To: Mike Smith and Jordan Camp
From: R. Weiss March 15, 1997
Concerning: Scattering in the Interferometer

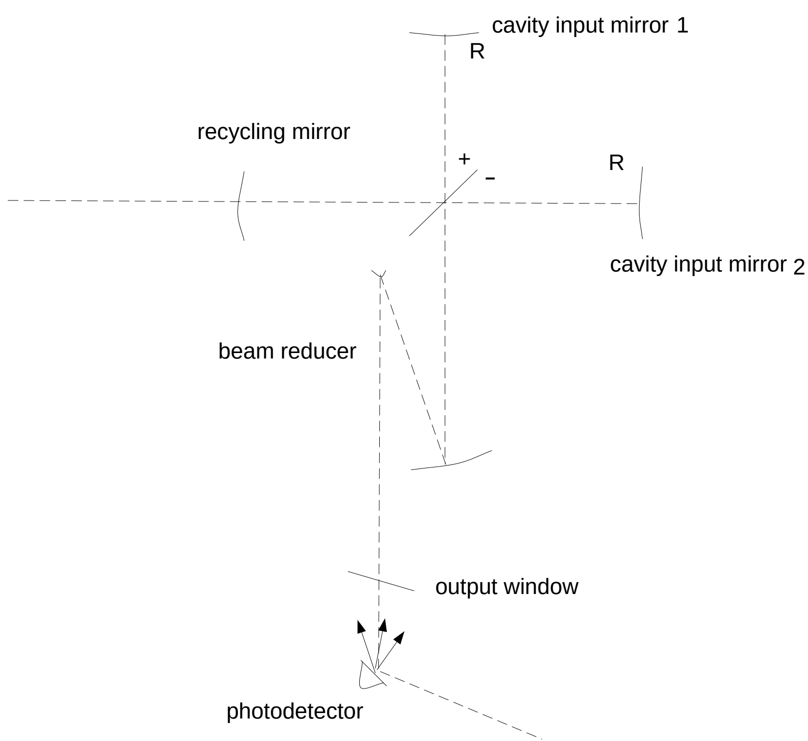
The things that I was going to look at after your visit were the following:

1) Estimate independently the influence of scattering by the photodetector at the antisymmetric port of the interferometer. This was the place you found most sensitive to feedback from backscattering in your calculation.

2) Attempt to estimate or measure the scattering by candidate photodetectors.

3) Provide some estimates for the scattering of standard optical surfaces.

Backscatter at the antisymmetric port



The optical fields at the antisymmetric port consist of :

1) the modulation sidebands that leak out due to the path length unbalance,

2) the balanced part of the carrier fields which has been reflected from the main cavities , carries the gravitational wave induced phase shift (differential phase shift proportional to the sine($\delta\varphi_g$) and when the interferometer is locked go to zero. The phase shift

is

$$\delta\varphi_g = \frac{d\varphi_g}{dh}\delta h = \frac{8\pi L\delta h}{T\lambda\sqrt{1+\left(\frac{f}{f_o}\right)^2}}$$

where L is the arm length, T the arm cavity input mirror transmission, f_0 the arm cavity corner frequency and f the gravitational wave frequency,

3) the unbalanced portion of the carrier field, which when the interferometer is locked, carries no information of the gravitational wave induced phase shifts (common mode phase shift proportional to the $\cos\delta\varphi_g$).

When the interferometer is locked the carrier power reaching the photodetector is

$P_{\text{det}} = \frac{1-c}{1+c}P_{\text{BS}}$ where c is the fringe contrast and P_{BS} is the carrier power on the beam splitter. The contrast defect is due to either carrier light in the wrong modes or carrier unbalance in the two arms; in the calculation it is assumed to be due to carrier unbalance. The sideband power is significant and can give a comparable contribution to the phase noise from the backscatter. Here the detailed mechanism is relevant since it is possible to have cancellations of the backscatter modulation if the sidebands and carrier are equally phase modulated by the moving scatterer. I assume no such cancellation in the estimate.

The backscatter into the interferometer by the photodetector or output window will be characterized by the differential scattering $B(\theta) = \frac{(dP_{\text{scat}}(\theta))/(d\Omega)}{P_{\text{inc}}}$ where θ is the angle from the specular and Ω is the solid angle for collecting the scattered light. The solid angle spanned by the TE_{00} mode of the interferometer in the recycling cavity (as well as the arm cavities) is $\Omega_0 = \frac{\lambda}{L}$, the coherence solid angle. To cause phase modulation of the interferometer, scattered light needs to fall into this solid angle in the full aperture beams. The optical transformation of the beam reducing telescope retains the solid angle * area (etendue or throughput). The solid angle into which scattered light is collected from the detector that falls within the coherence solid angle of the full aperature beam increases with the square of the magnification, m, of the telescope. The optical power due to scattering that can cause phase modulation of the interferometer is then given by the integral

$$P_{\text{eff}} = P_{\text{det}}\int_0^{\Omega_0 m^2} B(\theta)d\Omega$$

It is not generally true that the integral grows with m^2. If the scattering is from point like scatterers (dust or microcracks) it is a good approximation, on the other hand if the scattering is from microroughness with a power law spectrum in spatial frequencies, the assumption can make a gross overestimate; for example, estimate linear in m^2 while the truth lies closer to $\log(m^2)$. I assume as you have that components on the isolation stack are not moving significantly at the gravitational wave frequency and furthermore that the net rms displacement amplitude of the isolated components at low frequencies is less than a wavelength, so that it is not necessary to consider fringe “wrap around” up conversion.

Now on to the estimate which includes the following steps. The unbalanced carrier light is scattered by the detector surface, phase modulated by the motion of the detector $\delta\varphi_{\mathrm{scat}} = \frac{4\pi x_{\mathrm{scat}}}{\lambda}$, collected by the reducing telescope and a fraction enters the interferometer in the coherence solid angle. The beam splitter antisymmetrially sends the light toward the arm cavity input mirrors. The recombination with the main interferometer beams take place at these mirrors through reflection. The scattered beams do **not** build up in the recycling cavity beacuse of the antisymmetry of the beam splitter as seen from this port so the scattered beams reemerge to the detector where they are mixed with the RF sidebands. The photocurrent at the RF modulation frequency is compared to the photocurrent at the RF modulation frequency for a gravitational wave strain h. The tolerable amount of scattering is determined by allowing a noise equal to 1/10 (in amplitude) of the Initial Detector Science Requirement Document value.

Aside from constants that are eleminated by normalization to the gravitational wave, the photocurrent at the detector vibration frequency (after RF demodulation) is proportional to

$$i_{\mathrm{scat}}(f) \propto \sqrt{RP_{\mathrm{eff}}}\left(\frac{4\pi x(f)}{\lambda}\right)$$

while the photocurrent for the gravitational wave strain is proportional to

$$i_{\mathrm{g}}(f) \propto \sqrt{P_{\mathrm{bs}}}\left(\frac{8\pi L h(f)}{T\lambda\sqrt{1+(f/f_0)^2}}\right)$$

The desired condition is $\frac{i_{\mathrm{scat}}(f)}{i_{\mathrm{gSRD}}(f)} \le \frac{1}{10}$. which leads to a constraint on the detector scattering given by

$$\left(\sqrt{\frac{1-c}{1+c}\int_0^{\Omega_0 m^2} B(\theta)d\Omega} = \sqrt{\frac{P_{\mathrm{eff}}}{P_{\mathrm{BS}}}}\right) \le \left(\frac{1}{10}\right)\left(\frac{2Lh_{SRD}(f)}{Tx(f)\sqrt{R(1+(f/f_0)^2)}}\right)$$

The values I will use for the comparison

Table 1: Parameters used

h(100Hz)	3×10^{-23}	$1/\sqrt{\mathrm{Hz}}$
x(100Hz)	1×10^{-9}	$\mathrm{cm}/\sqrt{\mathrm{Hz}}$

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1 - c	1 x 10^{-3}	
R	0.97	
f_o	100	Hz
L	4 x 10^5	cm
Ω_0	2.5 x 10^{-10}	sr
m	72	

The result is a condition on the scattering integral

$$\int_0^{\Omega_0 m^2} B(\theta) d\Omega \leq 6.6\times10^{-12}$$

If the scattering from the detector were Lambertian (a pessimistic assumption) then $B(\theta) < 3.7\times10^{-4}$ sr^{-1} , a small but not hopelessly small value. We have been getting values in the range of 10^{-4} for pieces of glass at 55 degrees incidence.

You required a value $B(\theta) < 5.6\times10^{-5}$ sr^{-1} but you assumed a power buildup of the scattering of 35 in the recycling cavity and also included a Faraday isolator with a 30db (15db?) rejection. The power at the detector was 0.5 watts with a recycled power at the beam splitter of 350 watts leading to a contrast defect 1 - c = 2.85 x 10^{-3} . The factor of 3 ratio for the $B(\theta)$ is almost an accident, so some reconciliation of our estimating methods is needed.

Photodector scattering

Still at the measurements of a 3mm InGaAs detector, I only have upper limits at the moment since the beam is hitting parts of the detector other than the sensitive surface and there is scattering of the reflected light in the detector case. Will have to remove the detector from the can to improve the situation. Need agreement from Mike Zucker. to do this.
Upper limits so far at 1 micron:

$$B(20^o) \leq 2\times10^{-1} \text{ sr}^{-1}$$

$$B(55^o) \leq 1\times10^{-1}$$

Scattering from optical surfaces

A useful reference for surface roughness of standard optical components and the representation in terms of power spectra is *Power Spectrum Standard for Surface Roughness Part 1* D.J. Janeczko SPIE Vol 1165, P 175 (1989). I have adapted a formulation for fractal surfaces developed by Church, Takacs and Leonard in another article in the same volume and used this in a LIGO document on the Large Optics Specification which you may find useful. The scattering and the two dimensional power spectrum of the surface fluctuations are related

$$B(\theta) = \frac{16\pi^2}{\lambda^2} S_2(\nu)$$

where $S_2(\nu)$ is the two dimensional power spectrum expressed in waves^2 of λ /wavenumber $(\text{cm}^{-1})^2$ .The power spectrum can be modeled by an amplitude A in waves^2 /wavenumber, a correlation length l_{cor} , and a power law exponent c associated with the change in spectrum with spatial frequency ν in wavenumbers (cm^{-1}). The two dimensional power spectrum is modeled by

$$S_2(\nu) = \frac{\Gamma((c+1)/2)}{\Gamma(c/2)}\left(\frac{\pi^{1/2} l_{\text{cor}} A}{(1+(2\pi\nu l_{\text{cor}})^2)^{(c+1)/2}}\right)$$

where $\nu = \sqrt{\nu^2{}_x + \nu^2{}_y}$

The relation between the scattering angle and the spatial frequency is determined from the grating equation for **small** angles (this may invalidate some of the applications, check to make sure)

$$\theta \approx \nu\lambda$$

So that the scattering may be rewritten in terms of the power spectrum model parameters

$$B(\theta) = \frac{16\pi^{5/2}\Gamma((c+1)/2)}{\lambda^2\Gamma(c/2)}\left(\frac{l_{\text{cor}} A}{(1+((2\pi\theta l_{\text{cor}})/\lambda)^2)^{(c+1)/2}}\right)$$

The most commonly given information is the one dimensional power spectrum. This is determined from profilometry or one dimensional scans of phase maps. Most of our information concerning the core optics is given in one dimensional power spectra. The one dimensional power spectrum is modeled as

$$S_1(\nu_x) = \frac{A}{(1+(2\pi\nu_x l_{\text{cor}})^2)^{c/2}}$$

The surface variance is determined by integrating the power spectrum

$$\frac{\sigma^2}{\lambda^2} = \int_0^\infty S_1(\nu_x)d\nu_x = 2\pi\int_0^\infty S_2(\nu)d\nu$$

The total integrated scatter into all directions is

$$\frac{P_{scat}}{P_{inc}} = 16\pi^2\left(\frac{\sigma}{\lambda}\right)^2$$

Table 2: Power spectrum parameters

POLISH	A waves$(1\mu)^2$/cm^{-1}	l_{cor} cm	c	σ Angstrom
commercial	10^{-6} - 10^{-5}	0.1 - 1.0	1 - 2	10 - 30
precision	3 x 10^{-7} - 3 x 10^{-6}	0.1 - 1.0	1 - 2	2 - 10
super polish				0.5 - 2
LIGO COC CSIRO	3 - 8 x 10^{-7}	0.5 - 1	1	2 - 3

The uncertainties are large enough that we need to measure the scattering by the components we will be using.