amplitudes correspond to $\left|\Delta L_{\pm}^{S,T}\right|$ of equations (12) and (17).

Table 2: Earth-Tide Effects for Livingston[a]

Quantity	*Value*	*Units*
Site Properties:		
Site Longitude	90 46 27.3 W	deg, min, sec, E/W
Site Latitude	30 33 46.0 N	deg, min, sec, N/S
Bearing of Arm 1	S 18 E	N/S, deg, E/W
Bearing of Arm 2	S 72 W	N/S, deg, E/W
Diurnal (Tesseral) Wave:		
Common-Mode Amplitude	99.4	microns peak-max
Differential-Mode Amplitude	20.0	microns peak-max
Semi-Diurnal (Sectorial) Wave		
Common-Mode Amplitude	110.7	microns peak-max
Differential-Mode Amplitude	30.4	microns peak-max

a. Amplitudes correspond to $\left|\Delta L_{\pm}^{S,T}\right|$ of equations (12) and (17).

5 CONCLUSIONS

The earth-tide displacements presented here are estimates of the largest tides that could possibly occur at each site. The common-mode part of the earth tide affects the detuning between the wavelength of the laser light and the resonances of the arm cavities and can be zeroed by adjusting the laser frequency. The differential part of the tidal deformation resembles a gravitational-wave signal and must be removed by actuating the mirrors. Direct actuation of the mirrors presents a severe dynamic range problem unless that actuation occurs within the seismic-isolation system. This allows filtering of any high-frequency noise by subsequent seismic-isolation stages.

The maximum tidal corrections required are approximately 100 μm P-P in each of the interferometer arms with a shortest period of 12 hours. This task can be achieved by a number of actuation systems. However it is desirable to also reduce the microseismic noise which occurs with a 6-second period. This suggests the use of piezoelectric actuators to achieve the higher frequency of correction. Unfortunately, the maximum tidal displacements are near the maximum range of commercially available piezoelectric actuation systems, which can achieve approximately 120 μm P-P displacements. In addition to tidal and microseismic displacements, there are smaller thermally-induced displacements[6] that will need correction, putting additional demands on this actuator.

A more detailed tidal calculation that predicts actual, rather than maximal, tidal deformations should be made to estimate a more realistic requirement on the seismic-isolation actuators. This model could be combined with the microseismic statistics and a thermal-deformation model to predict the required range in the actuators to avoid loss of interferometer lock over a given time interval.

6 REFERENCES

[1] F. Raab and D. Coyne, *The Effect of Microseismic Noise on a LIGO Interferometer*, LIGO-T960187-01-D.

[2] Paul Melchior, The Tides of Planet Earth, Pergamon Press, Oxford, (1978).

[3] Reference [2], equation (3.67), pg 67.

[4] Combining equations (3.70) and (3.72) of reference [2].

[5] See pg 288 of reference [2].

[6] J. Berger, *J. Geophys. Res*., **80**, 274, (1975).