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Violin Mode Noise

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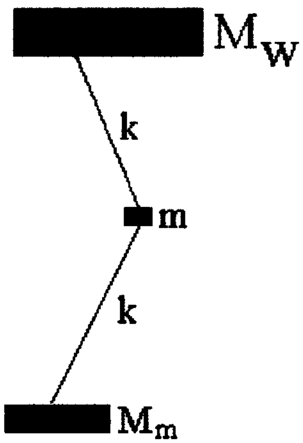

Introduction

A series of pulses in the driving force of a violin mode are in principle capable of reproducing any output data signal. This is a rather serious noise problem since an accidental series of such pulses can produce results that might be confused with a gravity wave. Fortunately the complete set needed to reproduce most signals is fairly large. An incomplete set has a signature in the form of spurious bumps in the data that do not correspond to the expected signal.

The differential equation

So that the equation for the signal from a violin mode in the time domain is

$$\ddot{d}(t)+4\pi w\dot{d}(t)+(2\pi f_0)^2 d(t)=\rho(t)$$

In the frequency domain this is

$$\left[-f^2-2jwf+f_0^2\right](2\pi)^2 D(f)=P(f)$$

In the case that $\rho(t)=\delta(t-t_0)$, $P(f)=\int_{-\infty}^{\infty}\delta(t-t_0)e^{j2\pi ft}\,dt=e^{j2\pi ft_0}$,this yields

$$D(f)=\frac{-e^{j2\pi ft_0}}{(2\pi)^2\left[f^2+2jwf-f_0^2\right]}$$

Two pulses canceling the center of the natural peak

Suppose I have a pulse at $t_0 = 0$ and a second at t_1 sometime later. Then

$$D_2(f) = \frac{-\left(1 + c_1 e^{j2\pi f t_1}\right)}{(2\pi)^2\left[f^2 + 2jwf - f_0^2\right]} = \frac{-\left(1 + c_1\cos(2\pi f t_1) + jc_1\sin(2\pi f t_1\right.}{(2\pi)^2\left[f^2 + 2jwf - f_0^2\right]}$$

The object is to make the central peak disappear. There are three parameters, $\mathrm{Re}(c_1)$, $\mathrm{Im}(c_1)$ and t_1. There are two conditions, that the real part and imaginary part of $D_1(f_0)$ equal zero. If I choose c_1 to be real, the imaginary condition is satisfied by

$t_1 = \dfrac{m}{2f_0}, \quad m = \text{any integer}$. Then the cosine is given by

$\cos(\pi m) = (-1)^m$ so that the real part of c_1 is given by

$c_1(-1)^m + 1 = 0; \quad c_1 = (-1)^{m+1}$. The value a short distance from f_0 can be found from the expansion

$$D_2(f) \cong D_2(f_0) + (f - f_0)\left.\frac{\partial D_2}{\partial f}\right)_{f=f_0} \cong j2\pi t_1 c_1 (f - f_0)(\cos(2\pi f_0 t_1))G(j$$

$$j2\pi t_1 c_1 (f - f_0)(-1)^m G(f_0)e^{-(f_0 w_t)^2} = -j2\pi m\frac{(f - f_0)}{2f_0}G(f_0)$$

The last term is the peak from a delta function response. The derivative of this with respect to f at $f=f_0$ is zero owing to f_0 being the value at the peak. Note that a two-term attempt to eliminate the peak will have the most success for small m. In fact m=0 is a positive and negative peak at exactly the same time, which totally eliminates the peak for all values of f.

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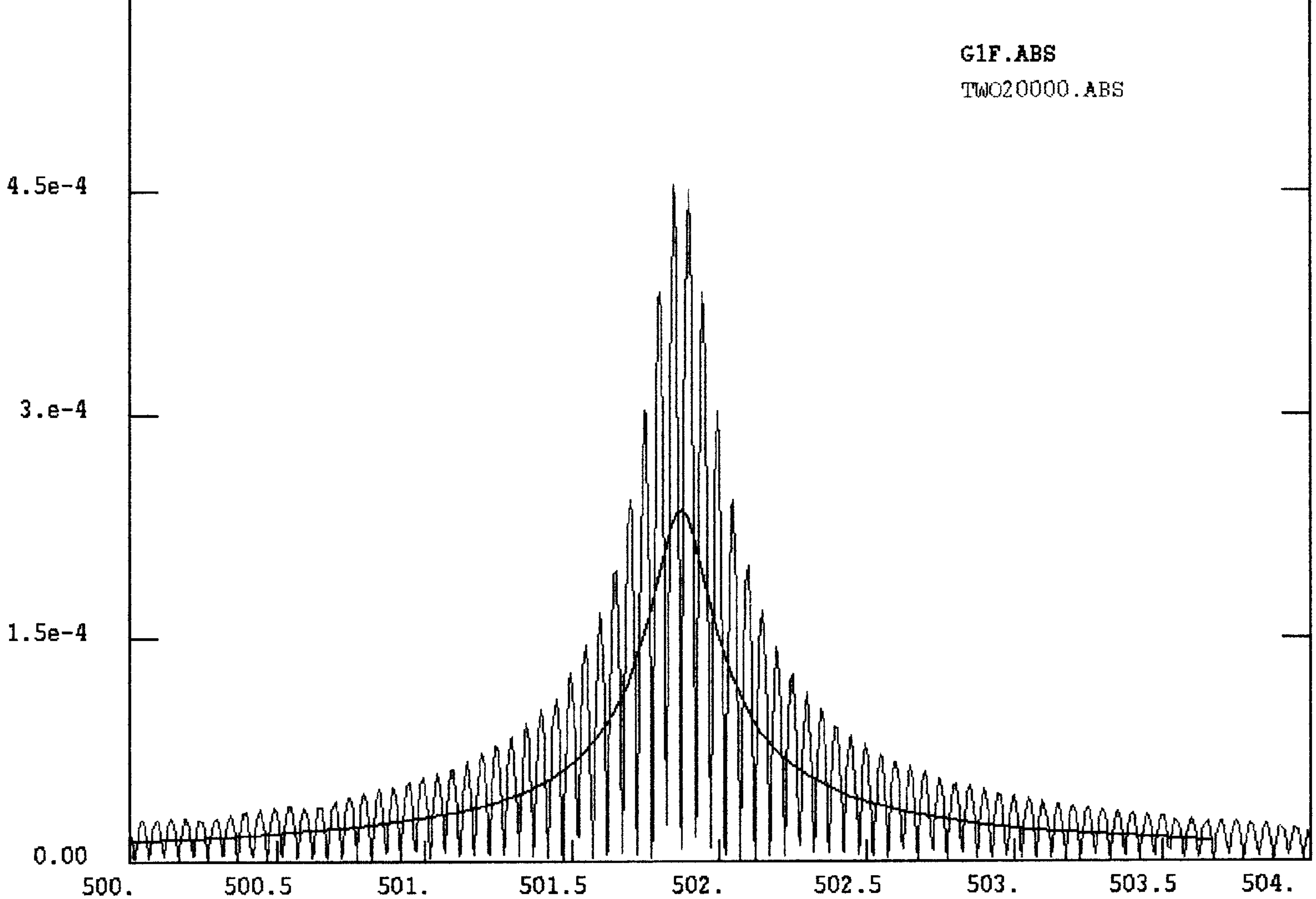


Figure 1 Two pulses ~40 seconds apart. The peaks are narrow relative to the size of the natural line.

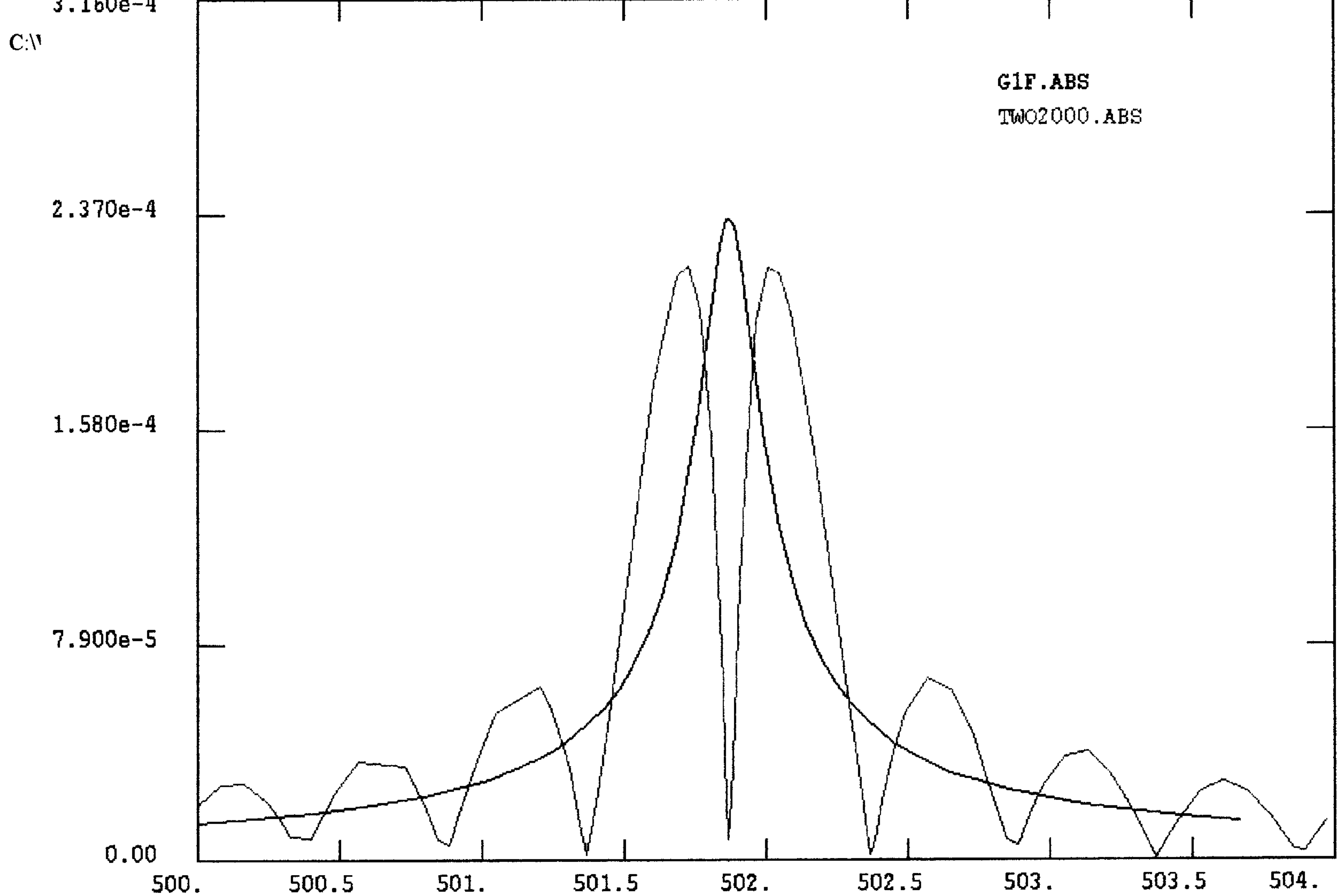


Figure 2 Two pulses ~4 seconds apart. The peaks are about the same width as those in the natural line.

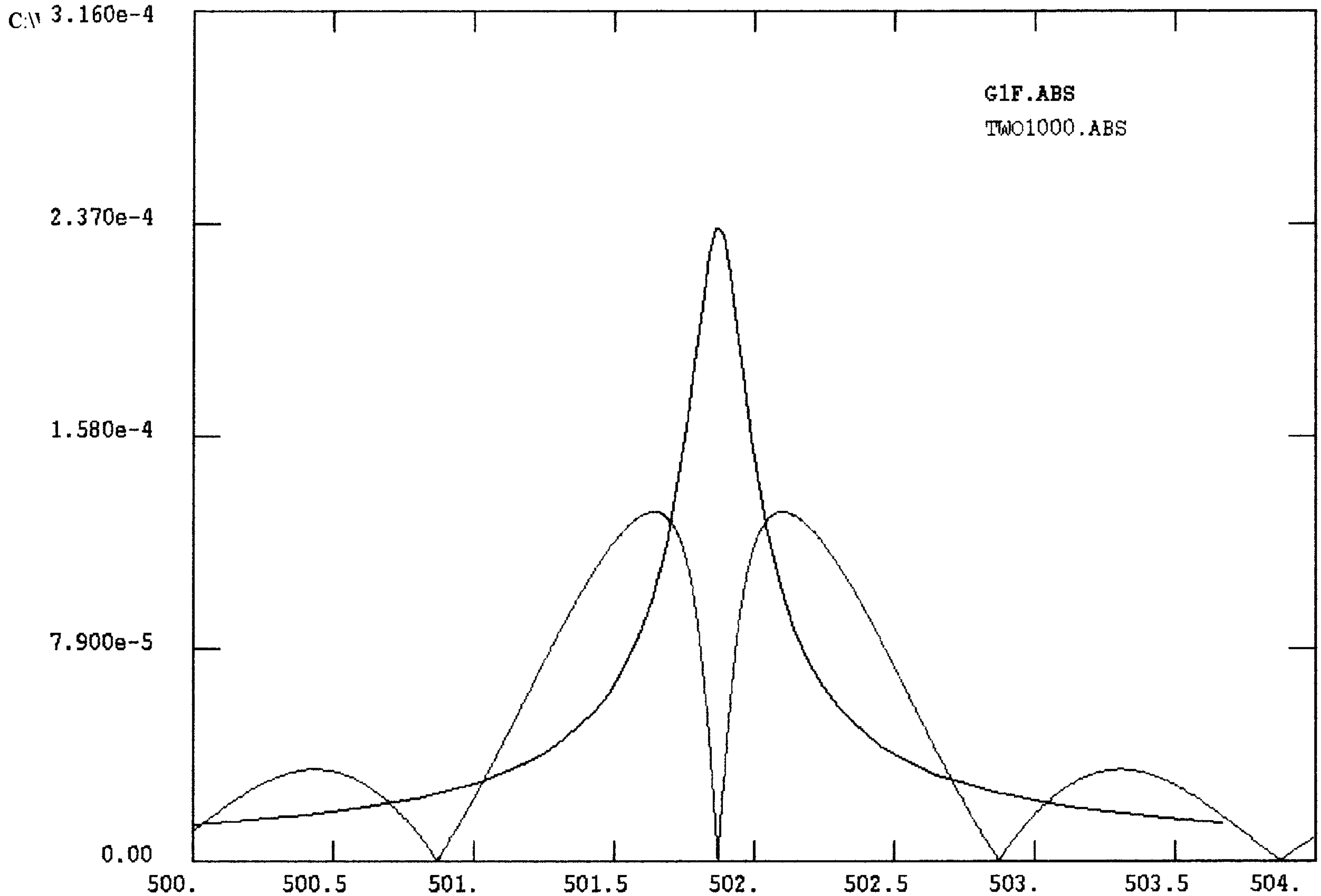


Figure 3 Two pulses on the order of 2 seconds apart are shown. The peaks are much wider than the natural shape.

The width of a pulse is the change in frequency required to go through π in the imaginary exponent. Thus $\Delta f\, t_0 = 1$ or $\Delta f = 1/t_0$, with a natural width of 0.1 cycles per second, the time required to get to this width is $t_0 = 10\sec$. Thus a good approximation to the peak may be a series of t_0's between 1 and 10 seconds. If the times are allowed to adjust so that $t_i = t_{0,i} + \delta_i$, the adjustments will change the value of $e^{j2\pi f\left(t_{i,0}+\delta_i\right)}$. These changes approximately repeat for $f\delta_i = 1$ meaning that the changes will be found to vary the value of δ_i only by 1/f.

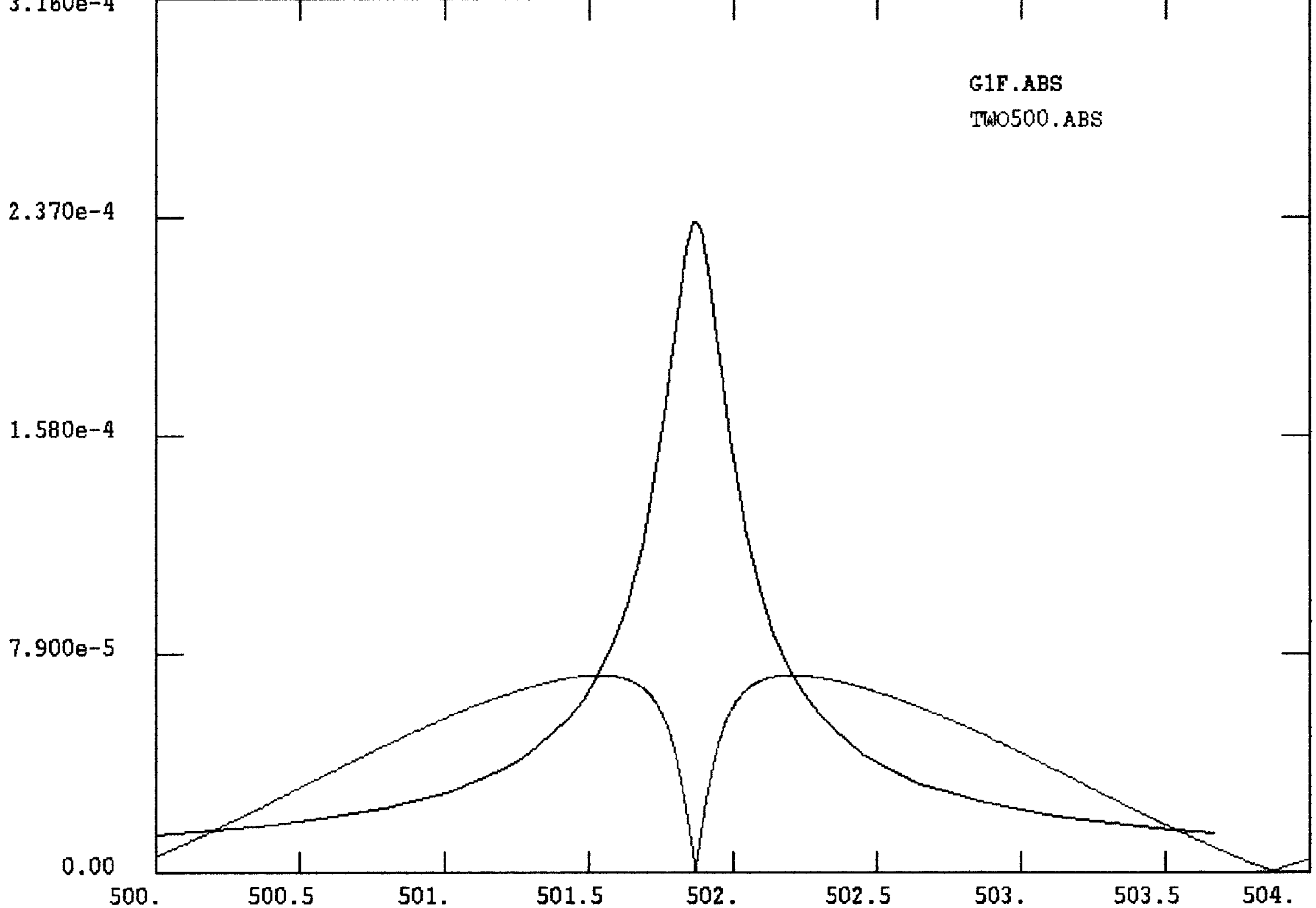


Figure 4 Two peaks ~1 second apart.

Finding $\rho(t)$

For any observed data D(f), there exists a P(f) given by

$$P(f) = -(2\pi)^2\left[f^2 + 2jwf - f_0^2\right]D(f)$$

Example 1 A displaced violin mode

In particular suppose that D(f) is a violin mode at a frequency other than the original

$$D(f) = e^{j2\pi t t_0}V(f; f_0', w')$$

$$\Rightarrow P(f; f_0, w) = e^{j2\pi f t_0}\frac{f_0'\left[(w - jf)^2 + f_0^2\right]}{f_0\left[(w' - jf)^2 + f_0'^2\right]}$$

This can be solved analytically to yield

$$\rho(t;t_0,f_0,f'_0,w,w') = \frac{f_0'}{f_0}\delta(t_0 - t)$$

$$-2\pi\theta(t-t_0)\frac{e^{-2\pi w(t-t_0)}}{f_0}\left\{\begin{matrix}\left[(w-w')^2 + f_0^2 - f_0'^2\right]\sin(2\pi f_0'(t-t_0)) \\ -\left[2jf_0'(w-w')\right]\cos(2\pi f_0'(t-t_0))\end{matrix}\right\}$$

The data can also be fitted as a series of delta functions. The fit below is to a series of 257 delta functions in which the heights and times are both varied.

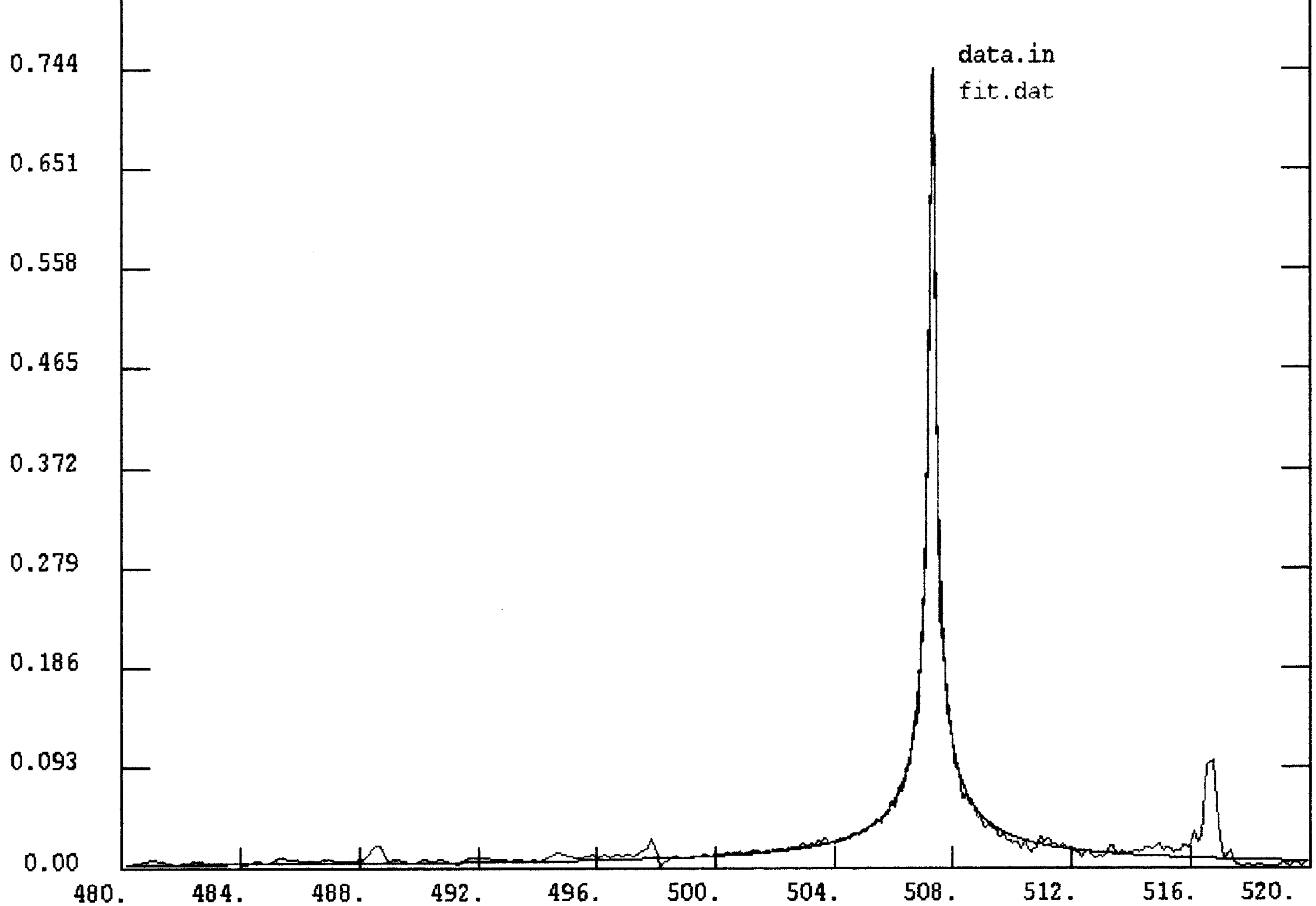


Figure 5 This is the fit produced by 257 starting times. The violin mode is at 500.87 cycles/second. The data is assumed to consist of a single violin mode at 507 cycles per second.

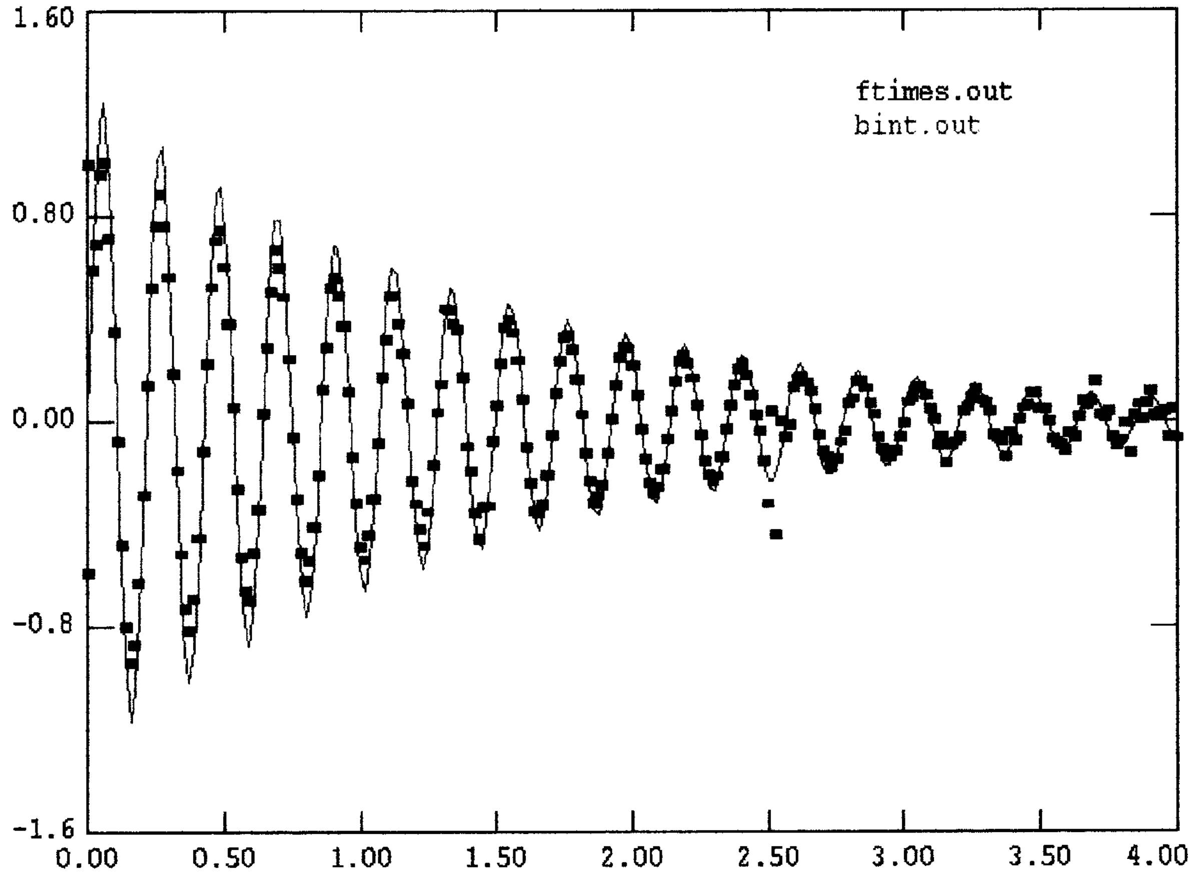


Figure 6 This is a plot rho(t) as found by the fit - Black dots and the analytical "result" line connecting points.

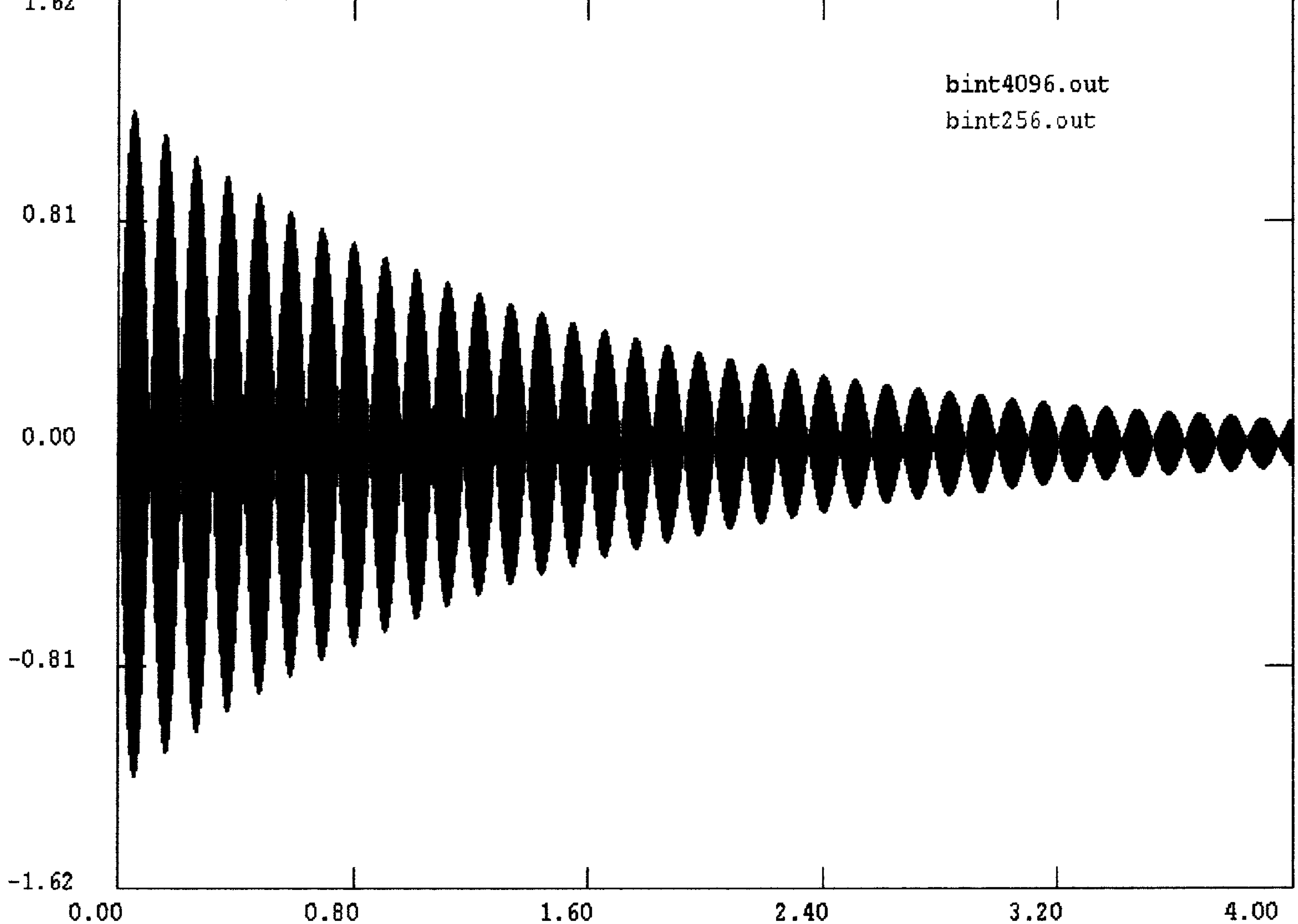


Figure 7 Now all of the points in the analytical result are shown, rather than just the uniformly spaced 257 points between 0 and 4.

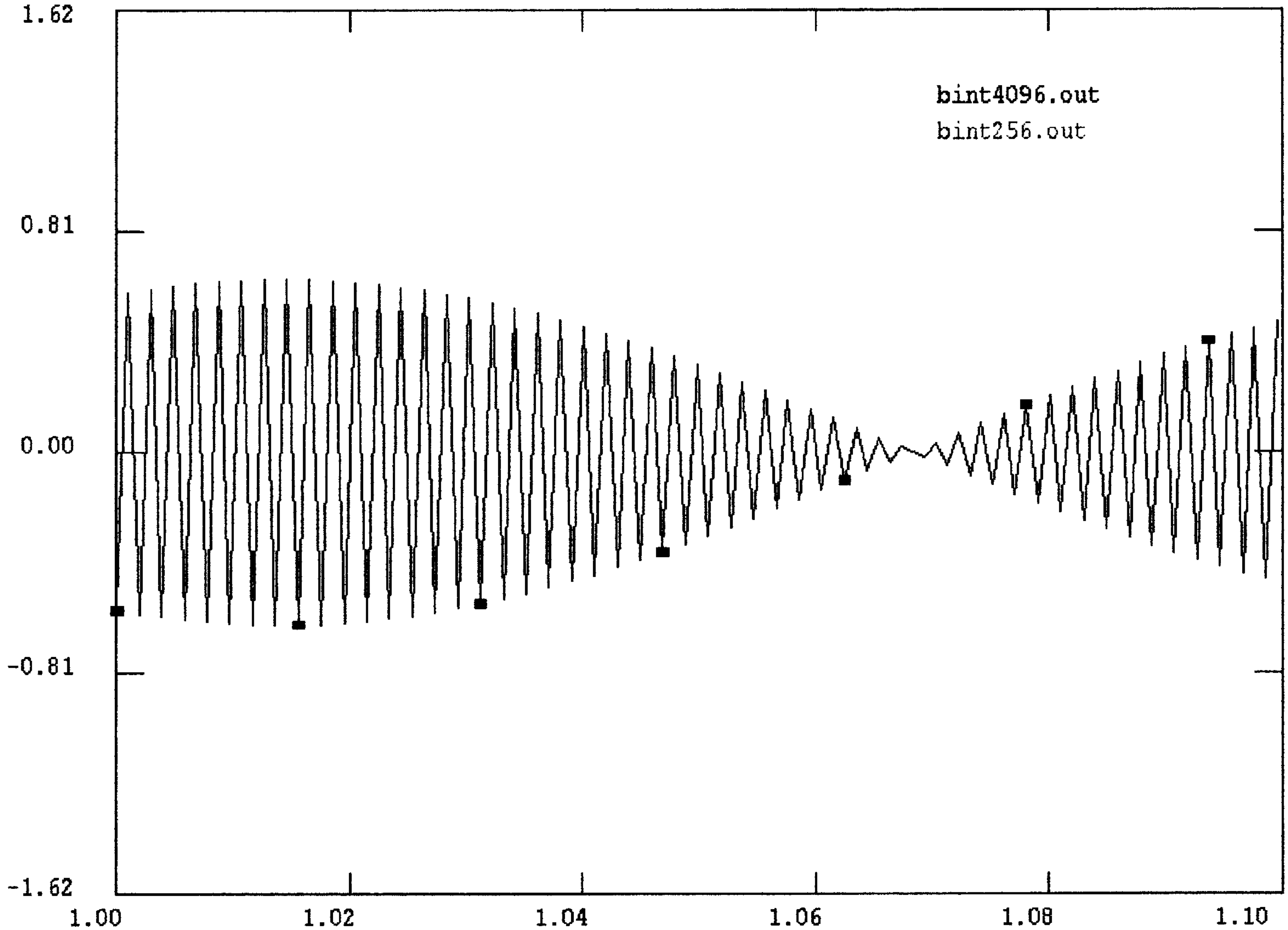
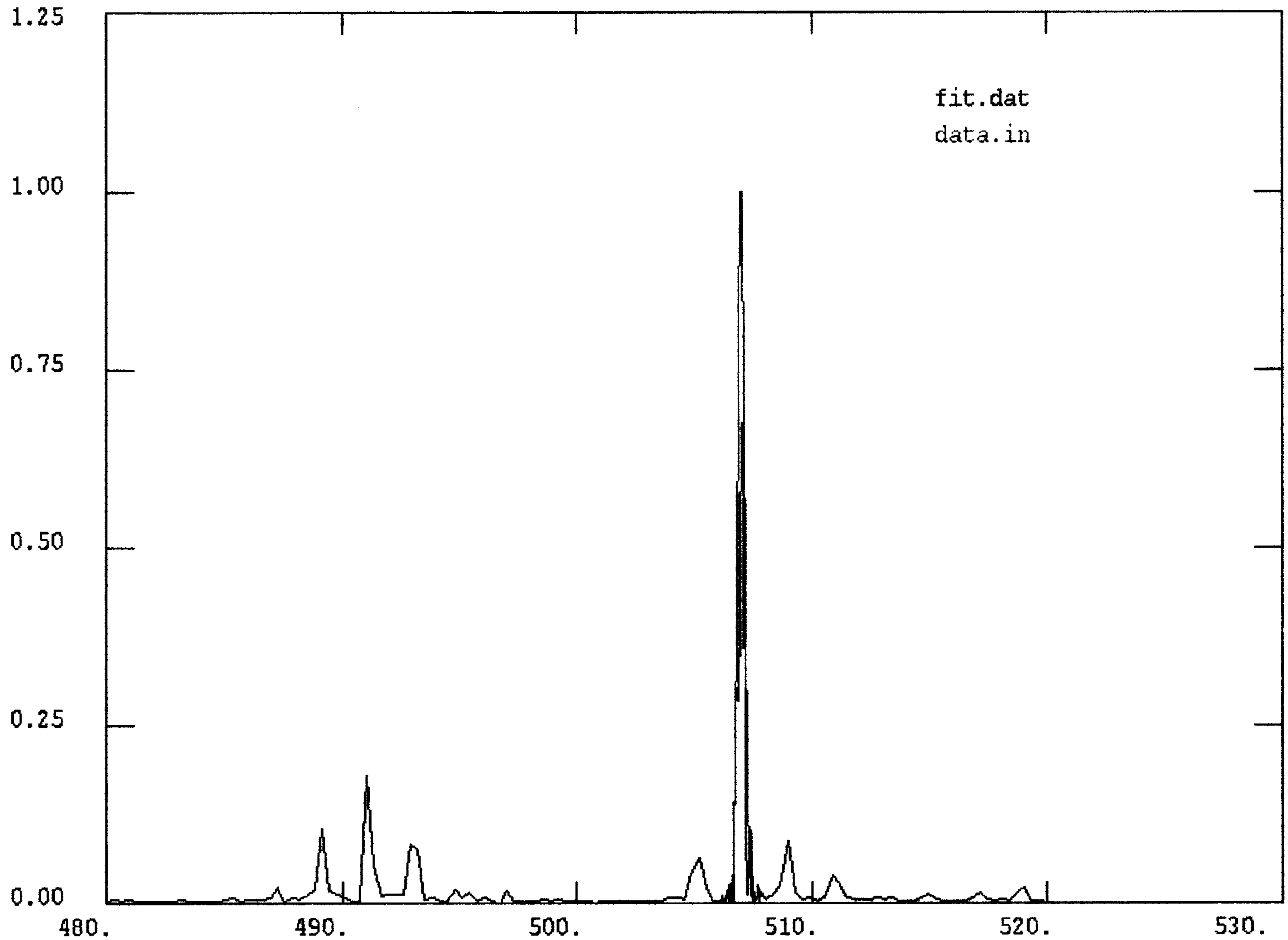
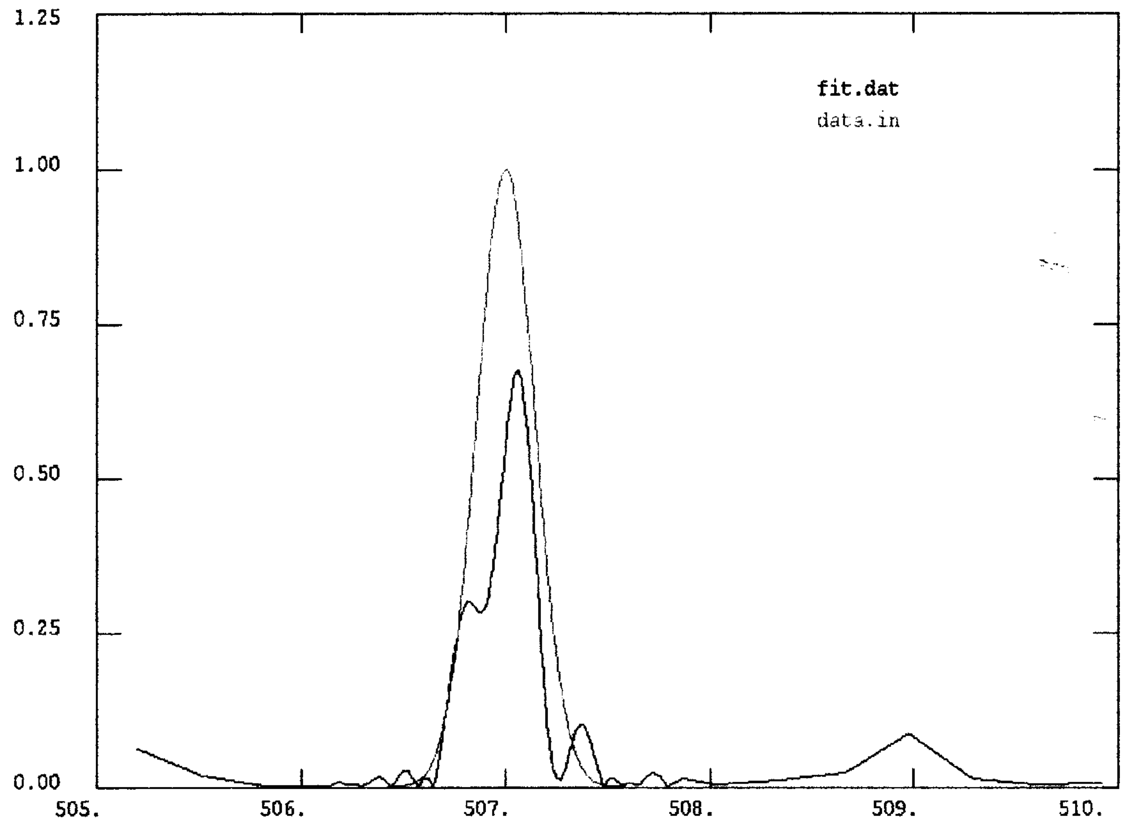


Figure 8 Greater detail of the fit results versus the analytical solution.

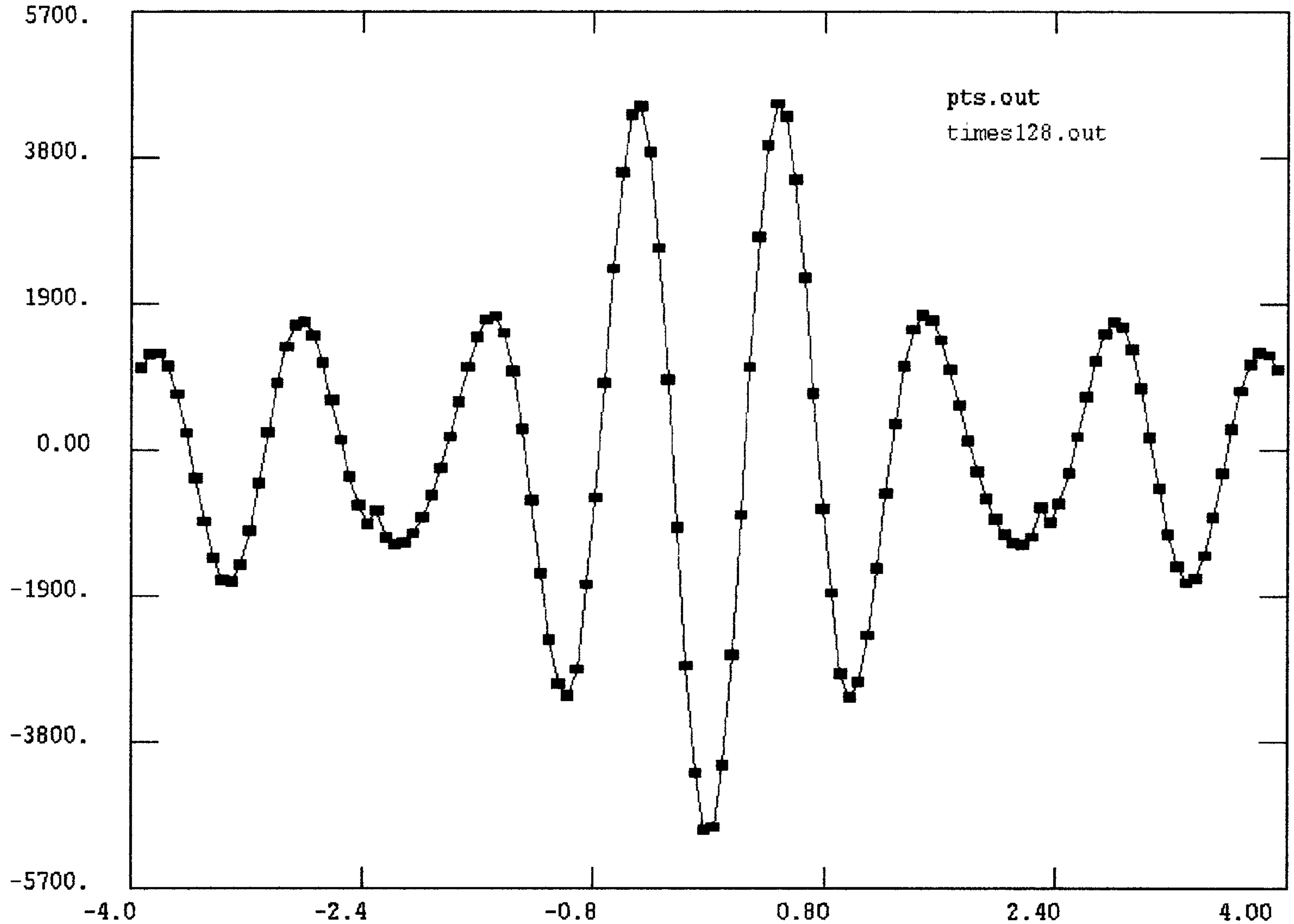
Example 2 A Gaussian data peak





In this case the data is a Gaussian peak centered at 507 cycles/sec near a violin mode at 500.87 cycles/second.

Figure 9 Expansion of the peak region.



Figue 10 Distribution of times leading to above fit.

Note 1, Linda Turner, 05/09/00 10:21:59 AM

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