



Cross-Correlation Searches for Periodic Gravitational Waves



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- Cross-corr method adapted to periodic GWs
- Uses signal model to correlate data @ diff times
- Tuning max time-lag btwn cross-correlated data allows tradeoff of sensitivity for computing time
- Can search for young NSs (e.g., SN1987A) (search over f_0 & braking model params)
- Can search for LMXBs (e.g., Sco X-1) (search over f_0 & binary orbit params)

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Abstract

Cross-correlation in gravitational wave (GW) data streams have been used to search for stochastic backgrounds, and the same technique was applied to look for periodic GWs from the low-mass X-ray binary (LMXB) Sco X-1. A technique has been developed which refines the cross-correlation scheme to take full advantage of the signal model for periodic gravitational waves from rotating neutron stars. By varying the time window over which data streams are combined, the search can "trade off" between parameter sensitivity and computational cost. Possible search targets include SN1987A remnant and Sco X-1.

Cross-Correlation for Stochastic Signals

Cross-correlation is a standard technique to search for faint signal in noise:

$$s(t) = n(t) + h(t) = n(t) + h_0 \cos(\omega t + \phi)$$

$$s_1(t) = n_1(t) + h_1(t) = n_1(t) + h_1 \cos(\omega t + \phi)$$

Application to stochastic background (S) expects value due to correlations in random signals

$$\langle S_1(t) S_2(t) \rangle = \langle n_1(t) n_2(t) \rangle + \langle h_1(t) h_2(t) \rangle$$

$\langle n_1(t) n_2(t) \rangle = 0$ (uncorrelated noise)
 $\langle h_1(t) h_2(t) \rangle = h_1 h_2 \cos(\omega t + \phi) \cos(\omega t + \phi)$

Optimally filtered statistic:

$$Y = \int dt \int dt' Q(t) S_1(t) S_2(t')$$

with optimal filter

$$Q(t) = \frac{1}{\sqrt{2}} \frac{S_1^*(t) S_2(t)}{\sqrt{S_1^*(t) S_2(t)}}$$

Used to search for periodic stochastic sources (e.g. including Scorpius X-1).

Cross-Correlation for Periodic Signals

Sco X-1 not random emitter: low-mass X-ray binary neutron star in binary orbit w/companion. GW signal from rotating neutron star:

$$h(t) = h_0 \left[\frac{1 + \cos^2 \iota}{2} \cos(2\pi f_0 t) + \frac{1 - \cos^2 \iota}{2} \cos(4\pi f_0 t) \right]$$

ι : inclination of NS spin
 $\phi(t)$: phase evolution in rest frame
 ν : Doppler mod from detector motion (binary orbit)
 Include features of signal in cross-corr method:
 • Long term coherence
 • Can cross-correlate data from different times
 • Doppler shift @ detector
 • Correlations peaked @ different times
 Note signal cross-correlation determinants:

$$\langle S_1(t) S_2(t') \rangle = \langle h_1(t) h_2(t') \rangle$$

$$= h_1 h_2 \int_{t_0}^{t_0 + T_{\text{dur}}} dt \int_{t_0}^{t_0 + T_{\text{dur}}} dt' \cos(2\pi f_0 t) \cos(2\pi f_0 t')$$

$\langle S_1(t) S_2(t') \rangle = 0$ if $|t - t'| > T_{\text{dur}}$

Parameter Dependence

neutron star

Figure 1: Illustration of the inclination and polarization angles relative to the neutron star spin axis to celestial coordinates defined by the Earth's rotation axis. These are amplitude parameters which do not have a large impact on the number of required templates.

Two kinds of parameters:

- Amplitude params: h_0 , inclination ι , polarization ν , initial phase ϕ_0
- Phase params: f_0 , spin-down, binary orbital parameters

Amplitude params don't pose challenges for filtering: h_0 is overall amplitude; ν drops out of cross-corr; can average ϕ_0 over $\sim 10^4$ for simplicity. Mismatch in phase params leads to cancellation in optimal statistic; need to search over them. Long coherent integration time can give unmanageable # of templates. Limiting allowed pairs $\sum_{i,j} \nu_{ij}$ by e.g. max time difference produces semi-coherent search w/manageable compute time.

Theoretical Sensitivity

Amplitude sensitivity of combined statistic:

$$h_{\text{min}}^2 \propto \frac{1}{\sum_{i,j} \nu_{ij}} \sqrt{\frac{1}{N_{\text{pairs}}}}$$

- If all pairs included, $N_{\text{pairs}} \propto N_{\text{am}}^2$
- Coherent search: $N_{\text{pairs}} \propto N_{\text{am}} N_{\text{ph}}$
- Only simultaneous pairs: $N_{\text{pairs}} \propto N_{\text{am}}$
- $N_{\text{pairs}} \propto N_{\text{am}}^{1.5}$ if $\nu_{ij} \propto \nu_{\text{max}}^2$
- If only pairs separated by T_{dur} or less, $N_{\text{pairs}} \propto N_{\text{am}} T_{\text{dur}}$

Figure 2: Geometrical factor $\langle S_1(t) S_2(t') \rangle$ appearing in the cross-correlation sensitivity, averaged over ν_{ij} and sidereal time, as a function of declination. The sky positions of the supernova 1987A remnant and Scorpius X-1 are shown for reference.

Application: Supernova 1987A Remnant

SN1987A remnant likely contains young neutron star, rapidly spinning down. Can search for GW w/cross-correlation method. Need to search over frequency and spin-downs; rather than searching f_0, \dot{f}_0, ν_{ij} , use phase model w/GW spin-down ν_{ij} ; EM spin-down \dot{f}_0 .

Search over $f_0, \dot{f}_0, \nu_{ij}, \phi_{\text{max}}$. Can ballpark sensitivity using initial LIGO design & assuming only simultaneous LIGO and LIGO data are used. Compares favorably to indirect age-based limit $h_0 < 1.1 \times 10^{-21}$.

Figure 3: Theoretical sensitivity to SN1987A remnant for 1 year simultaneous initial LIGO design data. 5% false alarm & 5% miss.

References

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