Coincidence gravitational wave burst experiments with wide-band detectors of differing sensitivity

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References:

[1] R.W.P. Drever, Y. Gursel, and M.Tinto, In preparation.

[2] R.W.P. Drever, Y. Gursel, and M.Tinto, In: Proceedings of the sixth Marcel Grossmann Meeting, Editors: H. Sato and T. Nakamura, World Scientific, Vol. 2, 1471 (1991)

[3] B.F. Schutz, In: *Detection of Gravitational Radiation*, Editor D. Blair (CUP, 1989)

[4] Y. Gursel, and M.Tinto, Phys. Rev. D, 40, 3884, 1989

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INTRODUCTION

- The possibility of coincidence experiments, involving three or more wide-band gravitational wave detectors, makes it important to identify the overall experiment sensitivity.
- LIGO will operate 3 interferometers in coincidence .
- 2 of them (with arm lengths differing by a factor of two) will be located at one site; the remaining interferometer at the other site will have an arm length equal to the arm length of the longer interferometer at the first site.
- The use of the half-length interferometer can give significant improvement in effective experiment sensitivity when the dominant noise sources do not depend on the arm length (spurious pulses from gas bursts).

INTRODUCTION (cont.)

- We will not consider such cases, where there are obvious advantages in use of an half-length interferometer.
- We will assume instead that the noise in each interferometer is Gaussian.
- We shall assume the following definition for the sensitivity of an experiment: *The sensitivity of a gravitational wave experiment is the amplitude of the gravitational wave which, with optimum polarization and direction of propagation, has a given probability (e.g. 50 %) of being detected by the experiment.*
- We will consider a simple threshold-crossing experiment.

INTRODUCTION (cont.)

- The three thresholds are chosen in such a way to give the above defined sensitivity, subject to the following two constraints on the accidental rates:
  - The maximum triple coincidental accidental rate for the whole experiment has been chosen to be 1 every 10 years
  - The single accidental rate in any of the interferometers has been assumed to be less than 1 %.

MATHEMATICAL DESCRIPTION

- In absence of the gravitational wave pulse, the output of detector *i* is assumed to be a Gaussian, band-limited noise of zero-mean and variance σ^2_i :

$$p\,(\sigma_i\,;\,x_i) = (2\,\pi\,\sigma^2_i)^{-1/2}\,\exp\,[\,-\,x^2_i/2\sigma^2_i]$$

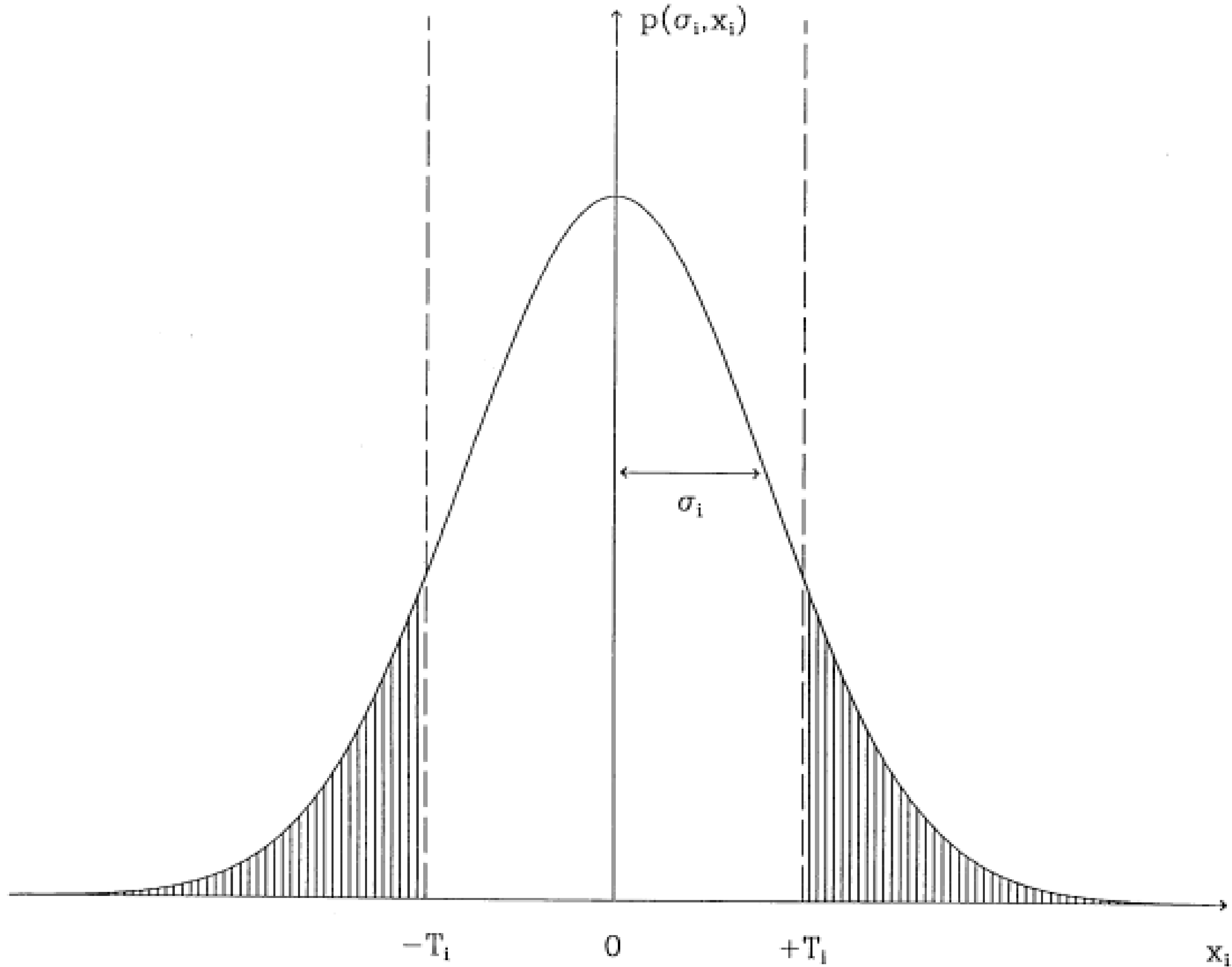
$$(2\,\pi\,\sigma^2_i)^{-1/2}\int_{-\infty}^{+\infty}\exp\,[\,-\,x^2_i/2\sigma^2_i]\,dx_i = 1$$

- The probability P that the output x_i of the detector exceeds a given (positive) threshold T_i is given by:

$$P(|x_i| > T_i) = 2\int_{Ti}^{+\infty} p\,(\sigma_i\,;\,z)\,dz$$

which can be written as:

$$P(|x_i| > T_i) = 1 - \mathrm{erf}\,[\,T_i/\,(2\sigma^2_i)^{1/2}\,]$$



p(σi,xi)
σi
−Ti
0
+Ti
xi

MATHEMATICAL DESCRIPTION (cont.)

where:

$$\text{erf}\,[x] = 2/(\pi)^{1/2} \int_0^x \exp[-t^2]\,dt \quad ; 0 <= x < \infty$$

$$\text{erf}\,[x] = 1 - [a_1 t + a_2 t^2 + a_3 t^3 + a_4 t^4 + a_5 t^5] \exp[- x^2] + \varepsilon$$

with : $t = 1/(1 + q\,x)$; q , a_k $k = 1, ..5$ are known coefficients, and

$$\varepsilon <= 1.5 \times 10^{-7} \quad \text{for} \quad 0 <= x < \infty$$

THE CONSTRAINT EQUATIONS

- Under the assumption that the output of an interferometer will be a band-limited, white, Gaussian noise with bandwidth f_c, we can write the following expression for the rate R_i at which the detector output x_i exceeds a given threshold T_i:

$$R_i = 2\, f_c\, P(|x_i| > T_i) \qquad (i = 1, 2, 3)$$

- The resolving time τ_1 for coincidence experiments between two detectors located at the same site is equal to the time between the digitized samples:

$$\tau_1 = 1/(2\, f_c)$$

while the resolving time τ_2 for coincidence experiments between detectors located at different sites is given by:

$\tau_2 = D + \tau_1$ (D = maximum time-delay between the two sites)

THE CONSTRAINT EQUATIONS (cont.)

- For tree interferometers (2 + 1) the accidental coincidence rate R_{acc} due entirely to Gaussian noise in the detectors is given by the following expression:

$$R_{acc} = 4\,\tau_1\,\tau_2\,R_1\,R_2\,R_3 \;\;\Rightarrow$$

$$P(|x_1| > T_1)\,P(|x_2| > T_2)\,P(|x_3| > T_3) = R_{acc}/(32\,f_c^3\,\tau_1\,\tau_2)$$

COMPUTATION OF THE OPTIMUM SENSITIVITY

- We assume that all the detectors are oriented parallel to each other.
- Consider the case of a gravitational wave of pulse amplitude h is impinging on the three detectors in the observatory:

$$x_i = h + \Lambda_i$$

- In a series of experiments in which h is held constant, the random variable x_i has a Gaussian distribution of mean h and variance σ^2_i

$$p_h (\sigma_i ; x_i) = (2 \pi \sigma^2_i)^{-1/2} \exp [- (x_i - h)^2/2\sigma^2_i]$$

COMPUTATION OF THE OPTIMUM SENSITIVITY (cont.)

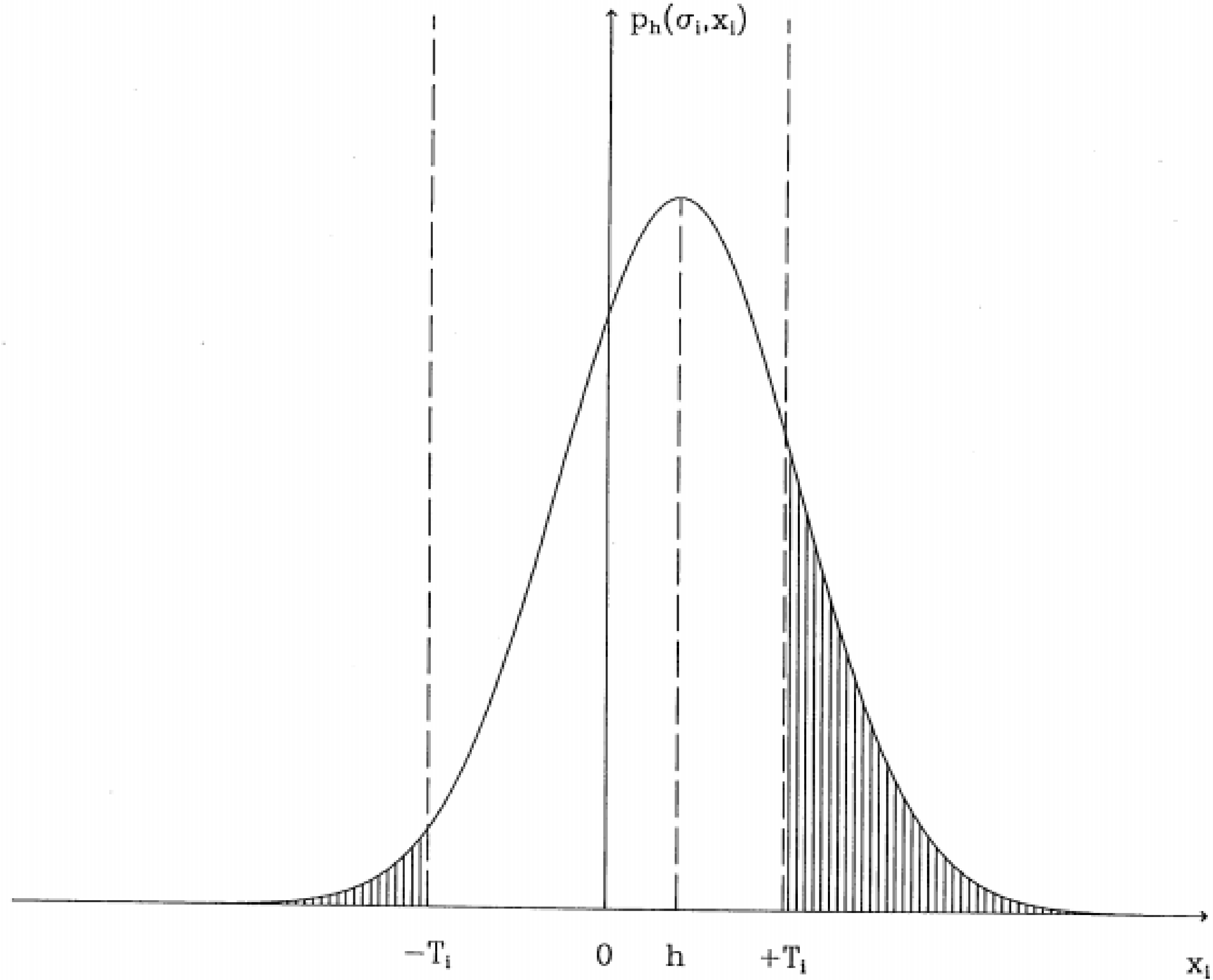
- With a gravitational wave present, the probability that the detector output x_i exceeds the threshold T_i is given by:

$$P_h(|x_i| > T_i) = 1 - 0.5 \{\mathrm{erf}[(h + T_i)/(2\sigma^2_i)^{1/2}] + \mathrm{erf}[(-h + T_i)/(2\sigma^2_i)^{1/2}]\}$$

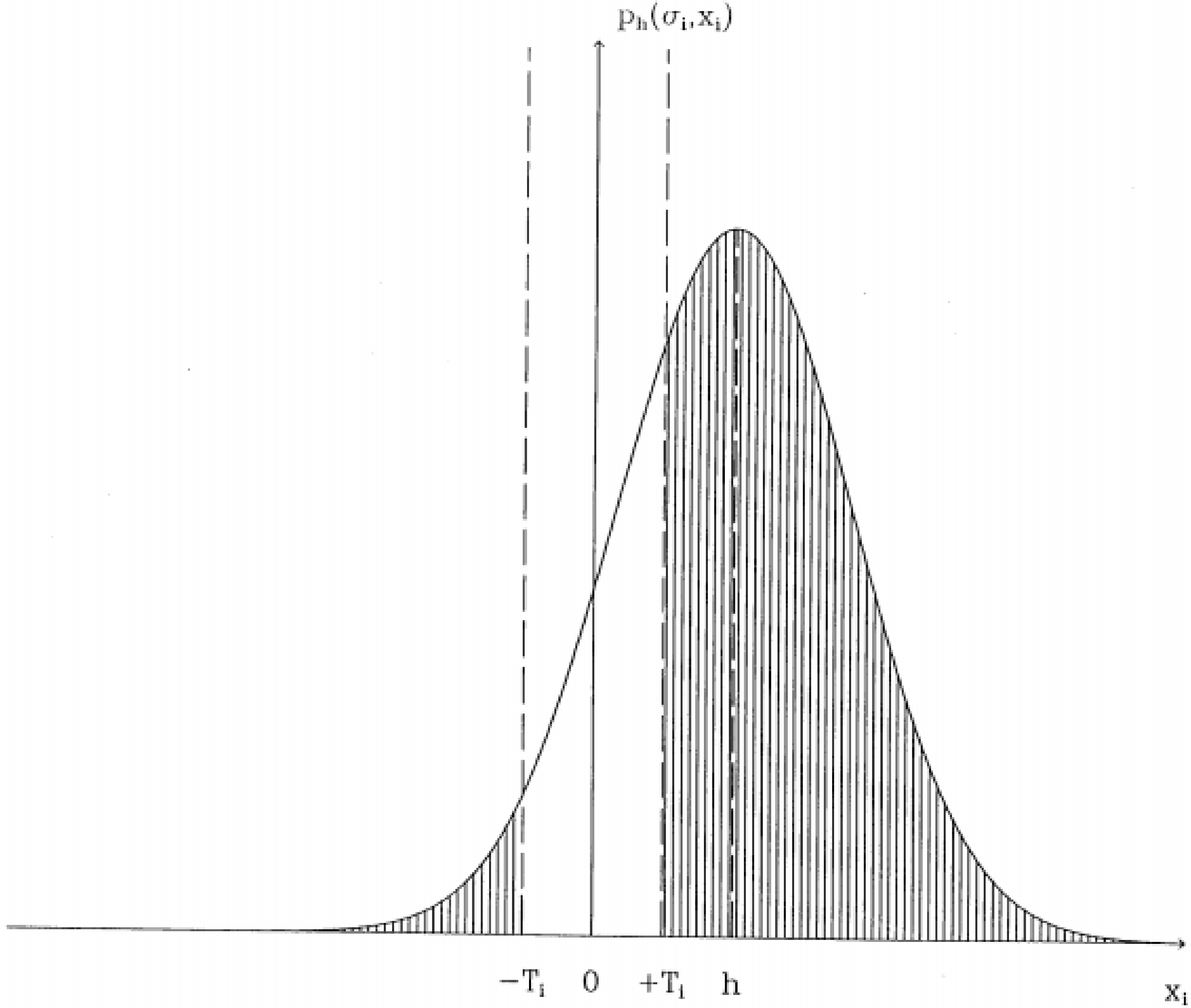
where: $h > 0, \quad T_i >= h$

$$P_h(|x_i| > T_i) = 1 - 0.5 \{\mathrm{erf}[(h + T_i)/(2\sigma^2_i)^{1/2}] - \mathrm{erf}[(-h + T_i)/(2\sigma^2_i)^{1/2}]\}$$

where: $h > 0, \quad 0 <= T_i < h$



$p_h(\sigma_i, x_i)$
$-T_i$
0
h
$+T_i$
x_i



$p_h(\sigma_i, x_i)$
$-T_i$
0
$+T_i$
h
x_i

COMPUTATION OF THE OPTIMUM SENSITIVITY (cont.)

- The gravitational wave amplitude $h(T_1, T_2, T_3)$, which has a 50% probability of being detected in coincidence by the three detectors, satisfies the following equation:

$$P_h (|x_1| > T_1)\, P_h (|x_2| > T_2)\, P_h (|x_3| > T_3) = 0.50$$

ANALYSIS

- We have considered three different configurations for triple coincidences:

(I) All the detectors have the same sensitivities

(II) (2+1) configuration, with the r.m.s. of the shorter detector equal to $\sqrt{2}$ of the r.m.s. of the longer (equal sensitivity) detectors

(III) (2+1) configuration, with the r.m.s. of the shorter detector equal to twice that of the longer (equal sensitivity) detectors

ANALYSIS (cont.)

- We have also considered three different configurations for coincidences with two separated detectors:

(I) The detectors have the same sensitivities

(II) The r.m.s. of the shorter detector is equal to $\sqrt{2}$ of the r.m.s. of the other detector

(III) The r.m.s. of the shorter detector is equal to twice that of the longer (equal sensitivity) detectors

RESULTS

Number of Detectors	σ_{half} / σ_{full}	f_c (Hz)	Min.Sens. / σ_{full}	Ratio
3	$\sqrt{2}$	1000	5.51	1.09
3	$\sqrt{2}$	200	5.25	1.09
3	$\sqrt{2}$	30	4.96	1.09
3	2	1000	5.75	1.21
3	2	200	5.48	1.24
3	2	30	5.18	1.28
2	$\sqrt{2}$	1000	6.65	1.15
2	$\sqrt{2}$	200	6.31	1.16
2	$\sqrt{2}$	30	5.94	1.16
2	2	1000	7.22	1.34
2	2	200	6.85	1.31
2	2	30	6.44	1.29