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Common mode rejection of laser frequency noise in the TNI

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1 Introduction

We will be using the wavelength of laser light as a yardstick to measure the length of a cavity, and any noise in this wavelength is going to look just like length noise in the cavity. We plan to use a common mode rejection scheme to separate noise in the test cavities from external sources, which include laser frequency noise. Whatever noise the cavities have in common ought to be due to some external cause, such as laser frequency, and the uncorrelated noise should be mostly thermal.

The question I want to address here is this, "What needs to be matched between the two test cavities, and how closely do they need to be matched?" Here, I will consider only laser frequency noise, saving the issue of seismic noise for later.

2 Quantitative model

From "Notes on the Pound-Drever-Hall technique," the error signal including both cavity and laser noise is

$$\varepsilon = -\frac{16\mathcal{F}L}{\lambda} J_0(\beta) J_1(\beta) P \left[\frac{\delta L}{L} + \frac{\delta f}{f}\right],$$

where $\mathcal{F}$ is the cavity finesse, L is the cavity length, λ is the wavelength of the laser light, β is the modulation depth, and P is the total laser power going into the cavity. The differential mean-squared error signal between two cavities is then, after some algebra,

$$\langle \Delta\varepsilon^2 \rangle \approx \left[\frac{16\mathcal{F}L}{\lambda} J_0(\beta) J_1(\beta) P\right]^2 \left\{ 2\frac{\langle \delta L^2 \rangle}{L^2} + \left(\frac{\Delta P}{P} + \frac{\Delta \mathcal{F}}{\mathcal{F}} + \frac{\Delta L}{L}\right)^2 \frac{\langle \delta f^2 \rangle}{f^2} \right\},$$

where ΔP is the difference in the power going into each cavity, $\Delta\mathcal{F}$ is the difference in cavity finesses, and ΔL is the difference in lengths. I have assumed that the cavities are nearly matched, so that $P \equiv P_1 \approx P_2$, *etc.* I have also assumed that the length noise in each cavity is uncorrelated with the length noise in the other cavity and with the laser frequency noise. This assumption may not be strictly true: There could be radiation pressure effects that couple the length of the cavity to the laser frequency. I have not yet done a calculation to estimate this effect, but I will ignore it for now, keeping the warning in mind.

We want ΔP, $\Delta\mathcal{F}$, and ΔL all to be small, so that laser frequency fluctuations get supressed in the differential measurement. Getting $\Delta P/P$ down to a few percent seems easy enough with a good $50/50$ beamsplitter. We are limited in $\Delta\mathcal{F}/\mathcal{F}$ to whatever REO gives us, however, and they expect that to be between 1% and 5%. If $\Delta\mathcal{F}/\mathcal{F} \sim 5\%$, there is not much to be gained from getting the cavity lengths matched to much better than 5%.

If the total mismatch $\Delta P/P + \Delta\mathcal{F}/\mathcal{F} + \Delta L/L \sim 10\%$, then we would get a common mode rejection ratio (rms) of about

$$\frac{\sqrt{2}}{\frac{\Delta P}{P} + \frac{\Delta\mathcal{F}}{\mathcal{F}} + \frac{\Delta L}{L}} \approx 14.$$

If $\delta L/L \sim \delta f/f$, then about 90% of the rms differential error signal will be due to length noise.

3 Mismatch budget

It seems reasonable, since we expect to get $\delta f/f$ down to something comparable to $\delta L/L$, that our initial mismatch budget should be

$$\Delta P/P + \Delta\mathcal{F}/\mathcal{F} + \Delta L/L \leq 10\%.$$

This will be entirely adequate for rejecting laser frequency noise.

The requirements for rejecting seismic noise should not affect these conclusions, since rejection of seismic noise involves matching different quantities.