LIGO-T970241-00-D

HYTEC-TN-LIGO-18
7/28/97

Temperature Effects on LIGO Damped Coil Springs

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July 28, 1997

Abstract

A simple method for estimating a damping material stiffness and loss factor as a function of temperature and frequency is presented. The method is used in the calculation of the LIGO coil spring damping and stiffness. Theoretical results are compared with measured data.

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LIGO PROJECT

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1. Summary

The intent of this work is to describe an analytical procedure for predicting the loss factor and axial stiffness of the LIGO spring designs as a function of temperature at constant frequencies. Published data for polymer Soundcoat DYAD 606 in the form of loss factor and stiffness as a function frequency at a constant temperature is converted into loss factor and stiffness as a function of temperature at a constant frequency. The converted data is then used in analytical expressions that calculate the loss factor and stiffness characteristics of the LIGO spring designs incorporating the polymer damping material in a spring sandwich design[1]. Results indicate that at the design frequency of 1.8 Hz, the polymer loss factor is a maximum at room temperature (72 degree F) and that damping for the spring design is maximum over a range of temperature from 65^o F to 72^o F. Theoretical values for a coil springs on epoxy seats are compared with experimental data obtained over a range of temperatures at frequencies from 0.93 Hz to 0.99 Hz.

2. Coil Spring Damping Material

2.1 Viscoelastic Material Damping Relations

A viscoelastic material deviates from perfect elasticity by a component of stress that lags strain expressed by

$$\sigma = E\varepsilon(1 + i\eta), \tag{1}$$

where σ, E, ε, i, and η are the stress, Young's modulus, strain, imaginary constant, and loss factor respectively. Assuming simple harmonic motion where strain can be represented as

$$\varepsilon = \varepsilon_o \sin(\omega t), \tag{2}$$

where ε_o, ω and t are the peak strain, circular frequency and time. The energy dissipated per cycle by a viscoelastic material is then (integrating the area contained within the stress strain curve)

$$\Delta E = \pi\eta E\varepsilon_o^2 . \tag{3}$$

The ratio of dissipated energy for a viscoelastic material to the stored energy for a perfectly elastic material is

$$\frac{\Delta E}{SE} = 2\pi\eta . \tag{4}$$

This can be compared to the conventional expression for a viscous damper where the ratio is approximately $4\pi\zeta$ so that

$$V = \frac{h}{2}, \tag{5}$$

where ζ is the critical damping ratio.

2.2 Chemical Kinetics

The rate at which chemical reactions take place is given by the Arrhenius expression [2]

$$k = Ae^{-E_a/RT}, \tag{6}$$

where k is the reaction rate, A is a constant, E_a is the activation energy in KJ per mole, R is the universal gas constant in KJ per mole Kelvin, and T is absolute temperature in Kelvin. Equation (6) can be expressed as a ratio of reaction rates by

$$ln\frac{k_1}{k_2} = \frac{E_a}{R}\left(\frac{1}{T_2} - \frac{1}{T_1}\right). \tag{7}$$

A chemical reaction will only occur when the translational kinetic energy exceeds the activation energy during a collision between two molecules. The number of molecules that exceed the critical activation energy is temperature dependent as shown in the Maxwell-Boltzman distribution of molecular kinetic energies.

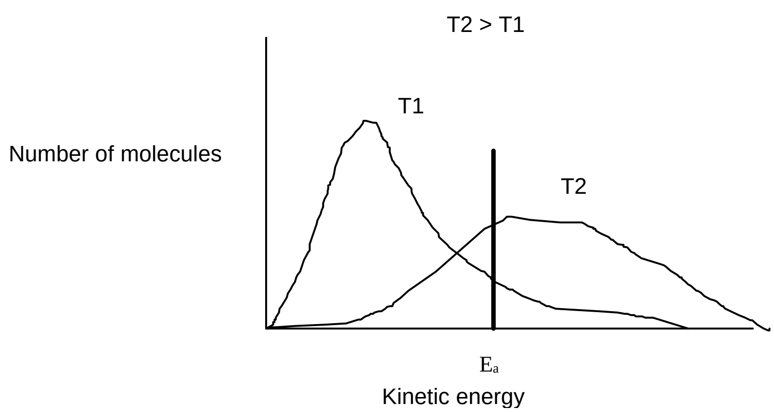

Figure 1: Maxwell-Boltzman distribution of molecular kinetic energies (E_a is the critical molecular energy and T denotes temperature).

2.3 Viscoelastic Damping Material DYAD 606 Data

Loss factor (h) and stiffness (K) data for DYAD 606 viscoelastic material is given as a function of frequency at a fixed temperatures of 55^0, 77^0, and 86^0F in Soundcoat Product Data Sheet - Bulletin 701[3]. A plot in the referenced bulletin shows loss factor and stiffness on the ordinate against frequency on the abscissa with shifts in the frequency axis for range of material temperatures. Figures 2 displays the loss factor and stiffness (N/m^2) for DYAD 606 at 77^0 F as a function of frequency from 0 to 10 Hz.

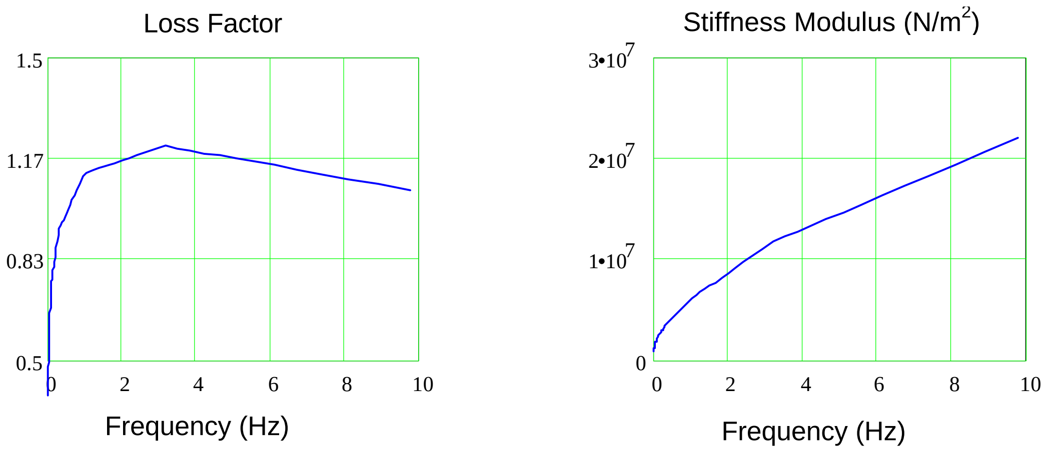

Figure 2: DAYD 606 loss factor and stiffness modulus as a function of frequency.

An expression for polymers similar to Equation (7) relating shifts in frequency to differences in temperature at a constant maximum loss factor is given by [4]

$$ln\frac{f_2}{f_1} = \frac{E_a}{R}\left(\frac{1}{T_1} - \frac{1}{T_2}\right), \tag{8}$$

where f is the frequency in Hz. Calculation for the material constant (E_a/R) is performed by selecting temperatures above and below a reference temperature, reading the frequencies at a constant loss factor for each temperature, and averaging the two computed constants from Equation (8). For temperatures of 50, 77, and 86^0 F, the frequencies at constant loss factor are 0.1, 3.162, and 10 Hz. Substituting these values into Equation (8) generates material constants of 19420 K and 20790 K for an average value of 20105 K. Equation(8) is derived from the assumption that the polymer relaxation time (τ) is related to the temperature by

$$\tau = \tau_o e^{E_a/RT}, \tag{9}$$

and that the energy loss per cycle is a maximum when $\omega\tau = 1$. If we assume that Equation (8) is valid for any loss factor (not just at the peak loss factor) and that it is also valid for storage modulus relations, the temperature effect at a fixed frequency can be determined as described in the following section.

2.4 Loss Factor as Function of Temperature at a Fixed Frequency

Loss factor can be schematically displayed as a function of frequency for different temperatures as shown in the Figure 3.

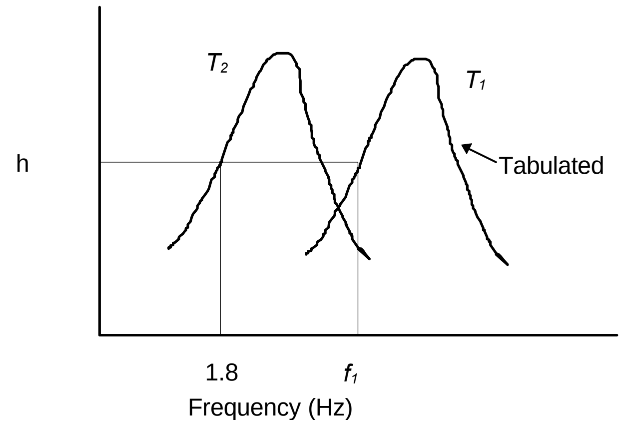

Figure 3: Sketch showing general relationship of frequency and temperature at a constant loss factor.

The loss factor curve for the reference temperature T_1 is tabulated as a function of frequency and we desire the loss factor as a function of temperature at some fixed frequency denoted by f_{fix}. Frequencies for a set of variable temperatures are calculated by rearranging (8)

$$f_1 = f_{fix}\, e^{\frac{E_a}{R}\left(\frac{1}{T_2}-\frac{1}{T_1}\right)}, \tag{10}$$

where the frequency f_2 has been replaced by f_{fix}. Thus the loss factors as a function of temperature at some constant frequency is found by first calculating the frequencies f_1 at a range of temperatures T_2 and then interpolating for h at f_1 on the tabulated reference temperature curve T_1. Stiffness values are found in the same manner using a tabulated curve at the reference temperature. Calculated loss factor and stiffness modulus (N/m^2) are shown in Figures 4 and 5 at fixed frequencies of 0.96 Hz (to be compared with subsequent test data) and 1.8 Hz (the calculated LIGO stack frequency).

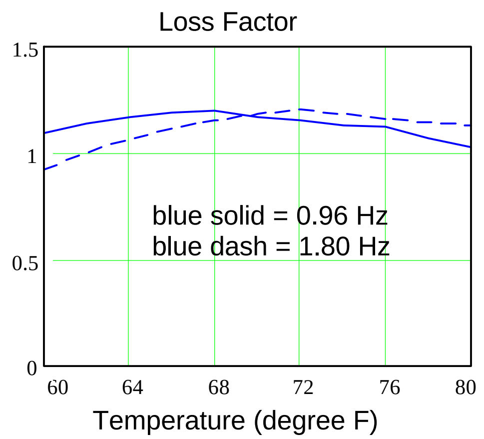

Figure 4: DYAD 606 calculated loss factor at 0.96 Hz and 1.80 Hz as a function of temperature.

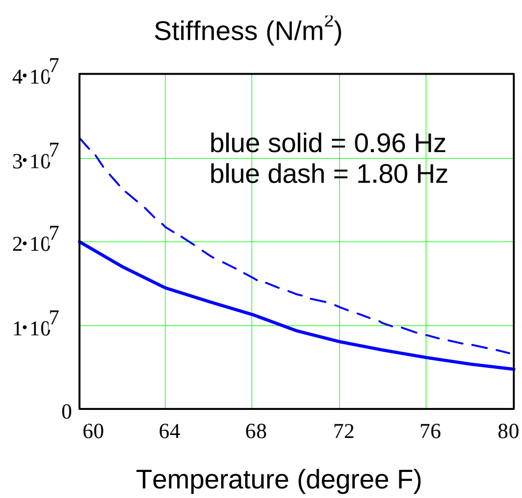

Figure 5: DYAD 606 calculated stiffness modulus at 0.96 Hz and 1.80 Hz as a function of temperature.

3. Spring Assembly Axial Loss Factor and Stiffness at 1.8 Hz

Closed form analytical expressions [5] are used to calculate the LIGO damped coil theoretical axial stiffness and loss factor as a function of temperature at a fixed frequency of 1.8 Hz. The loss factor and stiffness temperature relationships shown in previous figures are used for the damping material while the parameters listed in Table I are used for the spring. The theoretical expressions are [5]

$$\lambda = \sqrt{G_v'(1+i\eta)\frac{\pi d_v^3}{4t_v}\frac{G_wJ_w+G_cJ_c}{G_wJ_wG_cJ_c}}, \qquad (1)$$

$$k_t = \frac{G_w J_w + G_c J_c}{1 + \frac{2}{\lambda \ell} \frac{G_c J_c}{G_w J_w} tanh\left(\lambda \ell / 2\right)}, \qquad (2)$$

where G_v' is the damping material stiffness modulus, k_t the spring torsional stiffness and ℓ the spring free length (44.219 mm). In addition, $t_v = (d_w - D_c)/2$, $d_v = (d_w + D_c)/2$ and $i = \sqrt{-1}$. The loss factor is evaluated from $\eta_t = \Im m(k_t) / \Re e(k_t)$.

	Copper shell		Aluminum core	
	Symbol	Value	Symbol	Value
Outer diameter (mm)	D_w	9.520	D_c	6.735
Inner diameter (mm)	d_w	7.802	d_c	4.8952
Shear modulus (N/m^2)	G_w	4.482E10	G_c	26.0E9

Table I: Coil spring parameters for theoretical model.

The spring axial stiffness is calculated from

$$K_{ax} = \frac{4 k_t}{n \pi D^3}, \qquad (4)$$

where D is the coil pitch diameter (14 mm) and n the number of active turns (3.16). Figure 6 presents the theoretical results for the damped spring model at a constant frequency of 1.8 Hz.

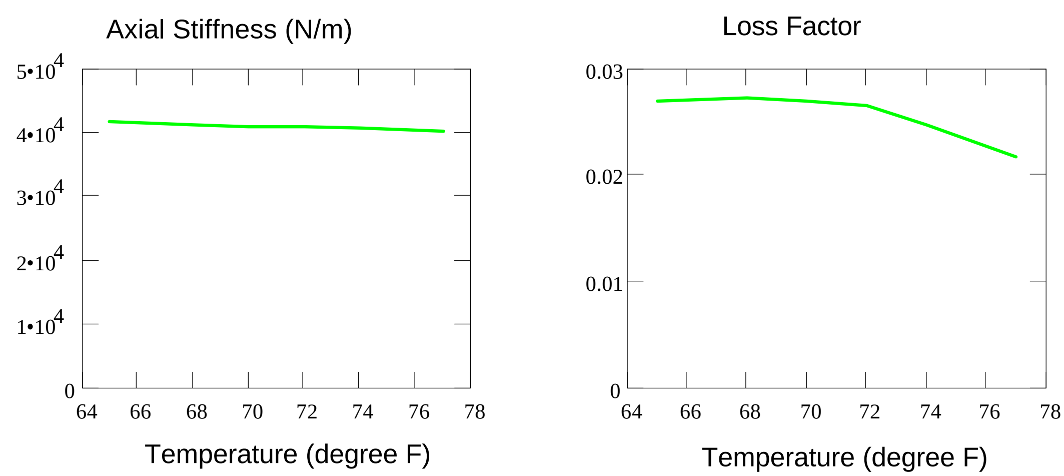

Figure 6: Theoretical axial stiffness and loss factor for damped coil on epoxy seats as a function of temperature at a fixed 1.8 Hz frequency.

4. Theoretical and Experimental Damped Coil Results

Axial tests to measure loss factor and stiffness at a range of temperatures were conducted for a damped coil on epoxy seats during a nine day time period (July 3 to July 9, 1997). Tests were conducted at the same temperature on different days to verify repeatability. Although measurements were not repeated at the lower temperatures, the desired repeatability at moderate to higher temperatures was achieved. Loss factors and stiffness are calculated for the coil assembly using the DYAD 606 material temperature properties shown in Figures 7 and 8 at 0.96 Hz. Test data was obtained in the range from 0.93 to 0.99 Hz.

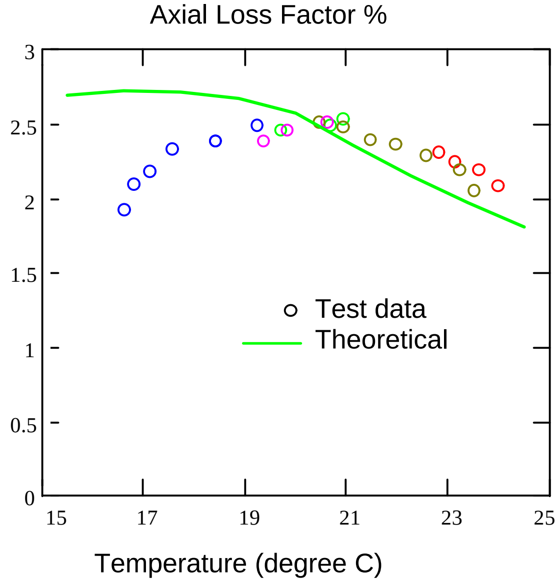

Figure 7: Damped coil on epoxy loss factor - theoretical results and test data (test data colors denote different days).

Inspection of Figure 7 reveals good agreement between test and the analysis model above 19^0 C and poor agreement below 19^0 C. Further examination into this discrepancy is desirable.

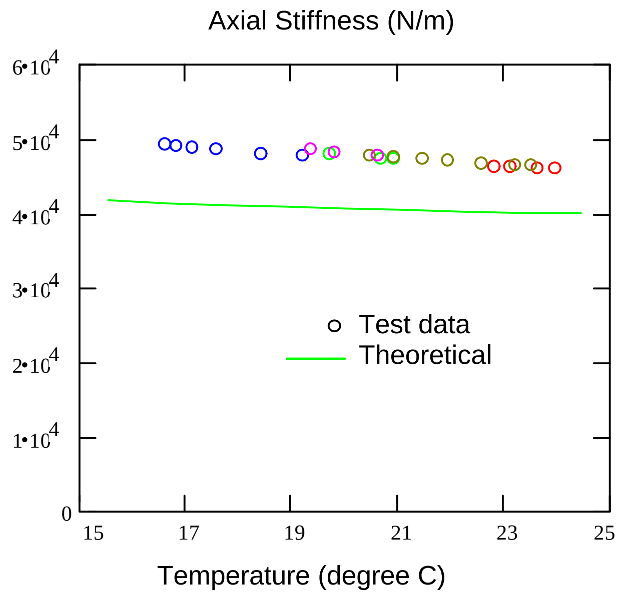

Figure 8: Damped coil on epoxy axial stiffness - theoretical results and test data (test data colors denote different days).

5. Conclusions

This document presents a simple procedure for converting published damping material loss factor and stiffness modulus data at specific frequency and temperature to other desired frequency and temperature. The method is used to determine the damping and stiffness as a function of temperature of the LIGO damped coil design using a previously developed theoretical model. Comparison of theoretical results with test data shows marginal agreement at lower temperatures with excellent agreement at moderate to higher temperatures.

6. References

1. E. Ponslet, *LIGO Coil Spring - Test Report*, HYTEC Inc., Los Alamos, NM, document HYTEC-TN-LIGO-14, February 1997.
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3. SOUNDCOAT, *Soundcoat Product Data Sheet,* Bulletin 701 "DYAD For use in thick plate vibration damping."
4. Gillam, *Materials Under Stress,* CRC Press, 1969
5. E. Ponslet, *Design of Vacuum Compatible Damped Metal Springs for Passive Isolation of The LIGO Detectors*, HYTEC Inc., Los Alamos, NM, document HYTEC-TN-LIGO-04a (revision *a*), January 1997.

Note 1, Linda Turner, 09/03/99 02:07:47 PM
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