Cross-Correlation Searches for Periodic Gravitational Waves

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Abstract

Cross-correlation of gravitational-wave (GW) data streams have been used to search for stochastic backgrounds, and the same technique was applied to look for periodic GWs from the low-mass X-ray binary (LMXB) Sco X-1. A technique has been developed which refines the cross-correlation scheme to take full advantage of the signal model for periodic gravitational waves from rotating neutron stars. By varying the time window over which data streams are correlated, the search can "trade off" between parameter sensitivity and computational cost. Possible search targets include SN1987A remnant and Sco X-1.

Cross-Correlation for Stochastic Signals

Cross-correlation is a standard technique to search for faint signal in noise:

$$x_1(t) = n_1(t) + h_1(t) = n_1(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}_1$$
$$x_2(t) = n_2(t) + h_2(t) = n_2(t) + \overleftrightarrow{h}(t) : \overleftrightarrow{d}_2$$

Application to stochastic background[1]; expectation value due to correlations in random signals

$$\langle \tilde{x}_1^*(f)\tilde{x}_2(f')\rangle = \langle \tilde{h}_1^*(f)\tilde{h}_2(f')\rangle = \delta(f-f')\,\gamma_{12}(f)\,\frac{S_{\text{gw}}(f)}{2}$$

- $S_{\text{gw}}(f)$ is GW spectrum
- "Overlap reduction function" $\gamma_{12}(f)$ encodes observing geom (detectors, sky distribution ...)

Optimally-filtered statistic:

$$Y = \int df\, \tilde{x}_1^*(f) Q(f) \tilde{x}_2(f)$$

with optimal filter

$$Q(f) \propto \frac{\gamma_{12}^*(f) S_{\text{gw}}^{\text{exp}}(f)}{S_{n1}(f) S_{n2}(f)}$$

Used to search for pointlike stochastic sources[2] including Scorpius X-1[3].

Cross-Correlation for Periodic Signals

Sco X-1 not random emitter; low-mass X-ray binary: neutron star in binary orbit w/companion. GW signal from rotating neutron star:

$$\overleftrightarrow{h}(t) = h_0 \left[\frac{1+\cos^2\iota}{2} \cos\Phi(\tau(t))\, \overleftrightarrow{e}_+ + \cos\iota \sin\Phi(\tau(t))\, \overleftrightarrow{e}_\times \right]$$

- ι: inclination of NS spin
- $\Phi(\tau)$: phase evolution in rest frame;
- $\tau(t)$: Doppler mod from detector motion (& binary orbit)

Include features of signal in cross-corr method:

- Long-term coherence:
  can cross-correlate data from different times
- Doppler shift @ detector:
  correlations peaked @ different freqs

Note signal cross-correlation deterministic

$$\langle \tilde{x}_I^*(f_I)\, \tilde{x}_J(f_J)\rangle = \tilde{h}_I^*(f_I)\, \tilde{h}_J(f_J)$$
$$= h_0^2 \tilde{\mathcal{G}}_{IJ}\, \delta_{T_{\text{sft}}}(f_0 - f_I - \delta f_I)\, \delta_{T_{\text{sft}}}(f_0 - f_J - \delta f_J)$$

- $\tilde{h}_I(f)$ is Short Fourier Transform, duration T_{sft}
- $\delta_{T_{\text{sft}}}(f-f') = \int_{-T_{\text{sft}}/2}^{T_{\text{sft}}/2} e^{i2\pi(f-f')t}\,dt$
- $\tilde{h}_I$ & $\tilde{h}_J$ can be same or diff times or detectors
- δf_I is relevant Doppler shift
- $\tilde{\mathcal{G}}_{IJ}$ is analogue to $\gamma_{12}(f)$

Optimal combination of SFT pairs:

$$Y = \sum_{IJ} Q_{IJ} \tilde{x}_I^*(f_0 - \delta f_I) \tilde{x}_J(f_0 - \delta f_J)$$

Optimal weights

$$Q_{IJ} \propto \frac{\tilde{\mathcal{G}}_{IJ}^*}{S_{n,I}(f_0) S_{n,J}(f_0)}$$

Parameter Dependence

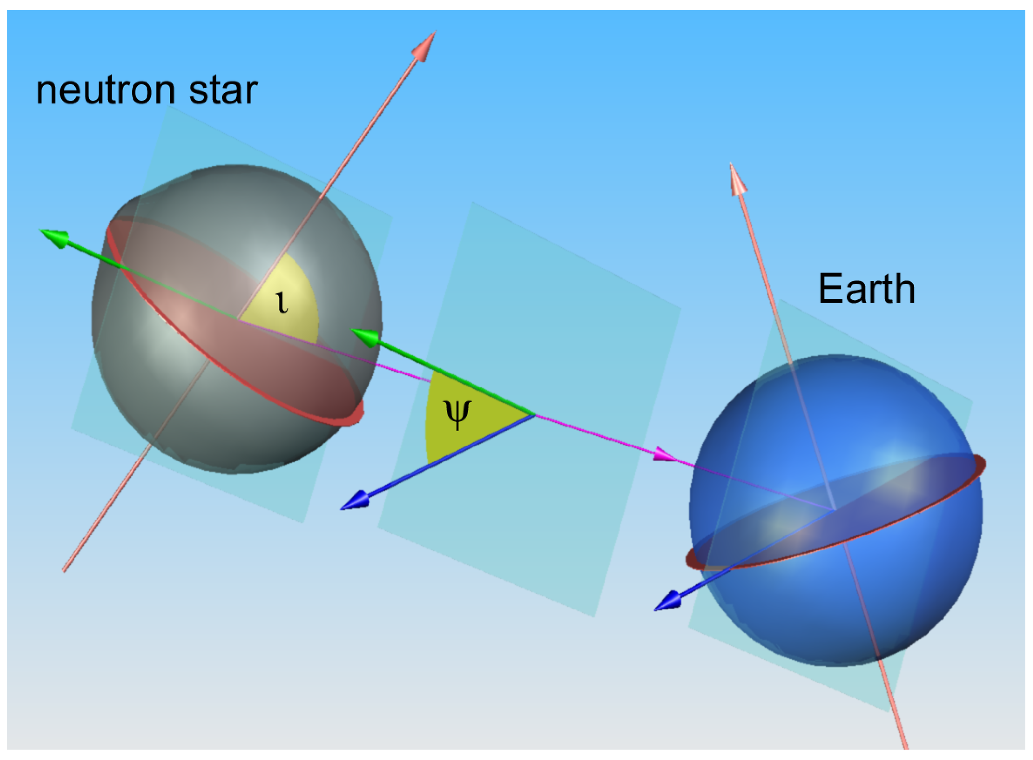

Figure 1: *Illustration of the inclination ι and polarization ψ angles relating the neutron star spin axis to celestial coördinates defined by the Earth's rotation axis. These are amplitude parameters which do not have a large impact on the number of required templates.*

Two kinds of parameters:

- Amplitude params:
  h_0, inclination ι, polarization ψ, initial phase Φ_0
- Phase params:
  f_0, spindowns, binary orbital parameters

Amplitude params don't pose challenges for filtering: h_0 is overall amplitude; Φ_0 drops out of cross-corr; can average $\tilde{\mathcal{G}}_{IJ}$ over $\cos\iota$ & ψ for simplicity. Mismatch in phase params leads to cancellation in optimal statistic; need to search over them. Long coherent integration time can give unmanageable # of templates. Limiting allowed pairs in $\sum_{IJ}$ by e.g., max time difference produces semi-coherent search w/manageable compute time.

Theoretical Sensitivity

Amplitude sensitivity of combined statistic:

$$h_0^{\text{th}} \propto \left(\sum_{IJ} |\tilde{\mathcal{G}}_{IJ}|^2\right)^{-1/4} \sqrt{\frac{S_n}{T_{\text{sft}}}}$$

- If all pairs included, $N_{\text{pairs}}^2 \propto N_{\text{sft}}$

$$h_0^{\text{th}} \propto (N_{\text{sft}} T_{\text{sft}})^{-1/2} = T_{\text{obs}}^{-1/2}$$

Coherent search

- If only simultaneous pairs, $N_{\text{pairs}} \propto N_{\text{sft}}$

$$h_0^{\text{th}} \propto N_{\text{sft}}^{-1/4} T_{\text{sft}}^{-1/2} = T_{\text{obs}}^{-1/4} T_{\text{sft}}^{-1/4}$$

- If only pairs separated by T_{lag} or less,

$$h_0^{\text{th}} \propto T_{\text{obs}}^{-1/4} T_{\text{lag}}^{-1/4}$$

Can simplify sensitivity estimates if observations uniformly distributed in sidereal time:

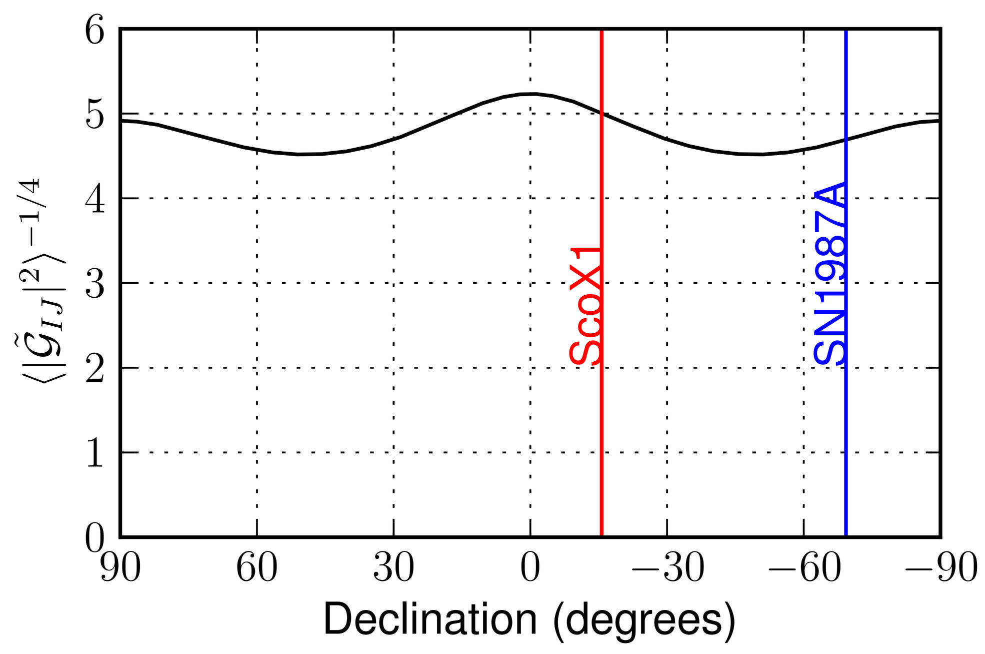

Figure 2: *Geometrical factor $\langle|\tilde{\mathcal{G}}_{IJ}|^2\rangle^{-1/4}$ appearing in the cross-correlation sensitivity, averaged over ψ, $\cos\iota$, and sidereal time, as a function of declination. The sky positions of the supernova 1987A remnant and Scorpius X-1 are shown for reference.*

Application: Supernova 1987A Remnant

SN1987A remnant likely contains young neutron star, rapidly spinning down[5, 6]. Can search for GW w/cross-correlation method. Need to search over frequency and spindowns; rather than searching f, $\dot{f}$, $\ddot{f}$, $\dddot{f}$, ..., use phase model w/GW spindown $\propto f^5$; EM spindown $\propto f^{\approx 3}$:

$$\frac{df}{d\tau} = Q_{\text{GW}} \left(\frac{f}{f_{\text{ref}}}\right)^5 + Q_{\text{EM}} \left(\frac{f}{f_{\text{ref}}}\right)^n$$

Search over f_0, Q_{GW}, Q_{EM}, n.
Can ballpark sensitivity using initial LIGO design & assuming only simultaneous LLO and LHO data are used. Compares favorably to indirect age-based limit $h_0 < 3.4 \times 10^{-25}$:

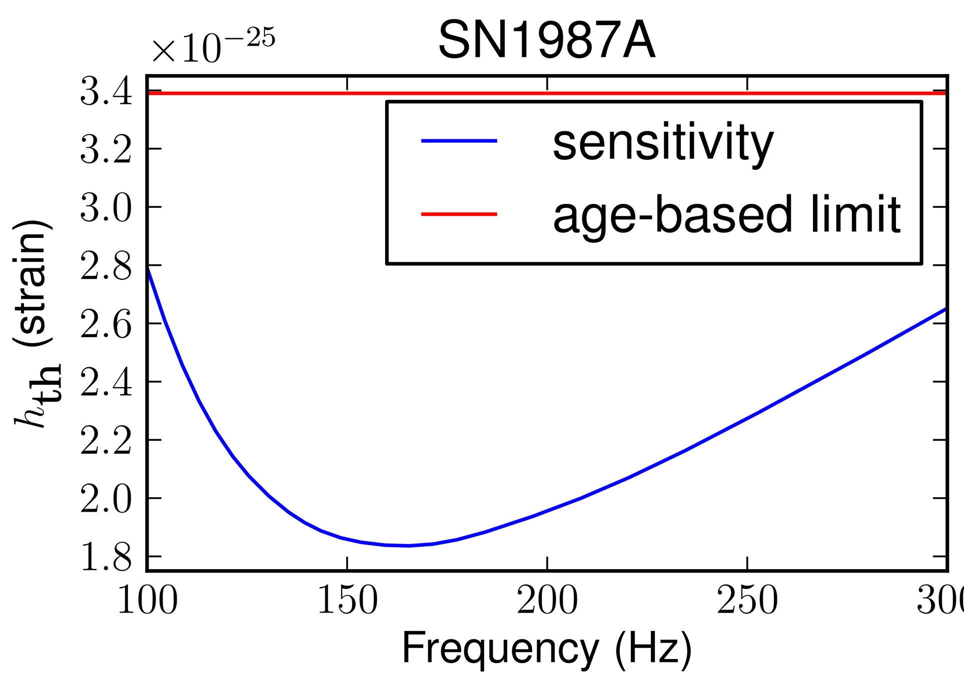

Figure 3: *Theoretical sensitivity to SN 1987A remnant for 1 year simultaneous initial LIGO design data, 5% false alarm & dismissal*

References

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