Scattered Light Control in LIGO

2000 ASPEN WINTER CONFERENCE ON GRAVITATIONAL WAVES

Presented by:

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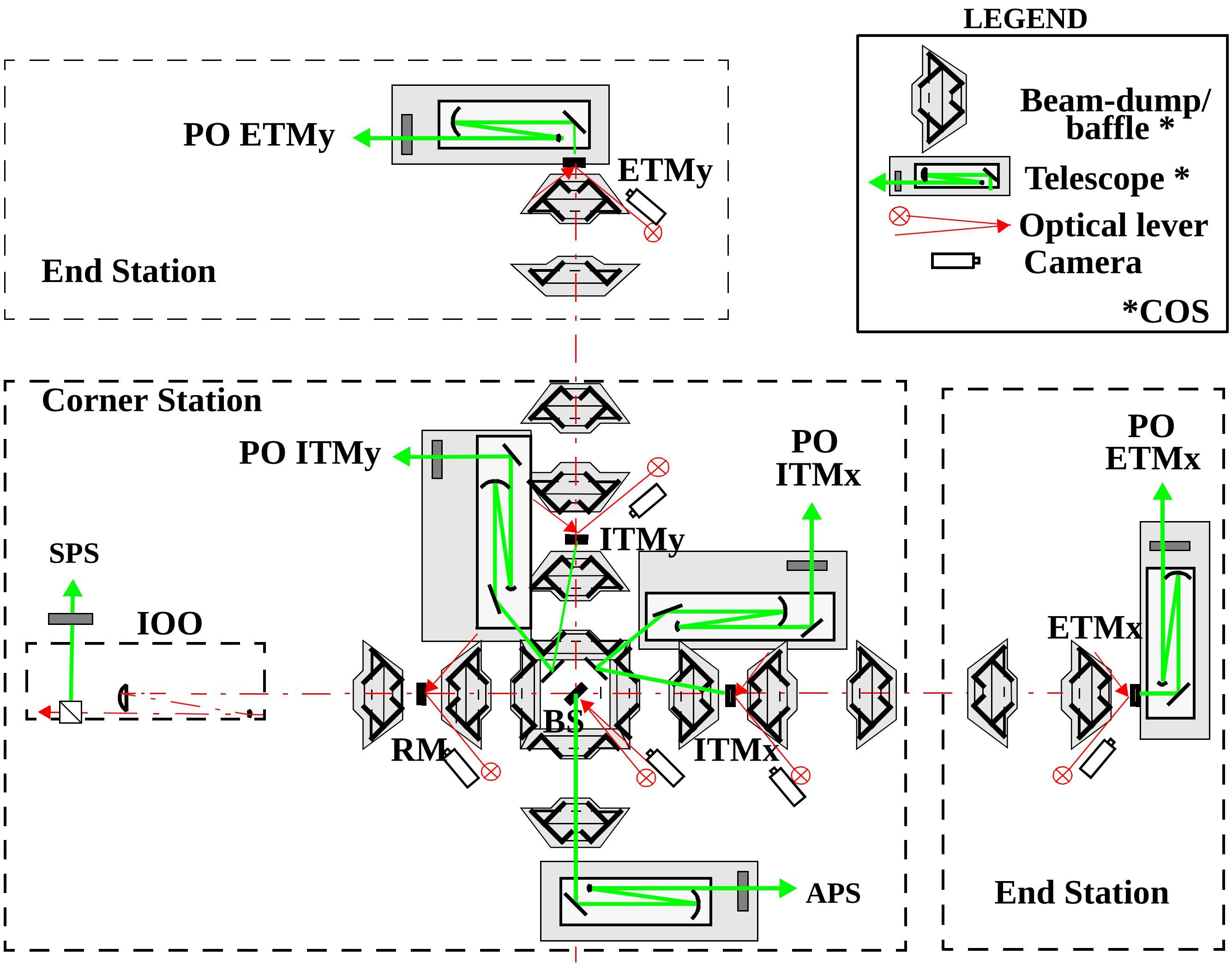
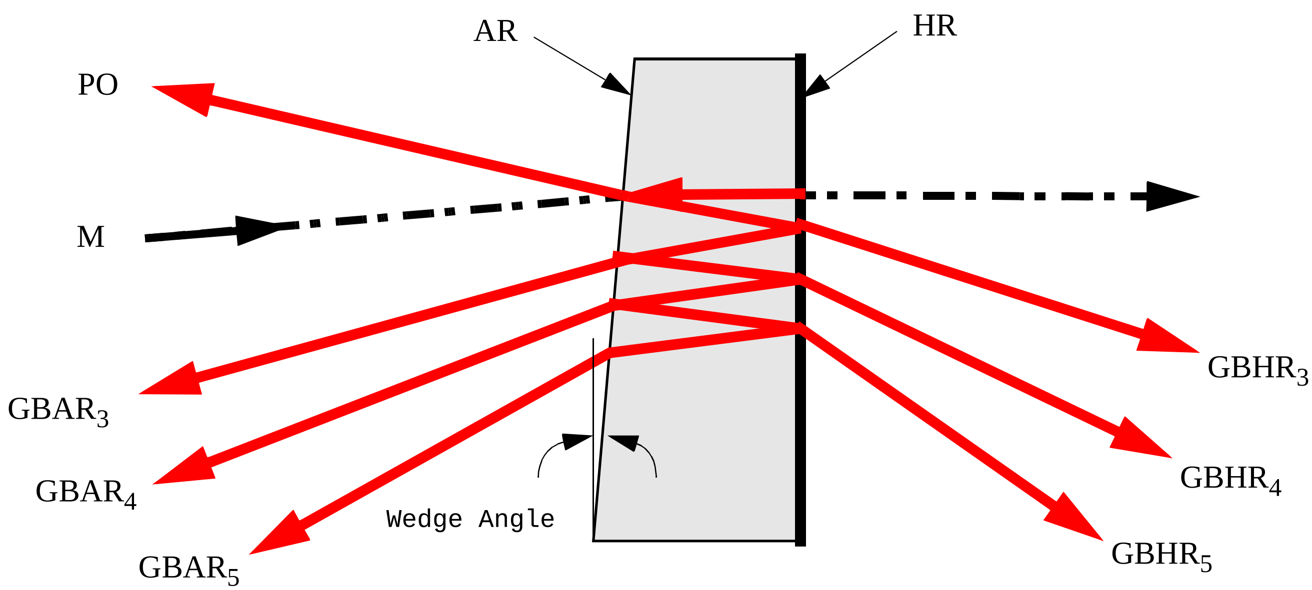
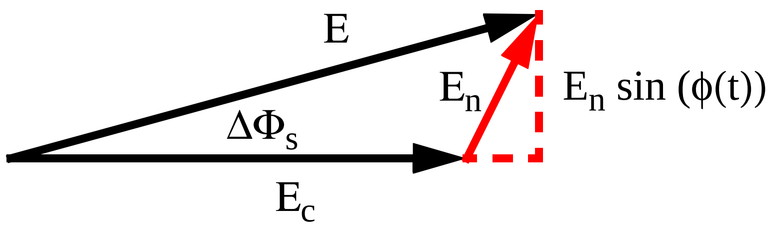
Schematic Layout of COS



Pick-off Beams and Ghost Beams



SCATTERED LIGHT PHASE NOISE



Carrier field phase shift due to scattered light noise in the IFO

$$\Delta\Phi_s = \frac{E_n \sin(\phi(t))}{E_c} = \sqrt{\frac{P_n}{P_c}} \sin(\phi(t))$$

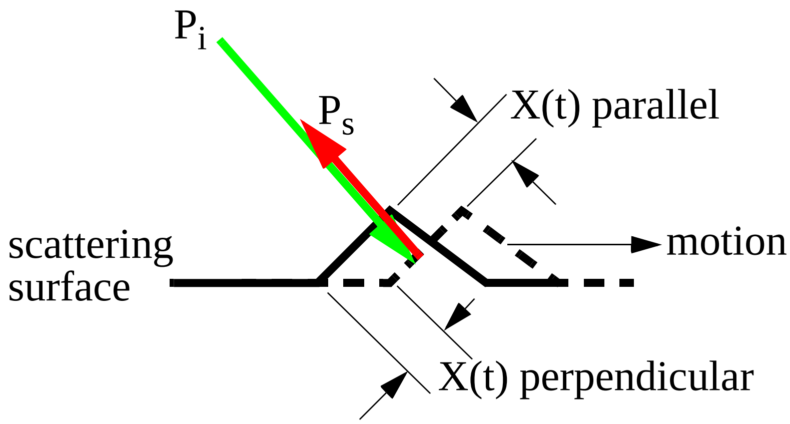
Indistinguishable from change in cavity arm length due to gravity wave

$$\Delta\Phi_s = 2kX(t)$$

Noise contribution to gravity wave strain:

$$\delta h_s(t) = \frac{X(t)}{L} = \frac{\Delta\Phi_s}{2kL} = \frac{\lambda}{4\pi L}\sqrt{\frac{P_n}{P_c}} \sin(\phi(t))$$

Scattered Light Phase Noise Spectral Distribution



Total phase shift caused by constant distance plus time varying scattering surface

$$\phi(t) = \phi_0 + \phi_s(t) = 2kx_0 + 2kx_s(t)$$

for $\phi_s(t) \ll \phi_0$

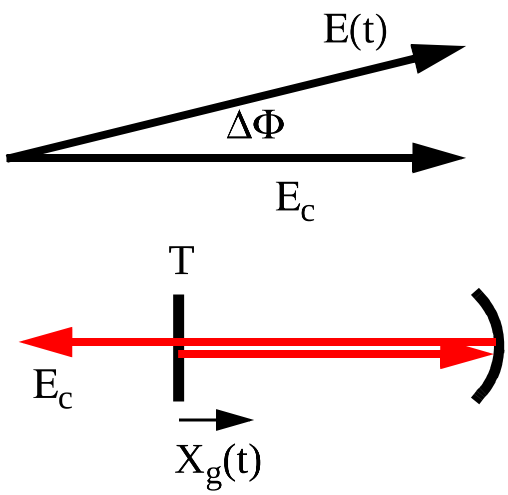
$$\sin(\phi(t)) = \sin\phi_0 + \phi_s(t)\cos\phi_0$$

Spectral power density of the time varying portion

$$\tilde{\sin\phi}(f)^2 = \overline{\cos\phi_0}^2\tilde{\phi_s}(f)^2 = \frac{1}{2}\tilde{\phi_s}(f)^2 = \frac{1}{2}\left(\frac{4\pi}{\lambda}\right)^2\tilde{x_s}(f)^2$$

$$\delta\tilde{h_s}(f) = \frac{\lambda}{4\pi L}\sqrt{\frac{P_n}{P_c}}\sin(\phi(t)) = \sqrt{\frac{P_n}{P_c}}\frac{1}{\sqrt{2}L}\tilde{x_s}(f)$$

Gravity Wave Strain Signal



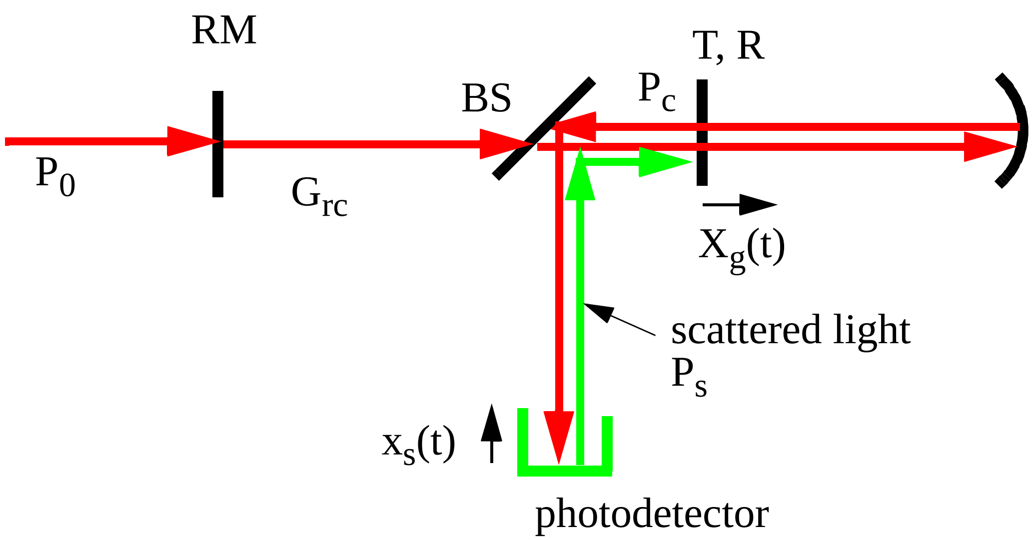
Phase shift, due to motion of arm cavity mirror, is increased by the number of passes through the cavity and reduced by excessive storage time in the cavity

$$\Delta\Phi_g(t) = \frac{4\pi X_g(t)}{\lambda}\frac{2}{T}\frac{1}{\sqrt{1+\left(\frac{f}{f_0}\right)^2}} \quad \text{and} \quad h_g(t) = \frac{X_g(t)}{L} = \frac{\Delta\Phi_g(t)}{2kL} = \frac{2}{TL}\frac{X_g(t)}{\sqrt{1+\left(\frac{f}{f_0}\right)^2}}$$

Gravity wave strain spectral density

$$\tilde{h}_g(f) = \frac{\tilde{X}_g(f)}{L} = \frac{\Delta\tilde{\Phi}_g(f)}{2kL} = \frac{2}{TL}\frac{\tilde{X}_g(f)}{\sqrt{1+\left(\frac{f}{f_0}\right)^2}}$$

APS Scattered Light, Noise/ Signal Ratio



Scattered light from APS (antisymmetric port) photodetector enters IFO, reflects from arm cavity, with reflectivity R, and recombines with carrier

$$P_n = P_s R$$

Scattered light noise/signal ratio

$$\frac{\delta \tilde{h}_s(f)}{\tilde{h}_g(f)} = \sqrt{\frac{RP_s}{P_c}} \frac{T\sqrt{1+\left(\frac{f}{f_0}\right)^2}}{2\sqrt{2}} \frac{\tilde{x}_s(f)}{\tilde{X}_g(f)}$$

K factor

in terms of the input laser power, P_0, and recycling gain, G_{rc},

$$P_c = P_0 G_{rc}$$

$$\frac{\delta \tilde{h}_s(f)}{\tilde{h}_g(f)} = K_{APS}\sqrt{\frac{P_s}{P_0}}$$

where,

$$K_{APS} = \frac{T}{2\sqrt{2}}\sqrt{\frac{R}{G_{rc}}}\sqrt{1+\left(\frac{f}{f_0}\right)^2}\frac{\tilde{x}_s(f)}{\tilde{X}_g(f)}$$

Scattered Light Noise Requirement

The scattered light phase noise must not exceed 1/10 of the LIGO I minimum displacement sensitivity,

$$\tilde{X_g(f)}|_{Req} = 1\times10^{-19}\frac{m}{\sqrt{Hz}}$$

$$\left.\frac{\delta\tilde{h}_s(f)}{\tilde{h}_g(f)}\right]_{Req} = K_{APS}\sqrt{\left.\frac{P_s}{P_0}\right|_{Req}} \le \frac{1}{10}$$

Numerical values:

$R = 0.97$, $P_0 = 6W$, $T = 0.03$, $f_0 = 100Hz$, $G_{rc} = 50$

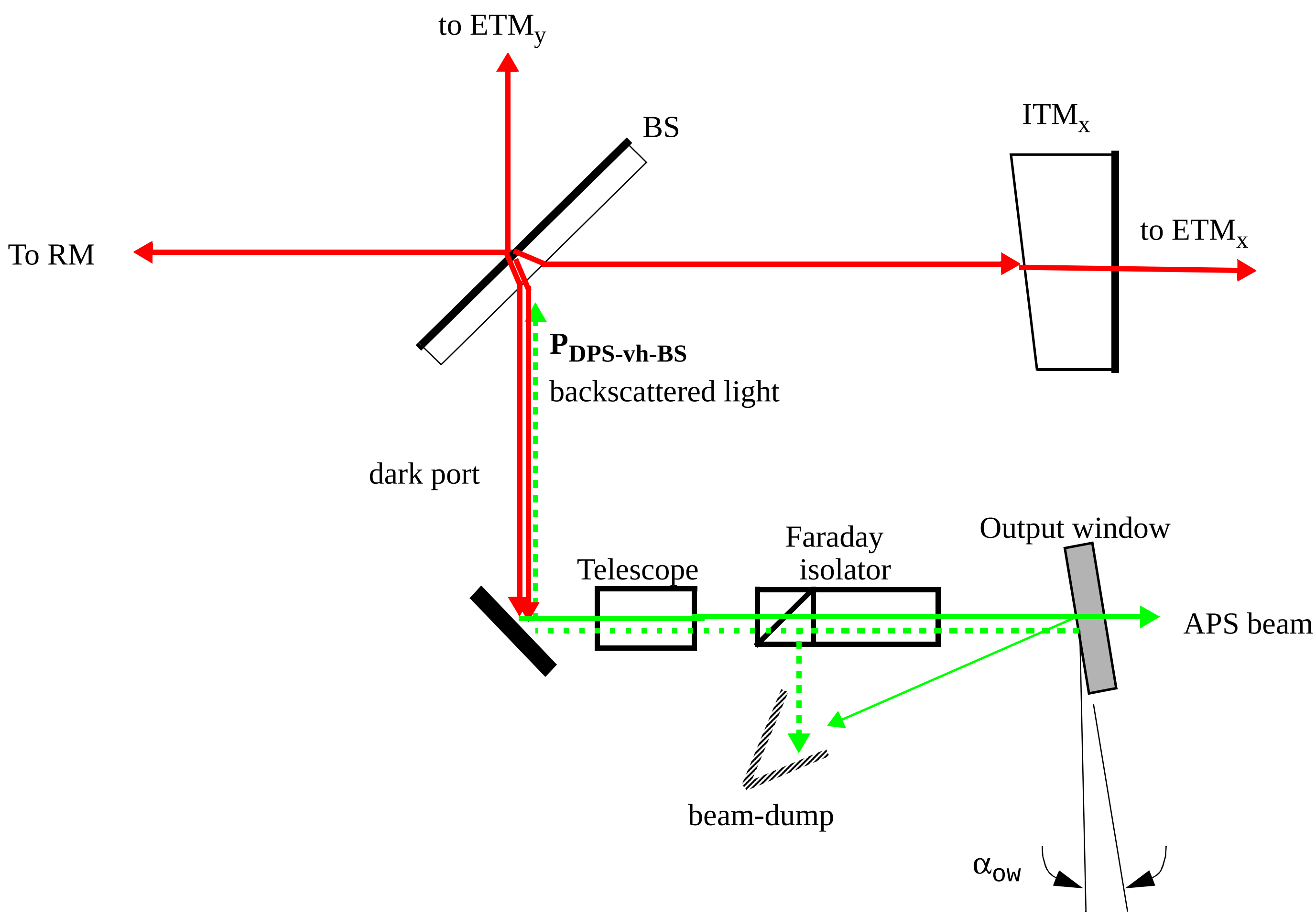
$\tilde{x}_s(f) = 1\times10^{-11}\frac{m}{\sqrt{Hz}}$

$K_{APS} = 2.1\times10^{5}$

$$\left.\frac{P_s}{P_0}\right|_{Req} \le 2.2\times10^{-13}$$

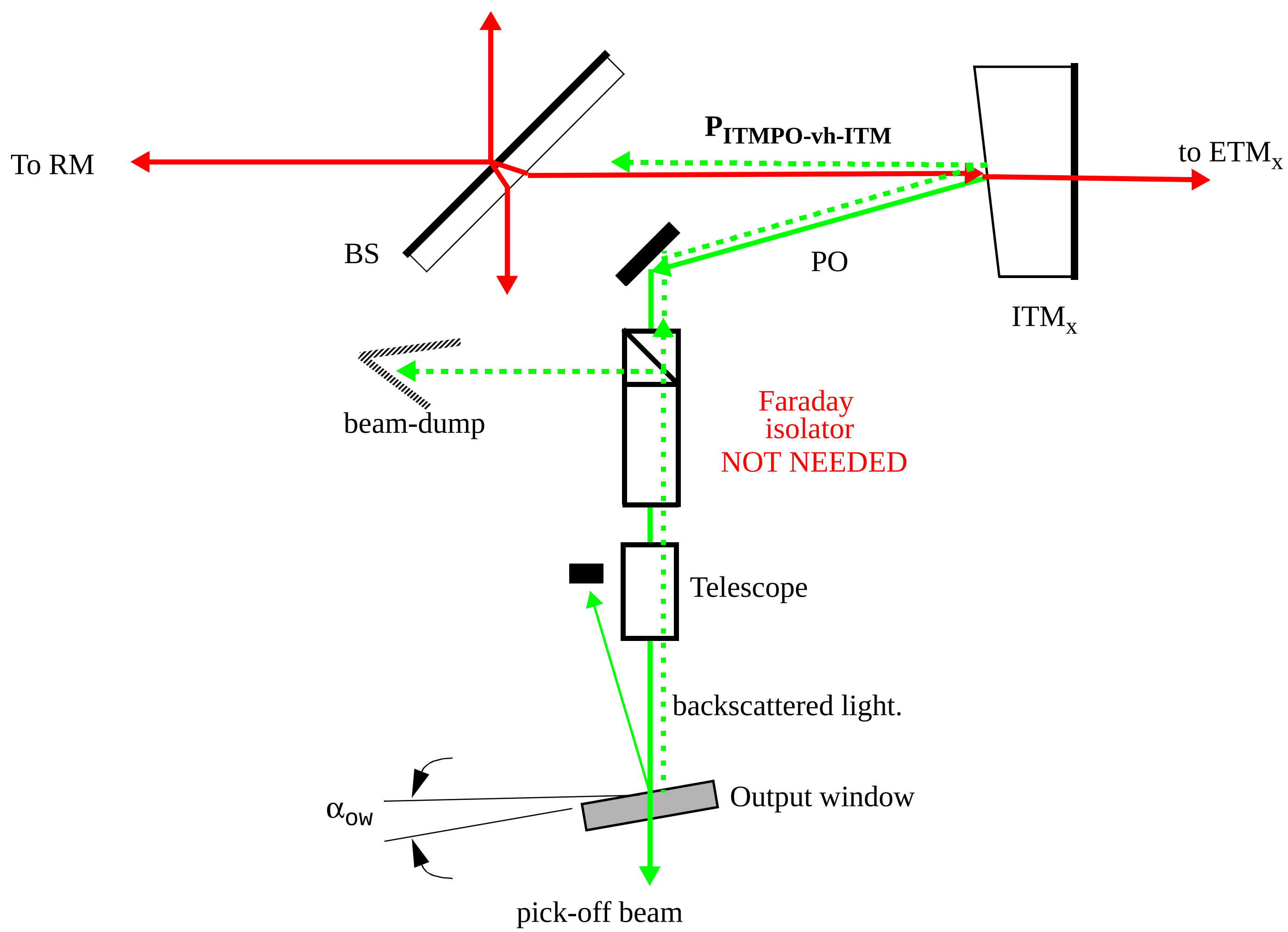
$$P_s|_{Req} \le 1.3\times10^{-12}W$$

Backscattered Power from APS Beam



$$P_{sDPS} = 2.0\times10^{-13} watts, \ BRDF \leq 8\times10^{-4} sr^{-1}$$

Backscattered Power from ITM PO Beam



$$P_{sITMPO} = 1.8\times10^{-13}\,watts,\quad BRDF \le 8\times10^{-4}\,sr^{-1}$$

K_i Values for Vacuum Housing and SEI Mounted Surfaces

- Generalized Scattered Light Noise/Signal Ratio

$$\frac{i_{si}}{i_g} = K_i \sqrt{\frac{P_{si}}{P_0}}$$

- K_i Values

Scattered Light Phase Noise Current Transfer Coefficient (K_i) for Scattering from Surfaces Mounted on Vacuum Housing and SEI Platform, for Initial LIGO Sensitivity

Surface Mount	*Scattering Path*	K_i @ 30Hz	K_i @ 100Hz	K_i @ 1000Hz
Vacuum housing	ITM PO to window on vac housing into recycling cavity	3×10^5	3×10^5	6×10^2
	APS to window on vac housing into BS	3×10^5	3×10^5	6×10^2
	ETM PO to window on vac housing into arm cavity	2×10^4	2×10^4	40
	SPS from vac housing into symmetric recycling cavity	3×10^3	3×10^3	6
SEI	ITM GB and BS GB to beam-dump on SEI into recycling cavity	300	50	0.3

Allocation of Noise Budget

- Noise Contributions from Scattering Paths
  - Noise contributed by an individual source

$$\left(\frac{is_i}{i_{gSRD}}\right)^2 = K_i^{\ 2} \cdot \frac{P_{si}}{P_0}$$

  - Total noise budget

$$\left(\frac{is}{i_{gSRD}}\right)^2 = N_1 \cdot K^2{}_1 \cdot \frac{P_{s1}}{P_0} + N_2 \cdot K^2{}_2 \cdot \frac{P_{s2}}{P_0} + \ldots + N_m \cdot K^2{}_m \cdot \frac{P_{sm}}{P_0} \le \left(\frac{1}{10}\right)^2$$

- Noise Allocation Factor

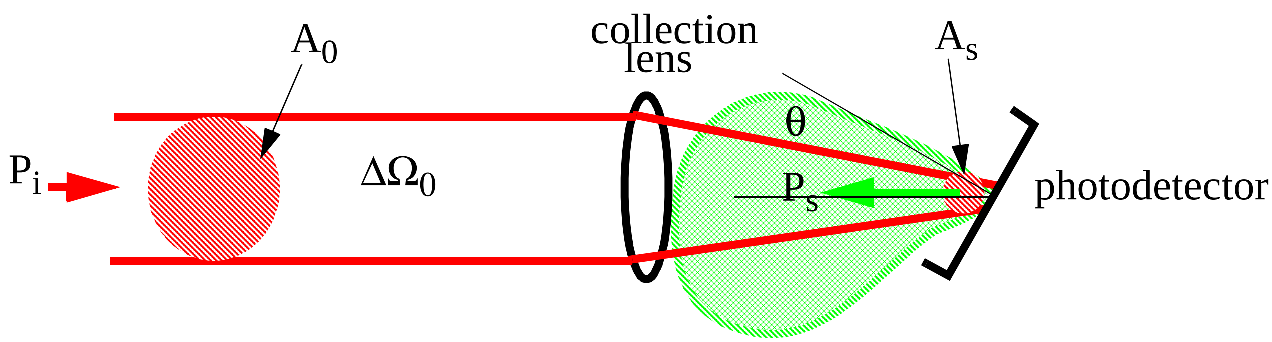
$$F_i = \frac{N_i \cdot (K_i)^2 \cdot P_{si}/P_0}{\sum N_i \cdot (K_i)^2 \cdot P_{si}/P_0} = \frac{N_i \cdot (K_i)^2 \cdot \left(\frac{P_{si}}{P_0}\right)_{REQ}}{\left(\frac{1}{10}\right)^2}.$$

- Scattered Light Requirement per Source

$$\left(\frac{P_{si}}{P_0}\right)_{REQ} \le \frac{F_i}{N_i \cdot K_i^2} \cdot \left(\frac{1}{10}\right)^2$$

Scattered Light from a Focussed Spot

Scattering into the IFO solid angle is enhanced by the collection lens

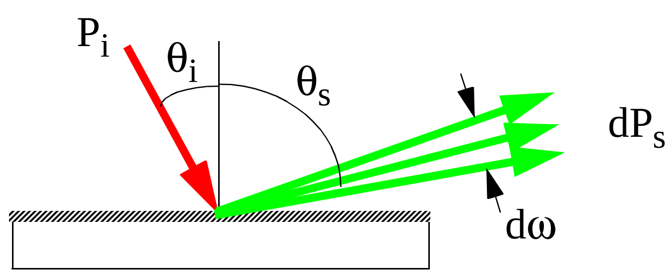


power incident on the photodetector	P_i
incidence angle	θ
IFO beam area	A_0
incident beam area	A_s
bi-directional reflectance distribution function, sr^{-1}	$BRDF$
IFO divergence solid angle, sr	$\Delta\Omega_0 = \frac{\lambda}{L}$

scattered light $P_s = P_i \cos(\theta) BRDF(\theta) \Delta\Omega_0 \frac{A_0}{A_s}$

Determination of BRDF

Direct measurement of scattered light

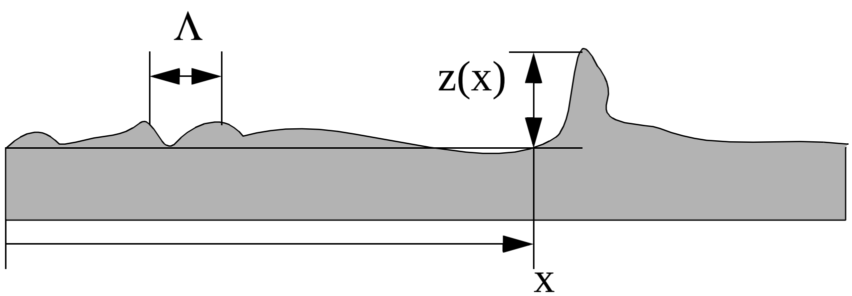


$$BRDF(\theta) = \frac{1}{P_i}\frac{dP_s(\theta)}{d\omega}$$

$$P_s = P_i \int_{\Delta\Omega} BRDF\, d\omega$$

Determination of BRDF

Surface Roughness Measurement



BRDF is related to the 2-dimensional power spectral density (PSD) of surface, $S_{2D}(k)$ waves^2-m^2 , where $k=2\pi/\Lambda$

$$BRDF(k) = \frac{16\pi^2}{\lambda^2} S_{2D}(k)$$

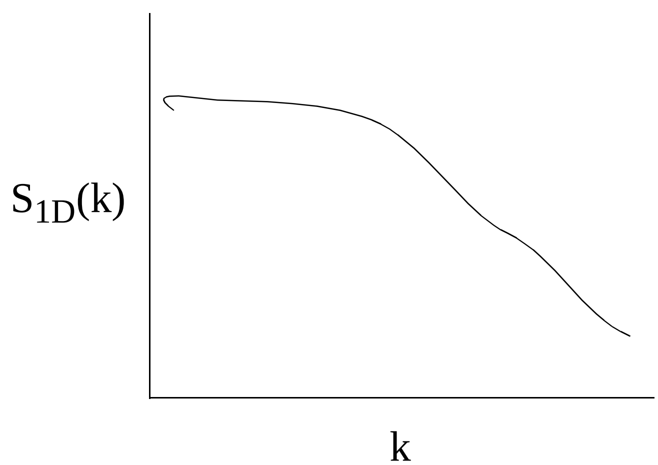
however, it is only practical to measure the 1-dimensional power spectral density of the surface, $S_{1D}(k)$ waves^2-m. The two are related as follows;

$$S_{1D}(k_x) = 2\int S_{2D}(k_x, k_y)\frac{1}{2\pi}dk_y$$

however, it is usually not possible to invert the integral.

Determination of BRDF

1-dimensional PSD is measured with phase maps (low spatial frequencies) and with surface profilometer (high spatial frequency roughness)



approximate $S_{1D}(k)$ with a fractal model

$$S_{1D}(f_x) = \frac{A}{[1 + (Bf_x)^2]^{\frac{C}{2}}} \text{ , where } f = \frac{1}{\Lambda}$$

fit data with constants, A, B, C, then $S_{2D}(f)$ is related to $S_{1D}(f)$ as follows:

$$S_{2D}(f) = AB\frac{\Gamma\left(\frac{C+1}{2}\right)}{2\sqrt{\pi}\Gamma\left(\frac{C}{2}\right)}\frac{1}{[1 + (Bf)^2]^{\frac{C+1}{2}}}$$

Determination of BRDF

then the BRDF can be determined from $S_{2D}(f)$

$$BRDF(k) = \frac{16\pi^2}{\lambda^2} S_{2D}(k)$$

in the alternative form:

$$BRDF(\theta) = BRDF(0) \frac{1}{[1 + b\theta^2]^{\frac{C+1}{2}}}$$

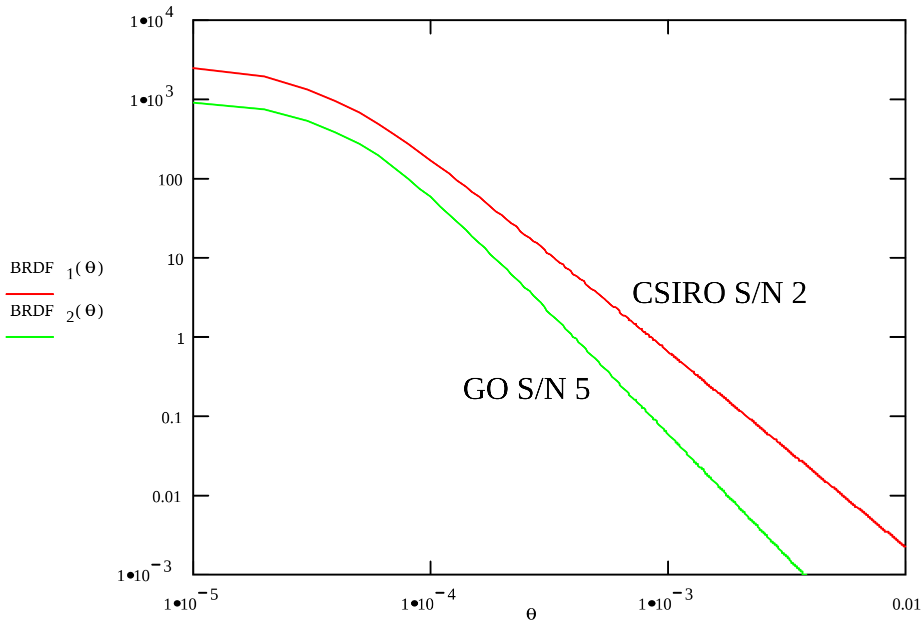
where the scattering angle is related to the spatial frequency and the wavelength of light by the Bragg condition,

$$\theta = \frac{\lambda}{\Lambda},$$

and $$b = \frac{B^2}{\lambda^2}$$

LIGO COC BRDF

Scattering data from two representative samples of superpolished fused silica mirror surfaces was obtained from the Pathfinder Optics.



Scattering angle, rad

GO S/N 5

$$BRDF_2(\theta) = \frac{1000}{(1 + 5.302\times10^8 \cdot \theta^2)^{1.55}}\ \mathrm{sr}^{-1}$$

CSIRO S/N 2

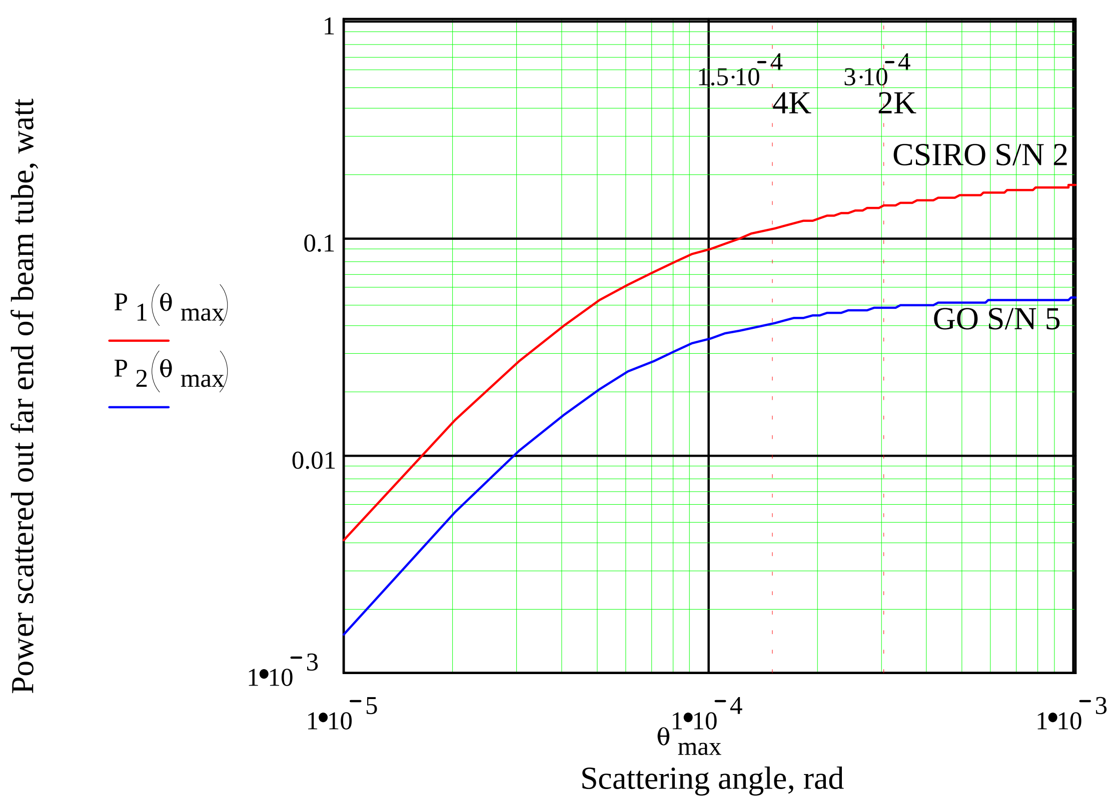
$$BRDF_1(\theta) = \frac{2755}{(1 + 8.508\times10^8 \cdot \theta^2)^{1.24}}$$

Small Angle Scattered Light from COC

A significant amount of small angle scattered light from the COC mirrors will pass out the far end of the beam tube. .

$$P_2(\theta_{max}) := P_i \cdot 2 \cdot \pi \cdot \int_0^{\theta_{max}} BRDF_2(\theta) \cdot \sin(\theta) \, d\theta$$

The calculation was made for the two representative BRDF functions described above, with 5000 watts in the arm cavity incident on the mirror.



Calculation of Backscattered PO and GB Power into IFO

- Backscattered Light Power

$$P_s = P_i \cdot T \cdot [\cos\theta_{iwo} \cdot BRDF_{wo}(\theta_s)] \cdot \Delta\Omega_0 \cdot \frac{1}{M^2} \cdot A_i$$

scattering solid angle $\Delta\Omega = \dfrac{\Delta\Omega_0}{M^2}$

P_i, incident power on scattering surface

T, transmission factor through COC into IFO

BRDF, bidirectional reflection distribution function

$\Delta\Omega_0$, solid angle of IFO beam

M, demagnification of incident beam

A_i, additional attenuation of scattered beam

- Implied BRDF of Scattering Surface

$$BRDF_i(\theta_s) = \left(\frac{P_i}{(P_s)_{REQ}} \cdot T \cdot [\cos\theta_i] \cdot \Delta\Omega_0 \cdot \frac{1}{M^2} \cdot A_i\right)^{-1}$$

Summary of Scattered Light Requirements

- Vacuum Housing Mounted Surfaces

Table 1: 4K IFO Scattered Light Requirements @ 100 Hz, P_{laser}=6w, G_{rc}=50, M=1/72.

Scattering path	*Power incident on surface, P_i, watt*	*Noise allocation factor*	*Scattered light requirement, $(P_s)_{REQ}$, watt*	*Attenuation of scattered light path*	*Implied BRDF of all surfaces in demagnified output beam, sr^{-1}*
$P_{APS\text{-}vh\text{-}BS}$	0.30	0.30	$<3.3\times10^{-13}$	$A_{FI} = 0.001$	$8\times10^{-4} sr^{-1}$
$P_{BSPO\text{-}vh\text{-}BS}$	0.06	0.13	$< 3.3\times10^{-15}$	$R_{ITM} = 1\times10^{-4}$	$8\times10^{-4} sr^{-1}$
$P_{ITMPO\text{-}vh\text{-}ITMxy}$	0.15	0.26	$<1.5\times10^{-13}$	$R_{ITM} = 1\times10^{-3}$	$8\times10^{-4} sr^{-1}$
$P_{ETMPO\text{-}vh\text{-}ETMxy}$	0.39	0.02	$<2.9\times10^{-12}$	$T^2_{ND} = 0.04$	$8\times10^{-4} sr^{-1}$

- SEI Mounted Surfaces

  ››backscattering from SEI mounted surfaces is 10^{-10} times smaller than the requirement for scattering from vacuum housing mounted surfaces and can be ignored.

- SUS Mounted Surfaces

  ››scattering from the surfaces of the COC can be ignored in comparison with scattering of PO beams from output windows.