Calibration of 40 m Interferometer Displacement Sensitivity
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1 Principle of Calibration

1. Calibration is routinely done while the interferometer is operating by applying a sinusoidal displacement Δl_{cal} to one of the test masses via the force actuators that also control the interferometer cavity length. The size of the calibration is adjusted to be much larger than the interferometer noise level but not so large as to induce non-linearities—typically 10^{-16} m rms. At a given frequency, the motion is proportional to the calibration voltage V_{coil} applied to the actuator (see Figure 1a). At frequencies of interest (above the 1 Hz pendulum resonance), the mass is approximately free. The force on the mass is proportional to V_{coil}; hence

   $$\Delta l_{\mathrm{cal}} = \beta \frac{V_{\mathrm{coil}}}{\omega^2}$$

   where $f = \omega/2\pi$ is the calibration frequency and the calibration constant β depends on the mass of the test mass, the strength of the actuator magnets, etc. β is not expected to change during the normal course of operation, and is measured only infrequently.

2. The interferometer responds to the induced calibration with a "readout chain" (Figure 1b) signal of magnitude V_{cal}, proportional to the drive:

   $$V_{\mathrm{cal}} = \alpha(f) V_{\mathrm{coil}}.$$

   (The readout chain signal is also proportional to the differential-mode feedback voltage.) The proportionality constant $\alpha(f)$ depends on the frequency of the calibration sinusoid; it is sensitive to system gains, and is measured frequently during data runs or tests. $\alpha(f)$ is the "swept sine" frequency response.

3. In normal operation, the calibration drive V_{coil} is removed, and the readout chain output is V_{sig}. This voltage is converted to the displacement signal by

   $$\begin{aligned} \Delta x &= \Delta l_{\mathrm{cal}} \frac{V_{\mathrm{sig}}}{V_{\mathrm{cal}}} \\ &= \beta \frac{V_{\mathrm{coil}}}{\omega^2} \frac{V_{\mathrm{sig}}}{V_{\mathrm{cal}}} \\ &= \beta V_{\mathrm{sig}} / [\alpha(f)\omega^2] \end{aligned}$$

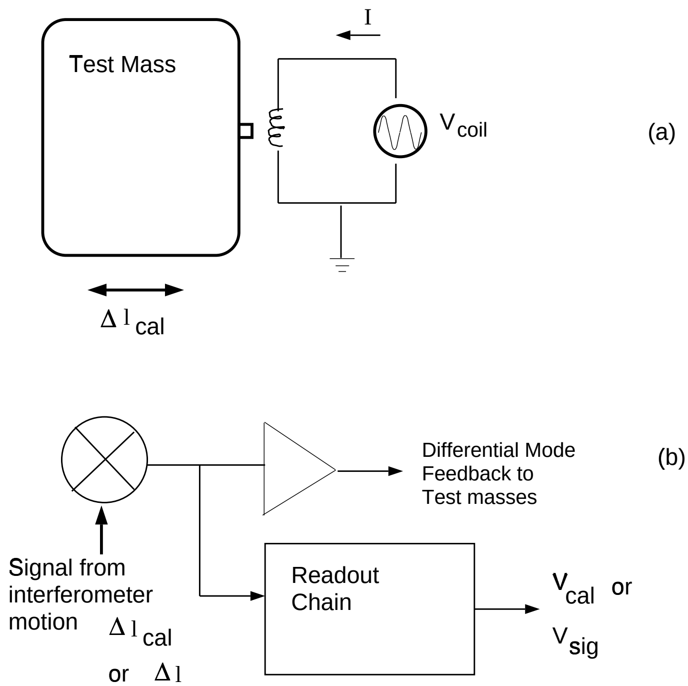

Figure 1: *(a)* Excitation of test mass position by sinusoidal current through calibration coil acting on test mass magnet. *(b)* Interferometer output signal, $V_{\rm cal}$ in response to calibration of end test mass, $V_{\rm sig}$ under normal operation.

2 Measurement of β

2.1 "Full Fringe" drive

The standard method for measuring β is to apply a large enough sinusoidal $V_{\rm coil,fringe}$ to drive the mass through adjacent "fringes," or successive resonant lengths separated by $\lambda/2$, where $\lambda = 514.5\,{\rm nm}$.

Typically to achieve this much motion the frequency of the drive, $f_{\rm fringe} = \omega_{\rm fringe}/(2\pi)$, must be low ($\approx$ 10 Hz), and the usual calibration addition circuitry must be bypassed. This requires calculating or separately measuring the attenuation factor $h_{\rm calib}$ of the calibration addition circuitry. (See Figure 3.)

Defining

$$\mu = \frac{f^2_{\rm fringe}}{V_{\rm coil,fringe}}$$

we have

$$\beta = (2\pi)^2 \mu \frac{\lambda}{2} h_{\rm calib}.$$

Numerical values:

Case	$1/\mu$ (Hz^2/V)	$h_{\rm calib}$ dB / lin	β (m-Hz^2/V)	$k(1\,{\rm kHz\ span})$ (V$\sqrt{\rm Hz}$/[m-Hz^2])
Aug 96 correct msrmnt	$(2.5\cdot 10^{-2}\,{\rm V/Hz^2})^{-1}$	-17.6 / 0.13	$5.36\cdot 10^{-5}$	25502
Incorrect msrmnt		-9.4 / 0.34		9930
Previously assumed				9570

The rightmost column, k, is the multiplier for the swept sine measurement that is entered into the spectrum analyzer manually when the "raw" spectrum is divided by the swept sine, for the case of a 1 kHz frequency

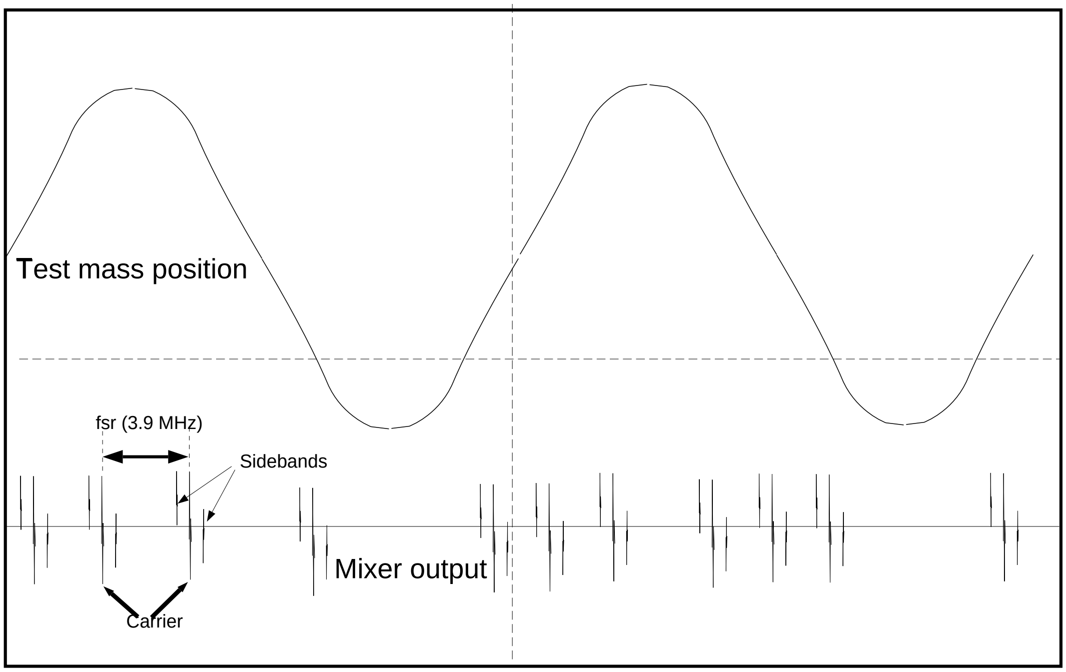

Figure 2: Sketch of oscilloscope output as test mass position (upper trance) is driven through "fringes" observed at the mixer output (lower trace).

span (1.87 Hz bandwidth). It is computed by

$$k(1\,\text{kHz span}) = \frac{\sqrt{1.87\,\text{Hz}}}{\beta}$$

The first row represents the most accurate estimate of β or k with the electronics installed for recombination; the second and third rows row are estimates based on an invalid measurement method for β that incorrectly assumes that a 50 ohm terminator supplied a path to ground for the calibration current. That terminator was removed when the electronics was modified from the nonrecombined to the recombined configuration[1] The correction factor that should be applied to all spectra since the start of recombination is

$$w = 9570/25502 = 0.38 = -8.5\,\text{dB}.$$

2.2 "Sideband Fringe"

Figure 2 shows the modulation sidebands visible at the demodulator output. Since the free spectral range is smaller than the modulation frequency, the sidebands are not associated with the carrier frequency that they are closest to; the nearness of the sidebands to the carrier is coincidental:

Free spectral range $f_{\text{fsr}} = c/2l = 3\cdot 10^8$ m/sec/2/38.5 m	Modulation frequency, f_{m}	$\Delta x_{\text{carrier}}$ (m) $\lambda/2$	$\Delta x_{\text{sideband}-\text{carrier}}$ (m) $(\lambda/2)(f_{\text{m}} \div f_{\text{fsr}})/f_{\text{m}}$
3.90 MHz	12.33 MHz	247 nm	41 nm

[1] Last spectrum before recombination 5 Dec 94. First spectrum with recombination 19 May 96.

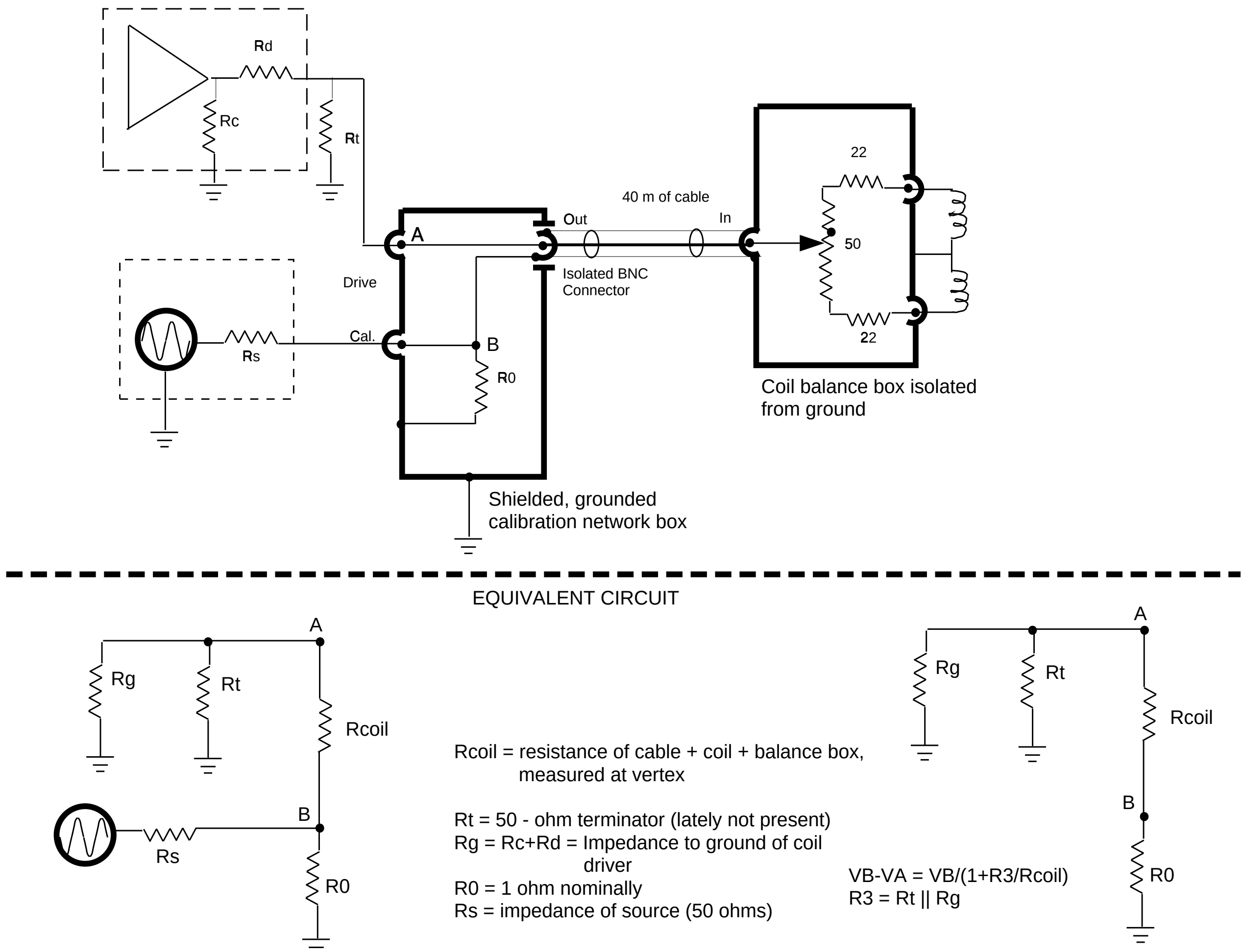

Figure 3: Addition/attenuation circuitry used to allow one mass to share the signals for length control and calibration.

2.3 "External Michelson"

The constant β can also be measured using the wavelength of an external (such as a low-power HeNe laser) as a standard, and setting up an external Michelson interferometer to measure the motion of the test mass in response to V_{coil}.