<u>Calibration of Optical Levers</u>

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Introduction

This is a Maple notebook about optical levers and how to calibrate and interpret their outputs. An optical lever (Fig. 1) shines a laser beam on a mirror whose angle we want to measure, and detects the transverse position of that beam's reflection some distance L away. This is done by directing the reflection onto a quadrant photodiode (QPD). The four segments of the QPD give four photocurrent signals, each of which is proportional to the amount of light falling in that particular quadrant. If the beam is precisely centered, all four give the same photocurrent; differences between the quadrant photocurrents tell us how far the beam is from the QPD center and in which direction, from which we can compute how the mirror has rotated.

[Insert Fig. 1 about here]

We are usually interested in the deviation of the optic's reflection along vertical and horizontal planes ("pitch" and "yaw"). Generally the quadrant diode is arranged in a "+" orientation to do this, meaning the divisions between the quadrants are vertical and horizontal. We can label the quadrants as follows:

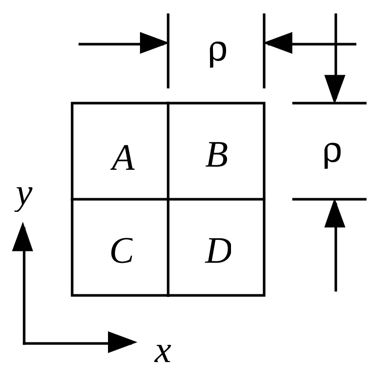


Fig. 2: Square quadrant photodiode nomenclature

The photocurrent developed in each quadrant will be proportional to the amount of optical power falling on that quadrant times the responsivity R of the diode. R has units of Ampere/Watt (A/W), and generally depends on the wavelength of the light. We'll assume that R is the same for all four quadrants (in reality, it can vary by 5% or more from quadrant to quadrant on a given diode). Each segment will feed a transimpedance amplifier which turns its photocurrent into a readout voltage; if the transimpedance used in these amplifiers is Z (in Ohms), the voltage out of each amplifier will be

$A = RZP_A$, $B = RZP_B$, $C = RZP_C$ and $D = RZP_D$

where P_A is the beam power falling within segment A, etc.

To determine the beam's displacements x and y from the center of the diode, we add and subtract the voltages from segments A through D to form new voltages X and Y as follows;

$X = (B+D) - (A+C) = (right - left)$

$Y = (A+B) - (C+D) = (top - bottom)$

We also add up the total of all the segment voltages,

$S = A + B + C + D.$

This is necessary because the laser power, diode responsivity or mirror reflectivity may change over time or from test to test. Some readout systems will also calculate the quantities

$$\Xi = \frac{X}{S}$$

and

$$\Psi = \frac{Y}{S}$$

for us, using analog divider circuits or in software. These quantities are independent of the total power, but when setting up they can be tricky, since they are nonsense when the beam falls off the diode or is blocked (see below). Some readouts give the S channel a different scale factor than X and Y readouts.

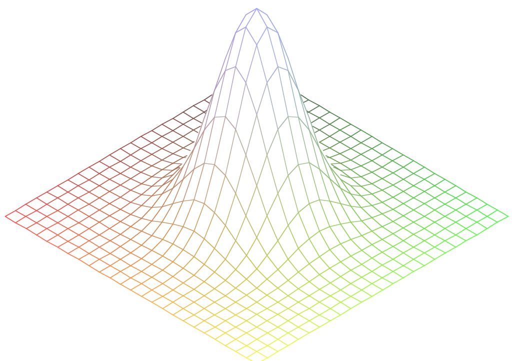
For simplicity we can assume that the laser beam hitting the diode has roughly a Gaussian intensity profile. This is how the laser beam usually starts out. By the time it hits the QPD it may have passed through distorted windows, clipped on edges, or suffered other insults so it may not end up a perfect Gaussian. Nevertheless, optical levers work fine as long as the beam falls mostly on the QPD and has only one peak in its intensity profile. The Gaussian profile is described by the beam radius ω, which is the radius where the intensity is
$\frac{1}{e^2} \sim 13.6\%$ of the peak intensity. Usually the laser is focussed to give a specific value of ω at the point where the beam hits the QPD. In the picture below, the intensity of a Gaussian beam is represented by the height of the surface; the square plot area is 3 mm x 3 mm in size and the beam radius ω is 1 mm.

Plot of Gaussian beam intensity profile:

```
> with(plots):
> omega := 'omega' : # about(omega); # waist parameter
> assume( omega > 0 ): # about(omega); # needed to expand
  expressions unambiguously
> E := (1/(sqrt(Pi*omega^2)))*exp(-(x^2+y^2)/(2*omega^2)): #
  electric field
> plot3d(subs(omega=1, E*E), x=-3..3, y=-3..3, title=`Gaussian beam
  intensity profile`);
>
```

```
> #nam := int(int(E*E, x=-infinity..infinity),
  y=-infinity..infinity); # normalization check
>
>
```

Gaussian beam intensity profile



```
>
```

Optical lever output signal properties

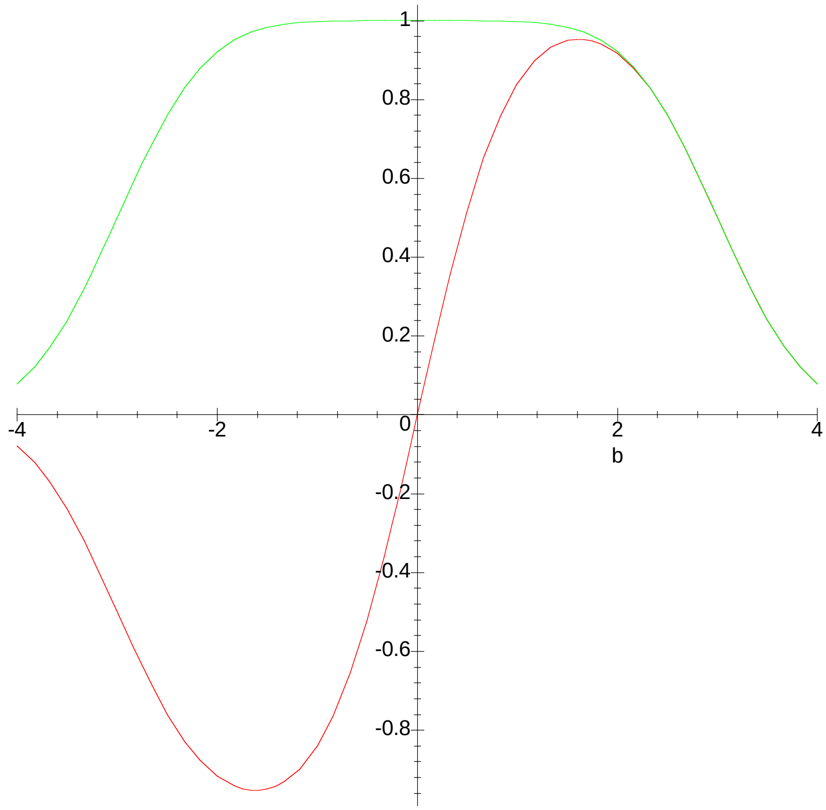
The formula for *X* above subtracts all the power falling on the right side of the QPD from all the power falling on the left side. (Similarly, the formula for *Y* subtracts bottom from top). If the beam is far from center, some of the light falls outside the edges of the diode and doesn't get detected by any segment. In the plot below, the lower curve is the *X* output voltage and the upper curve is the *S* output voltage as the beam moves across the diode in the *x* direction. Note that when the beam goes very far off center either way, *X* looks just the same as if it was perfectly centered (i.e., it goes to zero), but the *S* channel also vanishes at the same time showing that the beam has in fact left the QPD.

For the example plotted below, the horizontal axis reads in mm, we have chosen a beam radius $\omega = 1$ mm ($\frac{1}{e^2}$ diameter of 2 mm), and the QPD size is 6 mm wide from edge to edge.

```
> # diffedge := int(int(E*E, x=-infinity..infinity), y=-infinity..b)
  - int(int(E*E, x=-infinity..infinity), y=b..infinity):# difference
  between infinite half-planes
> # diffedgeplot := subs(omega=1, diffedge):
> diffedgefinite := int(int(E*E, x=-infinity..infinity), y=b-rho..b)
  - int(int(E*E, x=-infinity..infinity), y=b..b+rho): # difference
  between halves of size rho
> sumfinite := int(int(E*E, x=-infinity..infinity), y=b-rho..b+rho):
  # sum of both halves
> diffedgefiniteplot := subs(omega=1, rho=3, diffedgefinite):
> sumfiniteplot := subs(omega=1, rho=3, sumfinite):
> plot({sumfiniteplot, diffedgefiniteplot}, b = -4 .. 4,
  title=`Position and Sum Signals vs. Beam Displacement`);
>
```

>

Position and Sum Signals vs. Beam Displacement



>

Sensitivity of the optical lever

By *sensitivity* we mean how much the X or Y output voltage changes for a given physical movement x or y of the beam from the center of the QPD. This is the slope of the lower curve in the plot above. The sensitivity depends on the beam radius (i.e., the focus setting of the laser's output collimator) and the beam power (i.e., laser output power, reflectivity of the suspended mirror, and transmission of optics or windows in between them). If the polarization of the laser changes with time or temperature, the reflectivity of the suspended mirror will vary with the polarization, also affecting the power falling on the QPD.

By *null* we mean the point where X and Y outputs are close to zero. Initially we establish a reference for the mirror angle by moving a steering mirror just in front of the QPD to bring both outputs near null. The calibration is only constant for x and y within about $\pm\,\omega$ of null, where the output varies almost linearly with motion. As the plot above shows, beyond this range the relationship between beam displacement and output voltage is not simple.

For small beam motion x and y within this range, the output voltage will be

$$X = \frac{2\,RZP\,x}{\omega\sqrt{\pi}}$$

and

$$Y = \frac{2\, RZP\, y}{\omega \sqrt{\pi}}$$

where P is the total beam power (in Watts) , R is the diode responsivity (in Amperes/Watt), Z is the amplifier transimpedance (in Ohms), and ω is the Gaussian beam radius (in the same units as the displacement x and y, e.g., mm).

Note that whenever the beam is on the QPD (not necessarily exactly centered), the S (sum) voltage reads out the product RZP. So, if we can measure S_0 (sum voltage with the beam somewhere on the QPD) and we know just the beam radius ω, we can write

$$\frac{\delta X}{\delta x} = \frac{2\, S_0}{\omega \sqrt{\pi}} \sim 1.128 \frac{S_0}{\omega} \text{ V/mm}$$

and similarly for Y. This is useful for field setup since one doesn't have to measure the beam power separately.

To determine angular motion of the suspended mirror, we also need the optical path distance L from the mirror to the QPD (the distance from the laser to the mirror doesn't matter). The angular motion of the reflected *beam* is twice the angular motion of the suspended *mirror*, that is,

$$x = 2\, L\, \Theta_M \text{ and } y = 2\, L\, \Phi_M$$

where Θ_M and Φ_M are the horizontal and vertical mirror rotations in radians. Then the angular sensitivity (output voltage per unit mirror rotation) can be written as

$$\frac{\delta X}{\delta \Theta_M} = \frac{\delta Y}{\delta \Phi_M} = \frac{4\, L\, S_0}{\omega \sqrt{\pi}} \sim 2.257 \frac{L\, S_0}{\omega} \text{ V/rad.}$$

(Here, be careful to express L and ω in the same units!).

Example calculation of sensitivity

As an example, if we know $\omega = 1.5$ mm and measure $S_0 = 2.5$ V, and the optic is $L = 4.5$ m = 4500 mm away from the QPD, then

$$\frac{\delta X}{\delta x} = 1.88 \text{ V/mm}$$

and

$$\frac{\delta X}{\delta \Theta_M} = 16{,}928 \text{ V/rad} = 16.9 \text{ mV/}\mu\text{rad.}$$

Calibration of an optical lever using a tilted parallel glass plate

In a field installation the beam radius ω may not be known in advance. The beam may also have been distorted by windows or aperture stops along the optical path. If the beam isn't circular, the effective radius may even be different for x and y directions. As a result it is often required to calibrate the optical lever in place.

The most straightforward way to calibrate is just to move the QPD laterally a known amount and monitor the change in voltage. This is easy to do on a lab bench, but since the QPD is usually hard-mounted in the field it is hard to do without changing the mechanical setup significantly. An easier method, which does not involve disturbing the mechanical system, is to place a tilted flat glass plate of known thickness t in front of the QPD to displace the beam a known amount. We need to insert the plate twice; with its normal in the X plane to calibrate X, and with its normal in the Y plane to calibrate Y. Note that the plate must be uniformly thick, i.e. both surfaces parallel, to avoid unwanted angular deflection.

[insert Fig. 3 about here]

If the tilt angle of the plate is θ and the refractive index of the glass is n, then the lateral displacement of the beam is given by

$$d = \frac{t \sin\left(\theta - \arcsin\left(\frac{\sin(\theta)}{n}\right)\right)}{\sqrt{1 - \frac{\sin(\theta)^2}{n^2}}} .$$

Once we decide on an angle to use and refractive index and thickness for the window, we only need to calculate this once.

Deflection by a plate set at 45°

Usually it is convenient to set the plate at 45 ° to the beam, and the material used is almost always BK7 glass (n = 1.52) or fused silica or quartz (n = 1.46). In this case the deflection is

$$d = .336\, t$$

for BK7 glass, and

$$d = .316\, t$$

for fused silica. (If it isn't clear which material your plate is made of, the difference is less than 10%, so for most field work a good guess will be good enough).

```
> thetaprime:=arcsin(sin(theta)/n) :
> l := t/cos(thetaprime):
> d := l*sin(theta - thetaprime):
> evalf(subs(theta=Pi/4, n=1.46, d)); # fused silica
```

$$.3156686258\, t$$

```
> evalf(subs(theta=Pi/4, n=1.52, d)); # BK7 glass
```

$$.3355007672\, t$$

Correction for optical loss of calibration plate

Generally the calibration plate will also reflect some light off its surfaces, attenuating the beam. This loss depends on the angle, material and polarization. For accurate work, the effect of this complication can be removed from the result by also noting the change in the S (sum) signal when the plate is inserted.

If the S, X and Y signals are recorded before the plate is inserted (call these baseline values S_0, X_0 and Y_0 respectively), we can take the measured values S_X and X after the plate is inserted in the X direction, and S_Y and Y after the plate is inserted in the Y direction, and rescale them;

$$X_1 = \frac{X\,S_0}{S_X} \quad \text{and} \quad Y_1 = \frac{Y\,S_0}{S_Y}\,.$$

The corrected quantities X_1 and Y_1 represent what the outputs *would have been* if the beam power hadn't diminished. (Note that they revert to just the measured X and Y voltages if the calibration plate is perfectly transparent, such that $S_X = S_Y = S_0$.)

The X sensitivity is then found to be

$$\frac{\delta X}{\delta x} = \frac{X_1 - X_0}{d} = \frac{X\frac{S_0}{S_X} - X_0}{d}$$

with X and S_X measured with the plate tilted in the X direction, X_0 and S_0 being the original values without the plate.

Similarly,

$$\frac{\delta Y}{\delta y} = \frac{Y_1 - Y_0}{d} = \frac{Y\frac{S_0}{S_Y} - Y_0}{d}$$

where Y and S_Y are now measured with the plate tilted in the Y direction.

Since the laser beam is usually polarized, the loss due to the plate will often be different for the X and Y orientations. The sum signal S therefore has to be recorded separately for *each* orientation of the plate.

Example: calibration with a BK7 glass plate at 45°

We start out by measuring the thickness of the calibration plate with calipers; let's say we find

t = 1.0 mm.

From the expression for the beam displacement we get

d = 0.336 t = 0.336 mm

Next we turn on the optical lever laser and QPD power and check that both X and Y readouts are near null, and that S has a value indicating the beam is on the QPD. Suppose we record

$X_0 = .02\ V$,
$Y_0 = -.04\ V$, and
$S_0 = 2.7\ V$.

We then insert the plate tilted 45° in the x direction and measure

$X = .35\ V$ and
$S_X = 2.4\ V$.

Rotating the plate to the y direction, we next measure

$Y = -.14\ V$
$S_Y = 1.3\ V$.

(At this point it's good practice to remove the plate and double check that we recover the same X_0, Y_0 and S_0 values).

We can now calculate

$$\frac{\delta X}{\delta x} = \frac{X \frac{S_0}{S_X} - X_0}{d} = \frac{.35 \frac{2.7}{2.4} - .02}{.336} = 1.1 \text{ V/mm}$$

and

$$\frac{\delta Y}{\delta y} = \frac{Y \frac{S_0}{S_Y} - Y_0}{d} = \frac{-.14 \frac{2.7}{1.3} - (-.04)}{.336} = -\ 0.75 \text{ V/mm} .$$

In this example the measured X and Y sensitivities have slightly different magnitudes, even though we corrected for the higher attenuation of the test plate in the Y orientation. This might indicate that the beam hitting the QPD is not circularly symmetric, having a smaller effective radius ω along the X direction. The sign of the measured sensitivity depends on which direction the plate was oriented. To avoid confusion, determine and use the sign convention conforming to the standard "pitch" and "yaw" definitions for the optic being monitored.

```
> evalf((.35*(2.7/2.4)-.02)/.336);
> evalf((-0.14*(2.7/1.3)-(-0.04))/0.336);
```

1.112351190

-.7463369964

Susceptibility of plate calibration to incidence angle errors

If the calibration plate is not at precisely the angle we think it is to the beam, there will be an error in our assumed displacement d . This error works out to about 4% per degree the plate angle deviates from the nominal 45°. In a critical application it might be be worthwhile to measure the direction of the surface reflection coming off the plate, to verify this angle.

```
> dprime := diff(d, theta):
> evalf(subs(theta=Pi/4, n=1.52, (1/45)*dprime/d));
>
```

.04003805164

Use of normalized readouts Ξ and Ψ

Sometimes normalized outputs are useful. For example, the laser power or reflection coefficient of the optic (which depends on polarization) may change significantly over time, in which case interpreting trends in the *X* and *Y* voltage outputs will be tricky. The quantities

$$\Xi = \frac{X}{S}$$

and

$$\Psi = \frac{Y}{S}$$

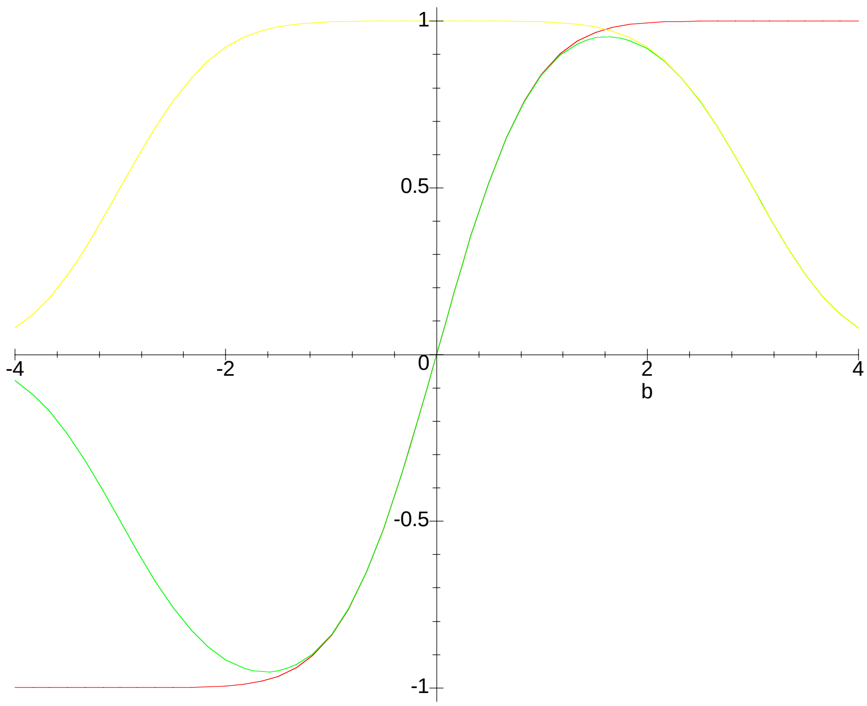
are independent of the received power on the diode and provide a better representation of the beam position if the power is variable. Sometimes these signals will be scaled by some factor which depends on the electronics. For example, in analog divider implementations it is customary to scale the quotient by 10 volts, i.e.

Ξ_{analog} = 10V * X/S and Ψ_{analog} = 10V * Y/S .

In what follows we'll assume the normalization is done digitally, so the normalized outputs Ξ and Ψ are pure numbers between -1 and +1 read from a data file or computer display. The following plot shows Ξ along with the *X* and *S* signals plotted against beam position *x*, for the case $S_0 = 1\ V$, $\omega = 1$ mm, and $\rho = 3$ mm (the vertical scale is ±1V for *X* and *S*, and ±1 for Ξ ; the horizontal scale is in mm).

```
> plot({diffedgefiniteplot/(sumfiniteplot), sumfiniteplot,
  diffedgefiniteplot }, b = -4 .. 4, title=`Sum, Position and
  Normalized Position vs. Beam Displacement`);
>
>
```

Sum, Position and Normalized Position vs. Beam Displacement



>

Sensitivity and Calibration of Normalized Readouts

Substituting the definitions for Ξ and Ψ into the expressions for predicted X and Y sensitivities above gives

$$\frac{\delta \Xi}{\delta x} = \frac{2}{\omega \sqrt{\pi}} = \frac{\delta \Psi}{\delta y}$$

which depend only on the beam radius ω . Similarly the angular sensitivities are

$$\frac{\delta \Xi}{\delta \Theta} = \frac{2 L}{\omega \sqrt{\pi}} = \frac{\delta \Psi}{\delta \Phi}$$

Since variations in the total power on the QPD are already divided out, calibration of Ξ and Ψ with a tilted glass plate does not require compensating for the plate's attenuation as did the raw position outputs. If the Ξ (Ψ) output with the plate inserted in the x (y) direction is called Ξ_x (Ψ_y), and the output with no plate is called Ξ_0 (Ψ_0), then the measured sensitivity is just

$$\frac{\delta \Xi}{\delta x} = \frac{\Xi_x - \Xi_0}{d}$$

and

$$\frac{\delta\Psi}{\delta y} = \frac{\Psi_y - \Psi_0}{d} .$$

>
>

Example: calibration of normalized outputs with tilted glass plate

As in the last example, we measure the thickness of our BK7 glass plate t = 1.0 mm, giving us a beam displacement d = 0.336 mm at 45°. Starting out with the beam roughly centered on the QPD (and checking that the sum S shows adequate power on the QPD) we record

$\Xi_0 = .015,\ \Psi_0 = -.003$

We insert the plate tilted in the x plane and measure

$\Xi_x = -.134$.

Then we rotate the plate to the y plane and find

$\Psi_y = .158$.

The measured sensitivities are then

$$\frac{\delta\Xi}{\delta x} = \frac{-.134 - .015}{.336} = -.443 / \mathrm{mm}$$

and

$$\frac{\delta\Psi}{\delta y} = \frac{.178 - (-.003)}{.336} = .479 / \mathrm{mm}$$

Again the x and y magnitudes are close but not identical, perhaps indicating the beam shape is not circularly symmetric.

The angular sensitivities are found by multiplying these linear sensitivities by twice the lever arm, as before; so if L = 4.5 m = 4,500 mm,

$$\frac{\delta\Xi}{\delta\Theta_M} = 2\,L\,\frac{\delta\Xi}{\delta x} = 9{,}000\ \mathrm{mm} * (-.443/\mathrm{mm}) = -3{,}991\ \mathrm{rad} = -3.99/\mathrm{mrad}$$

and

$$\frac{\delta\Psi}{\delta\Phi_M} = 2\,L\,\frac{\delta\Psi}{\delta y} = 9{,}000\ \mathrm{mm} * (.479/\mathrm{mm}) = 4{,}312/\ \mathrm{rad} = 4.312/\mathrm{mrad} .$$

```
> evalf(9000*(-.134-.015)/.336);
> evalf(9000*(.158-(-.003))/.336);
```

-3991.071429

4312.500000

>

Appendix A: exact expression for output of a square QPD vs. beam position

For a square diode of width 2ρ (i.e., each segment is ρ x ρ in size) the exact expression for the *X* output is

$$X = \left[\frac{2\, RZP\, x}{\omega \sqrt{\pi}} \right] \left(1 - \mathbf{e}^{\left(-\frac{\rho^2}{\omega^2} \right)} \right) .$$

This differs from the approximate expression used above by less than 2% as long as $2\,\omega < \rho$, i.e., whenever the full $\frac{1}{e^2}$ beam diameter fits within one segment of the QPD.

```
>
> diffedgeslope := diff(diffedgefinite, b):
> eval(subs(b=0,  diffedgeslope));
> evalf(subs(b=0, omega=1., rho=2, diffedgeslope));
> evalf(subs(b=0, omega=1, rho=20, diffedgeslope) - subs(b=0,
  omega=1, rho=2, diffedgeslope));
```

$$\frac{2}{\sqrt{\pi}\,\omega} - 2\,\frac{\mathbf{e}^{\left(-\frac{\rho^2}{\omega^2} \right)}}{\sqrt{\pi}\,\omega}$$

with assumptions on ω

1.107712182

.02066698536

Appendix B: Correcting for dark offsets and ambient light

When working with small beam power on the QPD, DC offsets in the preamplifiers or ambient light shining on the detector can cause significant errors. To test for this, measure the QPD outputs under normal conditions and then block the laser beam (preferably at the source) or turn off the laser. Ideally the "dark" quadrant output magnitudes *A*, *B*, *C*, and *D* should be no more than 1/20 their values with the beam on. If these aren't available separately, *S* should satisfy a similar criterion; however it is possible (not likely, but possible) for positive and negative quadrant offsets to cancel out in the sum *S*, giving a false sense of security.

If "dark" offsets are too large for the required measurement accuracy, but are constant, we can measure them once and subtract them from the raw measurements, forming

$$X_{corr} = X - X_{dark}$$

and

$$Y_{corr} = Y - Y_{dark} \, .$$

Since the calibration is based on subtracting values before and after inserting a test plate, offsets only affect the *null* and will not affect the measured *sensitivity*.

The absolute position of the diode center is usually not an issue, so small offsets can be ignored as long as they don't change. However, if offsets change over time they will introduce errors in returning an optic to the same orientation sometime in the future. This can happen if the electronics age or drift with ambient temperature, or if ambient light finding its way onto the QPD changes.

On the other hand, if the laser beam power on the QPD varies substantially, even a fixed voltage offset will have a varying fractional effect. For example, a 50 mV dark offset might have a negligible effect when the beam power gives a sum voltage of 5 V, but if the laser power or reflectance diminishes so that S falls to 0.5 V the same fixed offset will introduce a position error of 10% (i.e., 0.3 mm if $\omega = 1.5$ mm). As a result it is a good idea to block the beam and record the quadrant outputs "dark" each time the optical lever is serviced or calibrated. Use the offset-corrected values above whenever the beam power is small, or is liable to diminish significantly in the future.

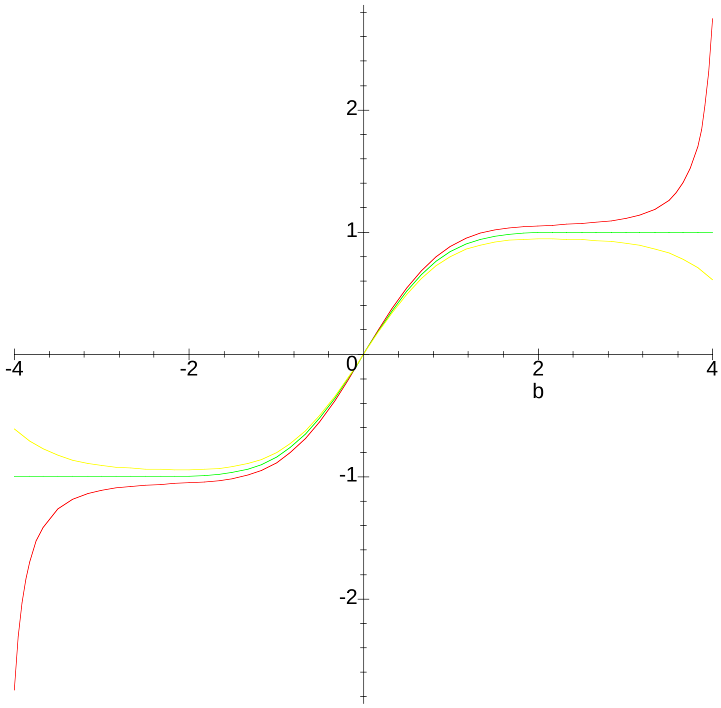
```
>
>
```

Appendix C: Effect of offsets and noise on normalized readouts

As the beam gets near the edges of the QPD, the "denominator" S may become small compared to noise and offset errors in the analog preamplifiers or in the analog-to-digital converter. Such errors can be amplified and lead to misleading results for normalized readouts Ξ and Ψ. For example, the effect of small DC offsets added to the sum S on the Ξ output is illustrated in the following plot. With a negative DC offset, Ξ diverges as the beam leaves the QPD; with a positive DC offset, Ξ falls back toward zero. If the errors in S or the individual quadrant signals take the form of noise, the Ξ output can behave erratically as the beam nears the edge.

```
> plot({diffedgefiniteplot/(sumfiniteplot +.05),
  diffedgefiniteplot/(sumfiniteplot -.05),
  diffedgefiniteplot/sumfiniteplot }, b = -4 .. 4, title=`Normalized
  position output with ± offset errors`);
>
>
```

Normalized position output with ± offset errors



Thus when using normalized outputs, it is still necessary to monitor the sum voltage *S* to insure the total power on the diode is adequate. A good rule of thumb is to see that the magnitude of *S* with the beam on the QPD is at least a factor of 20 larger than the "dark" value, measured with the beam blocked or the laser shut off.

>

>

>

>

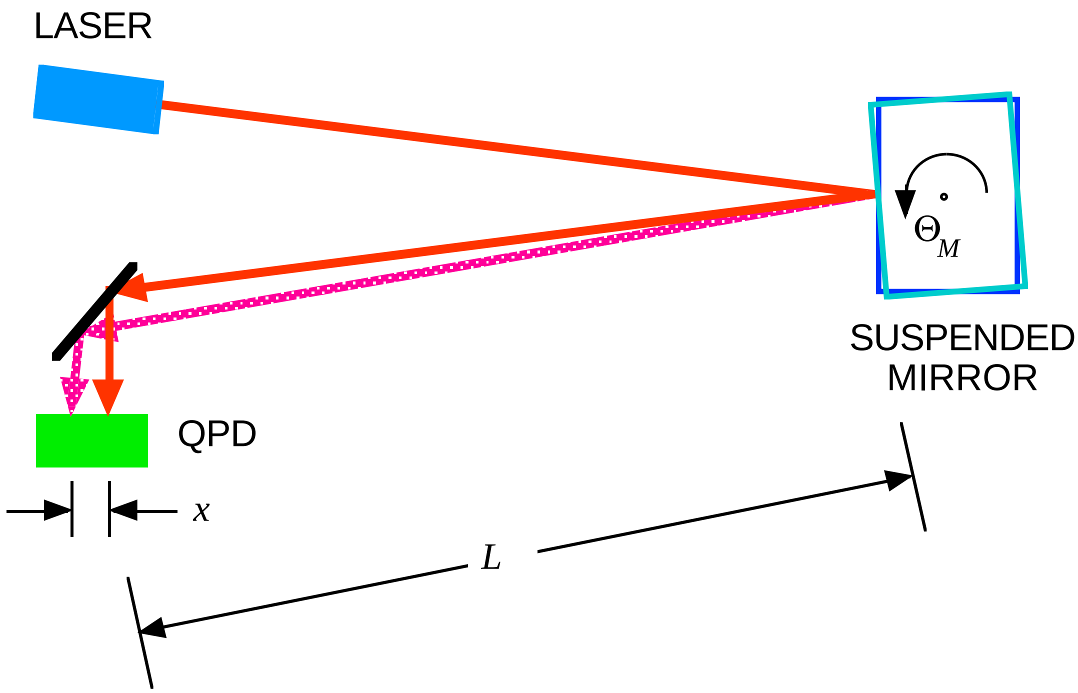


Fig. 1: Optical lever angle readout for a suspended mirror.

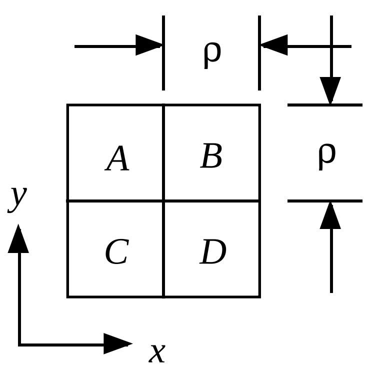


Fig. 2: Square quadrant photodiode nomenclature

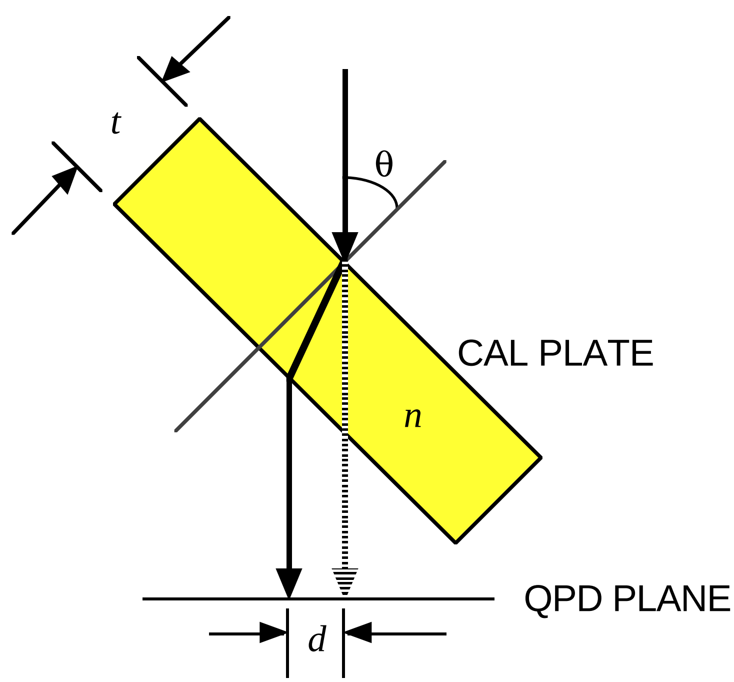


Fig. 3: Beam translation by a flat calibration plate placed at angle θ.