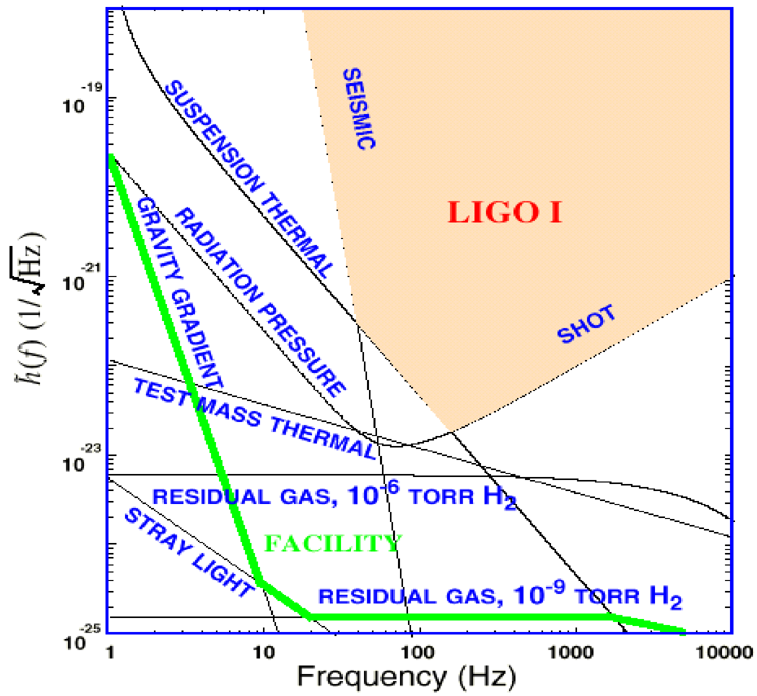
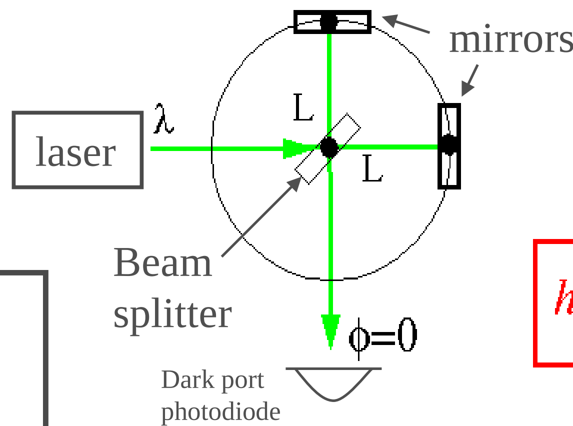
Physics of LIGO
Lecture 2

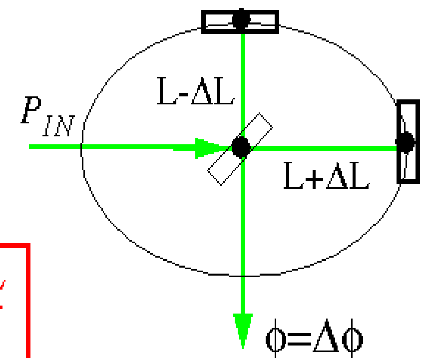
- Interferometers for GW detection
- Cavity optics
- Control systems



Interferometric detection of GWs

GW acts on freely falling masses:



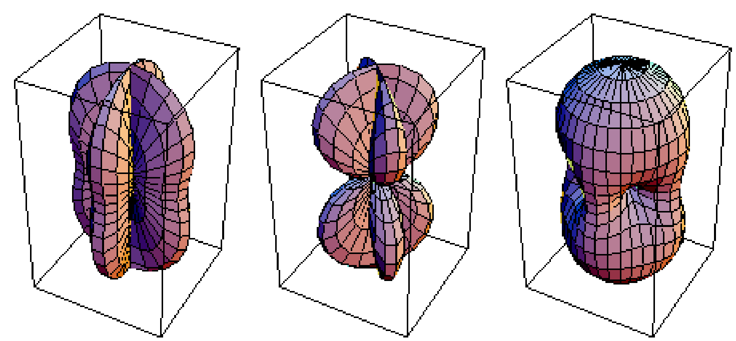


$$h = \frac{\Delta L}{L}$$

$$P_{out} = P_{in} \sin^2(2k\Delta L)$$

For fixed ability to measure *ΔL*, make *L* as big as possible!

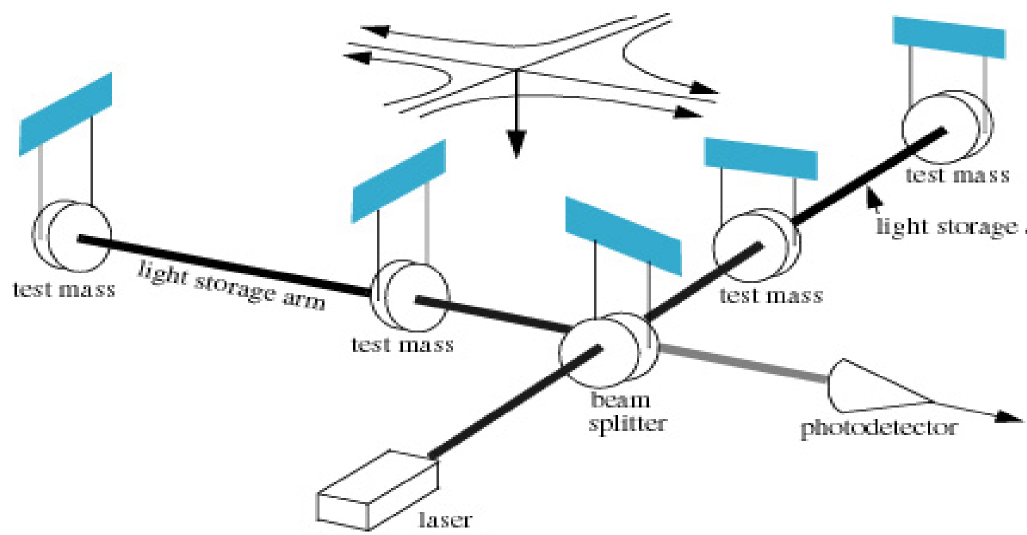
Antenna pattern:
(not very directional!)



+ polarization × polarization unpolarized

Interferometer for GWs

- The concept is to compare the time it takes light to travel in two orthogonal directions transverse to the gravitational waves.
- The gravitational wave causes the time difference to vary by stretching one arm and compressing the other.
- The interference pattern is measured (or the fringe is split) to one part in 10^{10}, in order to obtain the required sensitivity.



Suspended test masses

• To respond to the GW, test masses must be "free falling"

• On Earth, test masses must be supported against DC gravity field

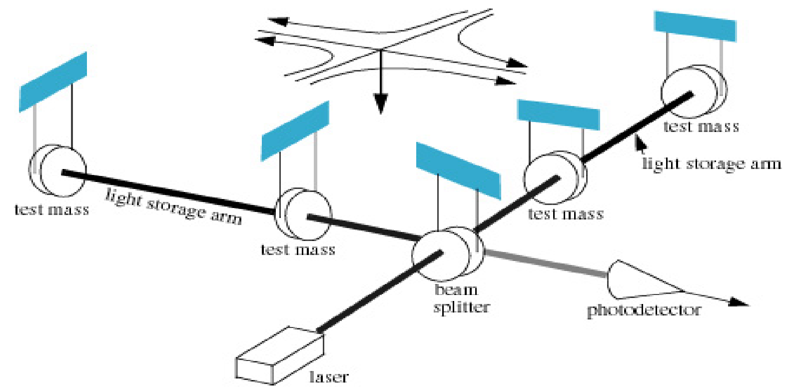
• The Earth, and the lab, is vibrating like mad at low frequencies (seismic, thermal, acoustic, electrical);

- •can't simply bolt the masses to the table (as in typical ifo's in physics labs)

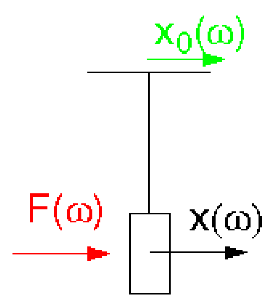
• So, IFO is insensitive to low frequency GW's

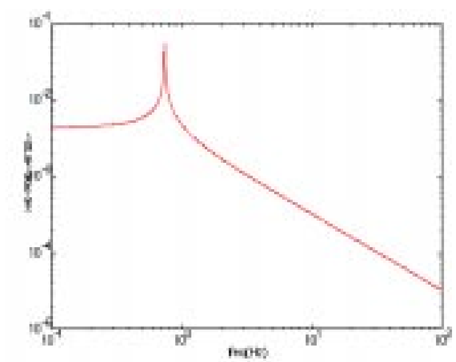
• Test masses are suspended on a pendulum resting on a seismic isolation stack

- •"fixed" against gravity at low frequencies, but
- •"free" to move at frequencies above ~ 100 Hz



"Free" mass:
pendulum at $f \gg f_0$





Pendulum dynamics

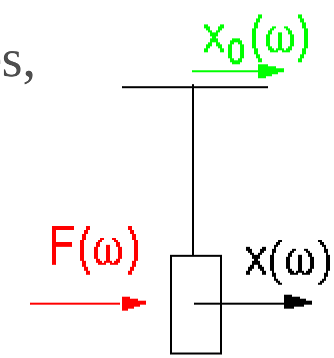
Pendula are wonderful mechanical filters

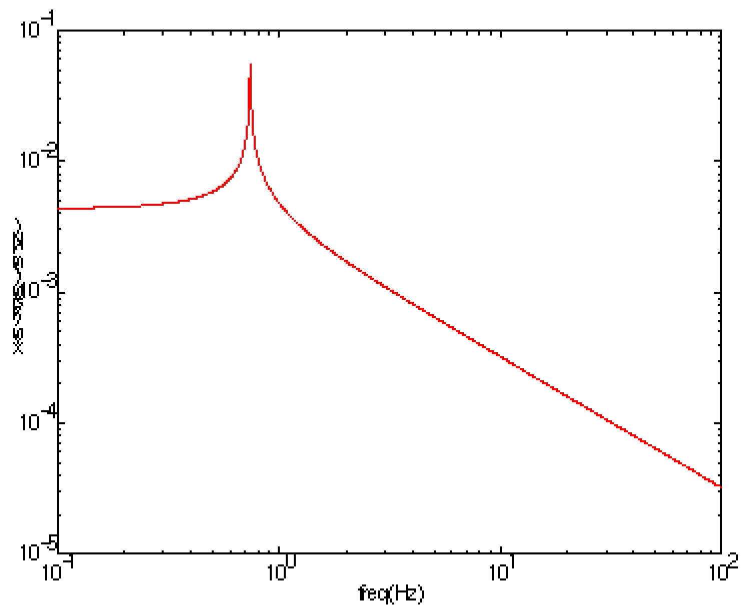
• They amplify the motion (seismic, thermal, environmental) from suspension point to mass at their resonant frequency f_0

• For frequencies $f >> f_0$, motion is suppressed like f^{-2} (can't yank the bob too fast)

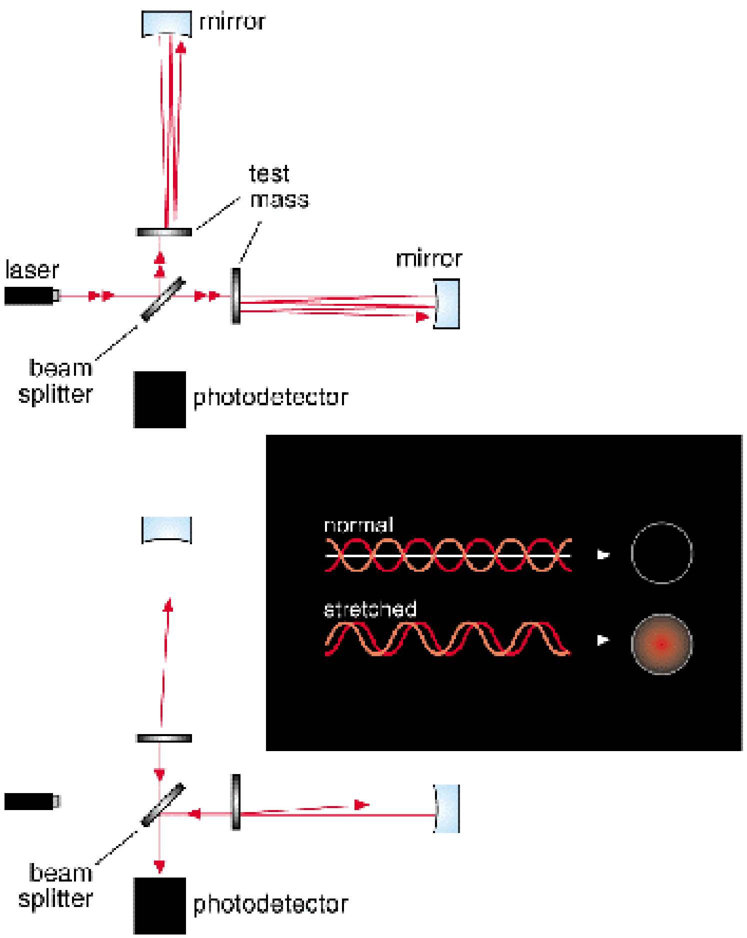
• At such high frequencies,

mass is quiet!





Interferometric phase difference



The effects of gravitational waves appear as a deviation in the phase differences between two orthogonal light paths of an interferometer.

For expected signal strengths, The effect is *tiny*:

Phase shift of ~10^{-10} radians

The longer the light path, the larger the phase shift…

Make the light path as long as possible!

A practical interferometer

- Using a pendulum of length l = 50 cm,
  f_0 ~ 1 Hz, so mass is "free" above ~ 100 Hz

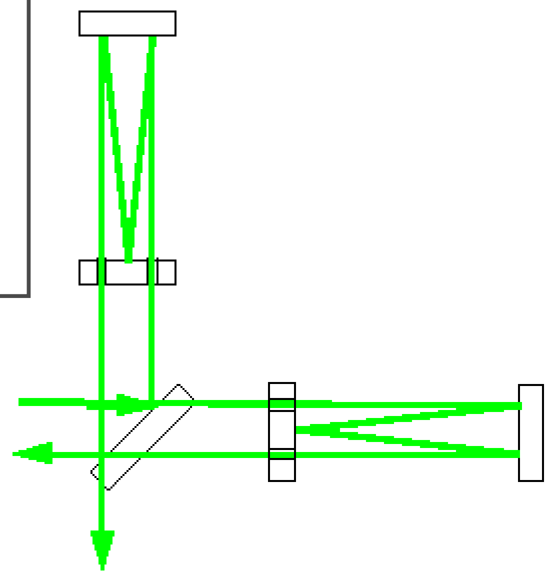
$$f_0 = \frac{1}{2\pi}\sqrt{\frac{g}{l}} \approx 0.7\,\mathrm{Hz}$$

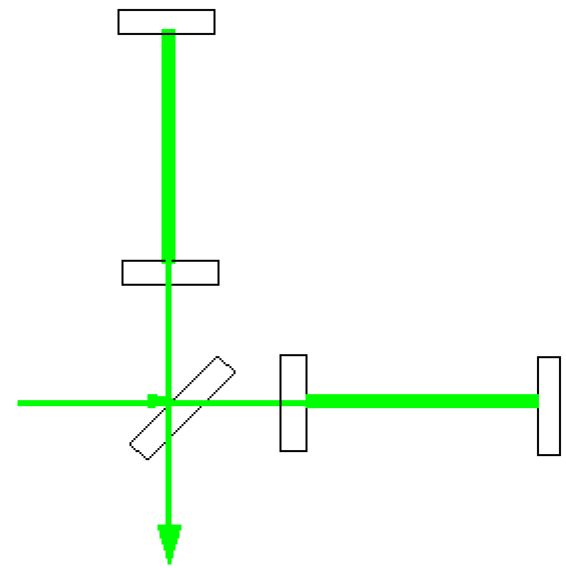
- A GW with $f_g \sim 100$ Hz $\Rightarrow \lambda_g \sim 3000$ km
  produces a tiny strain $h = \Delta L / L$
- *We measure* $\Delta\phi = 4\pi\, \Delta L/\lambda_{laser} = 4\pi\, L\, h / \lambda_{laser}$
  so to measure small *h*, need large *L*
- *But not too large!* If $L > \lambda_g/4$, GW changes sign while laser light is still in arms, cancelling effect on $\Delta\phi$
- Optimal: $L > \lambda_g/4 \sim 750$ km. But not very practical!
- For more practical length (L ~ 4 km), increase phase sensitivity:
  $\Delta\phi = 4\pi\, \Delta L/\lambda_{laser} \Rightarrow \Delta\phi = N(4\pi\, \Delta L/\lambda_{laser})$, *with* $N \sim 200$
- N : Increase number of times light beam hits mirror, so that the light is phase-shifted N times the single-pass length diff ΔL

Light storage: folding the arms

How to get long light paths without making *huge* detectors:

Fold the light path!





Simple, but requires large mirrors; limited τ_{stor}

(LIGO design) $\tau_{stor} \sim 3\ msec$

More compact, but harder to control

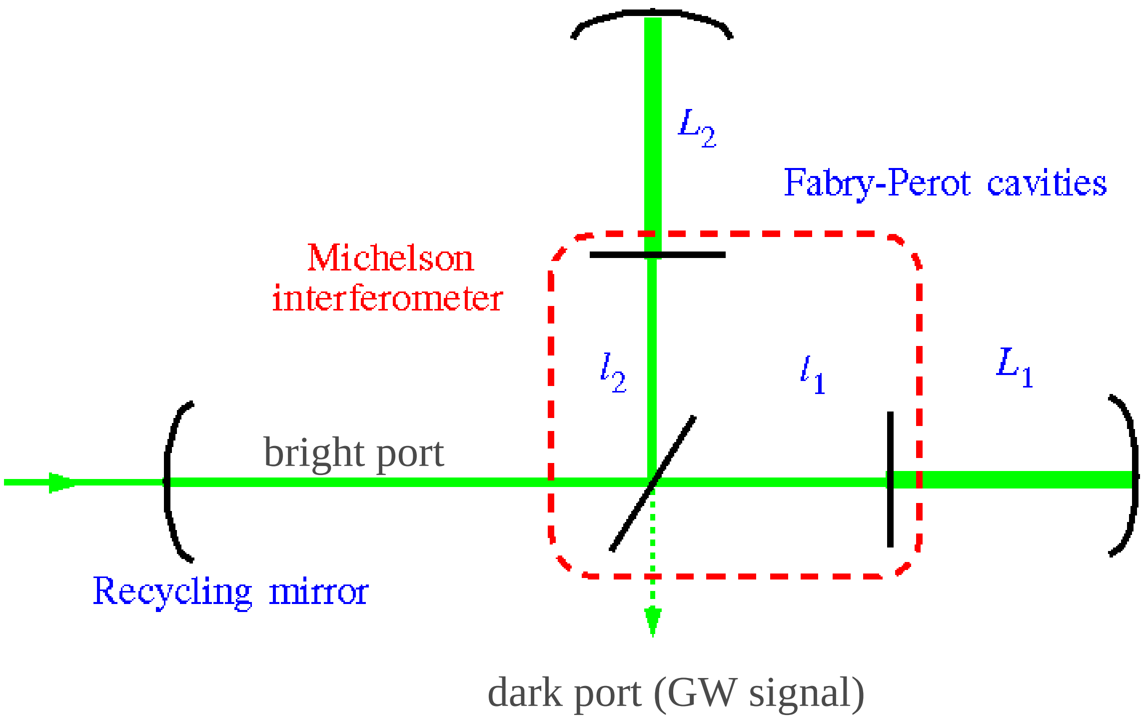
Limit to the path length

- Can't make the light path arbitrarily long!
- Time for light wave front to leave BS and return: $\tau_{stor} \sim 2NL/c$; N is number of round trips in arms
- During that time, GW may reverse sign, canceling the effect on the light phase: $\tau_{rev} \sim T_{period}/2 = 1/2f$
- LIGO is sensitive to GW's of frequency $f < 3000$ Hz but the best sensitivity is at $f_{pole} \sim 100$ Hz ("knee")
- So, keep $\tau_{stor} < 1/2f_{pole}$, or $N < c/4Lf_{pole} \approx 200$ for LIGO
- For LISA, $L=5x10^9$ m, $N=1 \Rightarrow f_{pole} \approx 0.01$ Hz

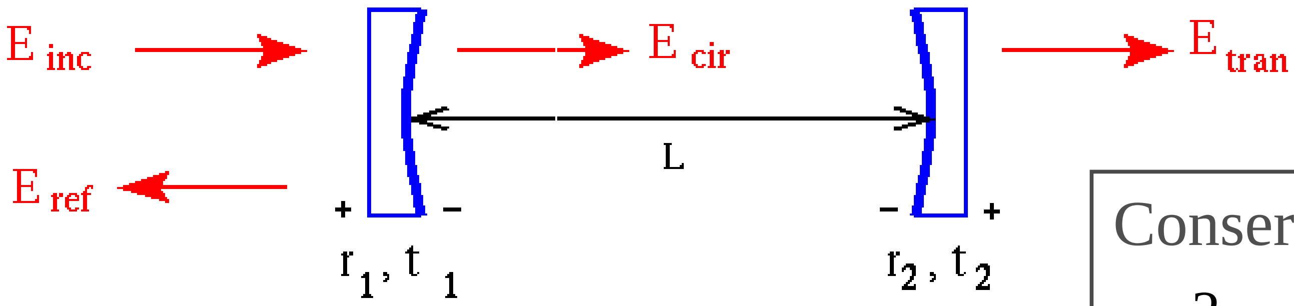
LIGO I configuration

Power-recycled Michelson with Fabry-Perot arms:

- Fabry-Perot optical cavities in the two arms store the light for many (~200) round trips
- Michelson interferometer: change in arm lengths destroy destructive interference, light emerges from dark port
- Normally, light returns to laser at bright port
- Power recycling mirror sends the light back in (coherently!) to be reused



Fabry-Perot Cavities



$$E_{cir} = t_1 E_{inc} + r_1 r_2 e^{-2ikL} E_{cir} = \frac{t_1}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

$$E_{ref} = r_1 E_{inc} - t_1 r_2 e^{-2ikL} E_{cir} = \frac{r_1 - r_2 (1-L) e^{-2ikL}}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

$$E_{tran} = t_2 \ e^{-ikL} E_{cir} = \frac{t_1 t_2 e^{-ikL}}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

Conservation of energy:

$$r_i^2 + t_i^2 + L_i = 1$$

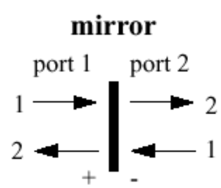
$$R_i + T_i + L_i = 1$$

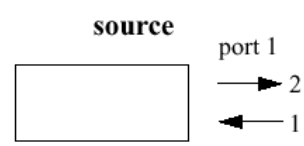
When $2kL = n(2\pi)$, (ie, $L = n\lambda/2$),

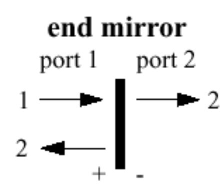
E_{cir} , E_{tran} maximized $\Rightarrow$ resonance!

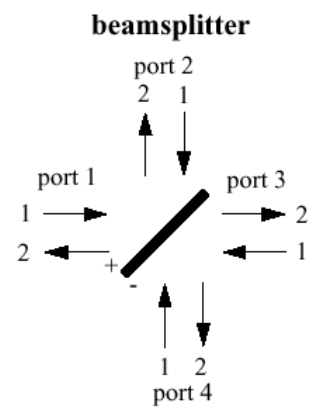
Field equations, dynamics

- Any arbitrary configuration of mirrors, beam splitters, sources, defines a set of static fields, and a set of linear relations between them (which depend on phase advances, reflectivities and transmissivities, etc)
- It is thus easy to solve for all the static fields in any configuration
- Dynamics: shake a mirror (or wiggle a source field) at frequency f, and all the fields respond with a wiggle at that frequency.
- Can then calculate the (complex) transfer function between any mirror and any field
- M. Regehr, ***Twiddle***









$$T(\tilde{x}_{mirr}(f) \rightarrow \tilde{E}_{port}(f))$$

Cavity coupling

$$E_{ref} = \frac{r_1 - r_2(1-L)e^{-2ikL}}{1 - r_1 r_2 e^{-2ikL}} E_{inc}$$

- if $r_1 = r_2(1-L)$, $E_{ref} = 0$ on resonance; optimal coupling
- if $r_1 > r_2(1-L)$, $E_{ref} > 0$ on resonance; under-coupling
- if $r_1 < r_2(1-L)$, $E_{ref} < 0$ on resonance; over-coupling

Free Spectral Range: $f_{FSR} = c/2L$

(eg, for 4 km arms, $f_{FSR} = 37.5\ kHz$)

LIGO: carrier is resonant in arms, sidebands not; f_{SB} far from f_{FSR}

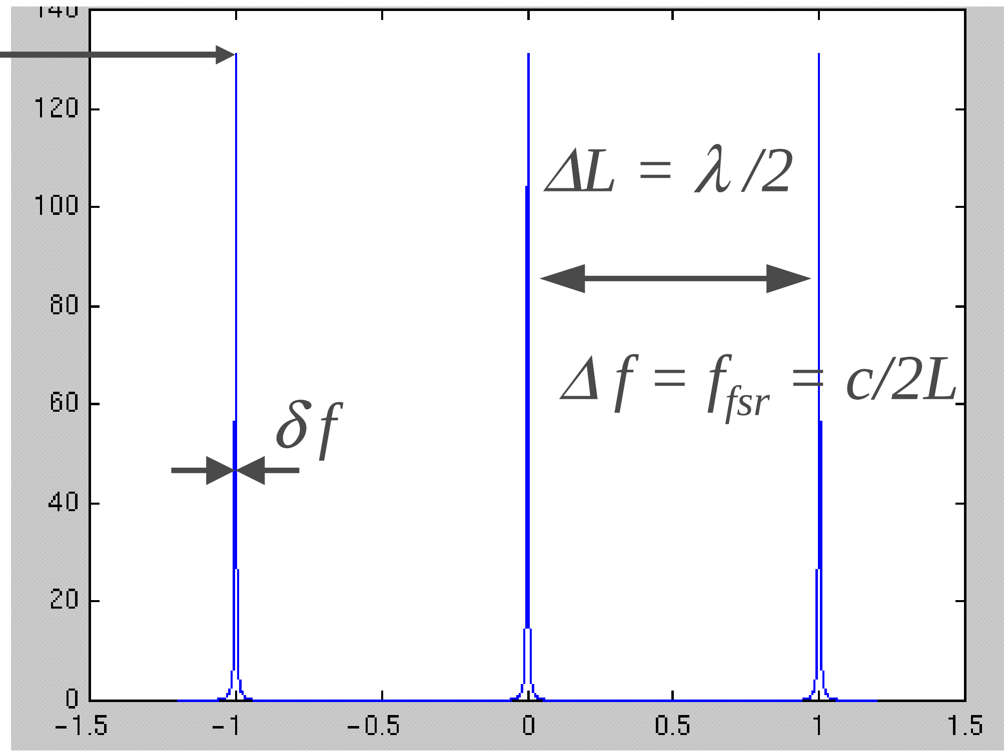
More cavity parameters

- Finesse: peak separation / full width of peak
- Finesse = $F = \frac{\pi\sqrt{r_1 r_2}}{1 - r_1 r_2}$ = 208 for LIGO 4km arms
- Light storage time = $\tau_{stor} = \frac{L}{c}\frac{\sqrt{r_1 r_2}}{(1 - r_1 r_2)}$ = 870 μsec for LIGO arms
- Cavity pole = $f_{pole} = 1/(4\pi\tau_{stor})$ = 91 Hz for LIGO arms
- Cavity gain = $G_{cav} = \left(\frac{t_1}{1 - r_1 r_2}\right)^2$ = 130 for LIGO arms
- Visibility: $V = 1 - P_{min}/P_{max}$, Power in/out of lock
- LIGO 4km arms: $t_1^2 = 0.03$, $r_2^2 \approx 0.99997$

FP circulating field

Power Gain

$$\left|\frac{E_{circ}}{E_{in}}\right|^2$$



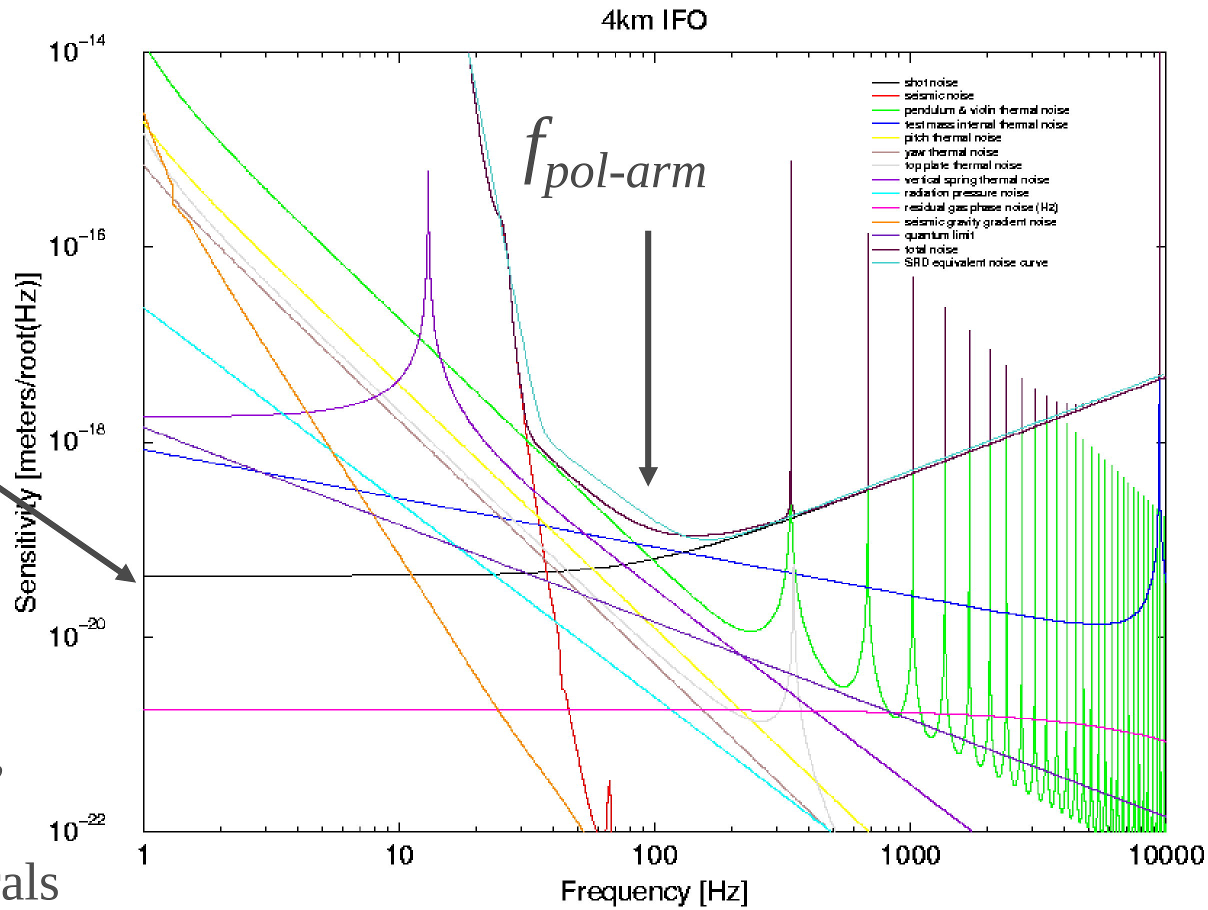
$$Finesse = \delta f / f_{fsr}$$

$$\Delta\nu = \Delta(2kL)/2\pi = \Delta f/f_{fsr} = \Delta L/(\lambda/2)$$

Arm cavity parameters and LIGO sensitivity

As r_{ITM} is increased,
G_{arm} is increased,
$f_{pol\text{-}arm}$ is decreased.

$$h_{dc} \sim 1/\sqrt{G_{arm} P_{laser}}$$



Given other noise sources (seismic, thermal), choose r_{ITM} to optimize Sensitivity to binary inspirals

Contrast

- Contrast is a measure of how perfectly light interferes at beamsplitter

$$C = \frac{P_B - P_D}{P_B + P_D}$$

- P_D is minimum carrier power at dark port with both arms in lock
- P_B is maximum carrier power at bright port with both arms out of lock
- Contrast defect 1-C is non-zero due to mode mismatch between arms; imperfect mirrors; etc
- This produces excess noise at GW output, reducing S/N

Power recycling

Optimal sensitivity requires high laser power

• predicted sources require shot noise of ~300 W on BS

• suitable lasers produce ~10 W, only ~6W at IFO input

Power Recycling: Make resonant cavity of IFO and *recycling* mirror

• use IFO at `dark fringe'; then input power reflected back

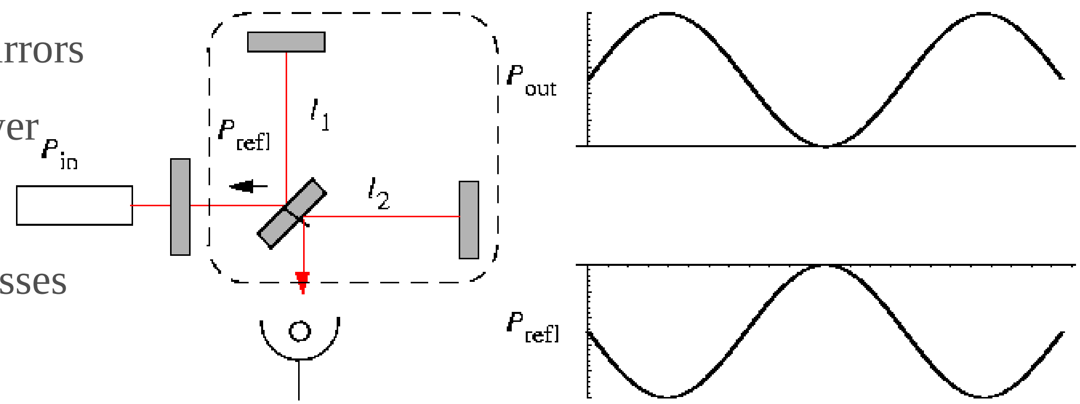
• known as Recycling of light (Drever, Schilling)

• Gain of ~40 possible, with losses in real mirrors

• allows present lasers to deliver needed power

• increases stored energy

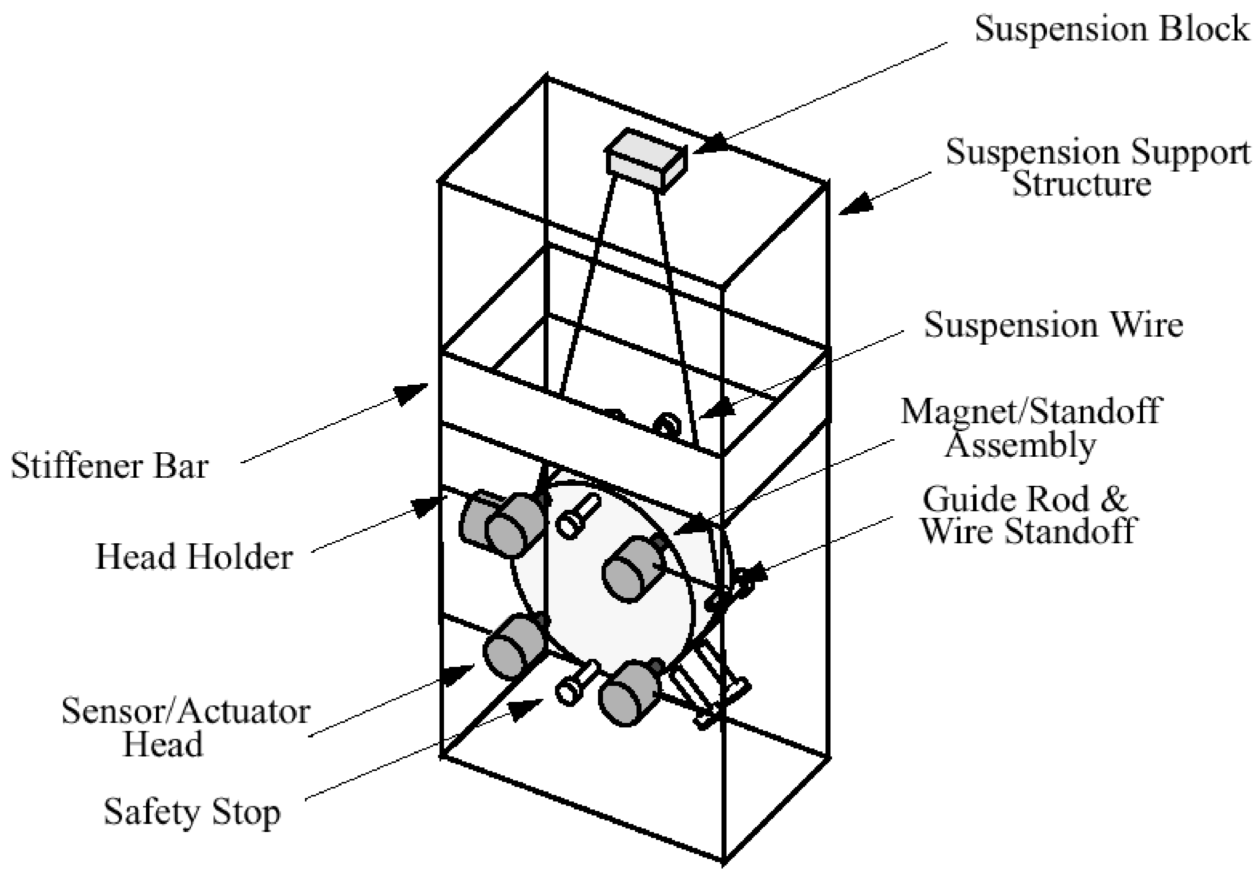
• just extract small amount (or so) if GW passes



Mirror control

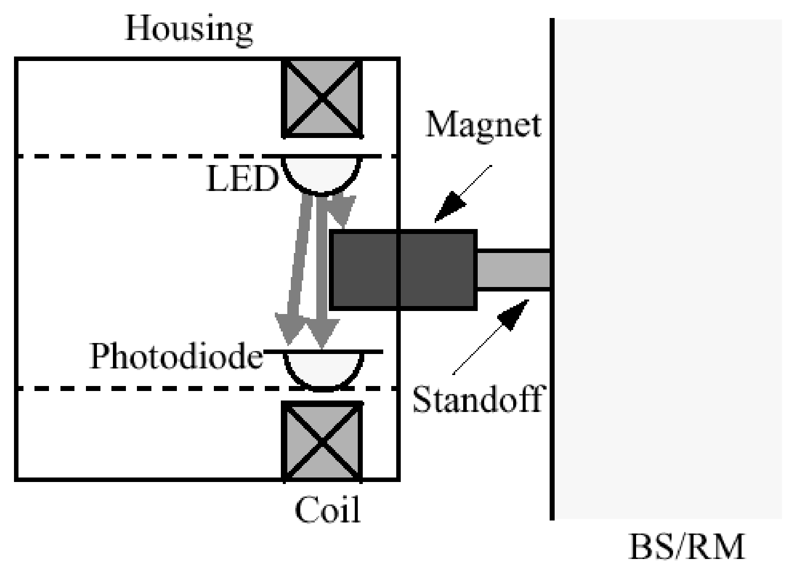
- Seismic isolation system, and pendulum, keep the mirror motion to a minimum.
- Now the mirrors are not being kicked around by the environment (at high frequencies);
- But, being free, they may not be where you need them to be!
- Need active control system to keep mirrors at set points (at/near DC), to keep F-P cavities resonant,
- Without injecting noise at high frequencies
- $\Rightarrow$ Carefully designed feedback servo loops

LIGO I Suspensions



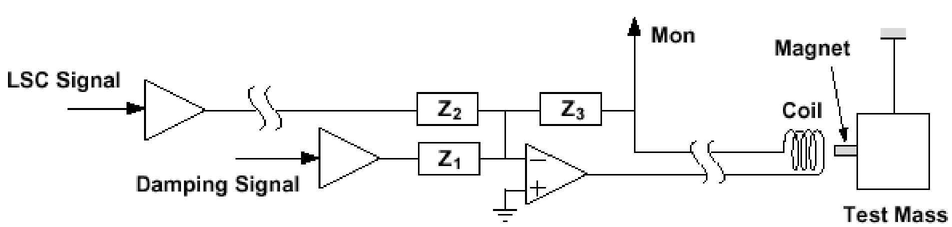
OSEMs

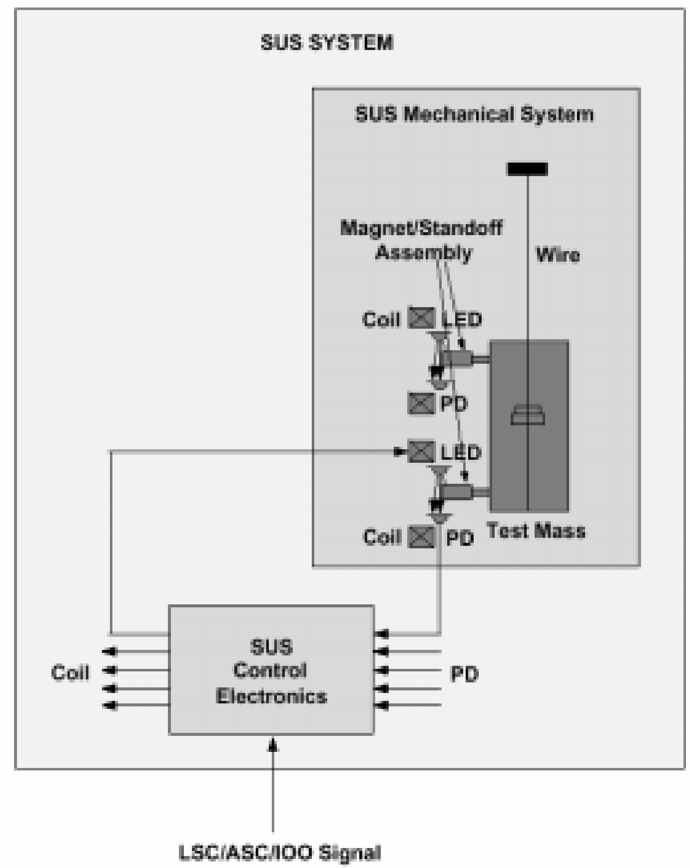
- Five magnets glued to fused Si optic
    - (this ruins the thermal noise properties of the optic – a big problem!)
- LED/PD pair senses position
- Coil pushes/pulls on magnet, against pendulum



Suspension control system

- Each suspension controller handles one suspension (5 OSEMs)
- Local velocity damping
- Input from LSC and ASC to fix absolute position, pitch, yaw of mirror to keep cavity in resonance

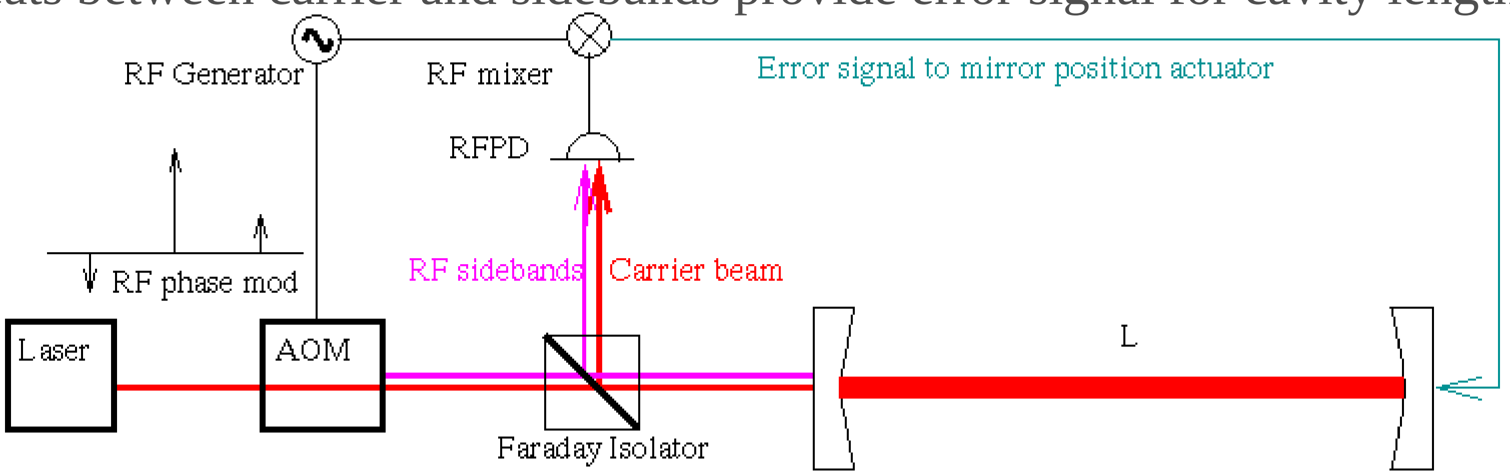




Cavity control

Pound-Drever (reflection) locking used to control lengths of all the optical cavities in LIGO

- Phase modulate incoming laser light, producing RF sidebands
- Carrier is resonant in cavity, sidebands are not
- Beats between carrier and sidebands provide error signal for cavity length



Phase modulation of input beam

Phase modulation adds sidebands to the beam:

$$E_{inc} = E_{laser} e^{i(\omega t + \Gamma \cos \Omega t)} \approx E_{laser} e^{i\omega t} \left(J_0(\Gamma) + J_{+1}(\Gamma) e^{i\Omega t} + J_{-1}(\Gamma) e^{-i\Omega t} \right)$$

Ω = RF modulation frequency ($\Omega /2\pi \sim 30$ MHz)

Γ = modulation depth

J_i = Bessel functions; $J_{\pm 1} \approx \pm \Gamma/2$ for $\Gamma<1$

$$E_{ref} = \left(E_0^{ref} + E_{+1}^{ref} e^{i\Omega t} + E_{-1}^{ref} e^{-i\Omega t} \right) e^{i\omega t}$$

Arrange the length of the cavity, and the value of Ω, so that

•carrier is resonant in FP cavity, sidebands are not,

•so they have different reflection coefficients

•phase of carrier is sensitive to length changes in cavity, sidebands are not

Demodulation

$$S_{ref} = \left(\left|E_0\right|^2 + \left|E_+\right|^2 + \left|E_-\right|^2 \right) + 2\,\mathrm{Re}\left(\left(E_0^* E_+ + E_0 E_-^* \right) e^{i\Omega t} \right) + 2\,\mathrm{Re}\left(E_+^* E_- e^{i2\Omega t} \right)$$

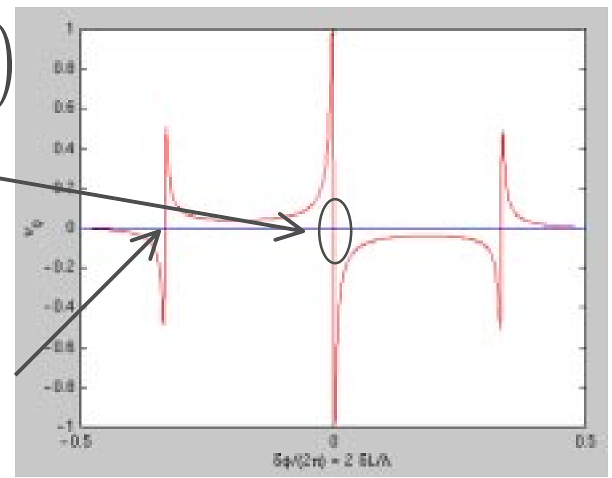
Use an electronic "mixer" to multiply this by

cosΩt or sinΩt, average over many RF cycles, to get:

- In-phase demodulated signal $\quad v_I = 2\,\mathrm{Re}\left(E_0^* E_+ + E_0 E_-^* \right)$
- Quad-phase demodulated signal $\quad v_Q = 2\,\mathrm{Im}\left(E_0^* E_+ + E_0 E_-^* \right)$

Which are sensitive to length of cavity (very near resonance)

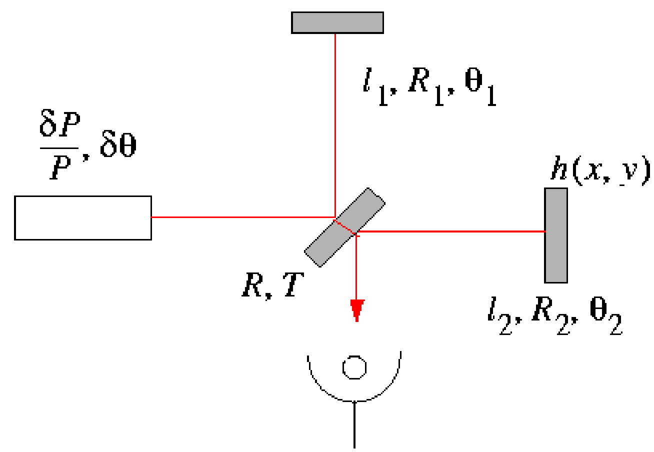
And can be used as an *error signal* to control cavity length



Sideband resonant - error signal has wrong sign

Schnupp Asymmetry

GW signal (L_-) is measured using light *transmitted* to dark port (**Schnupp locking**, as opposed to reflection locking)



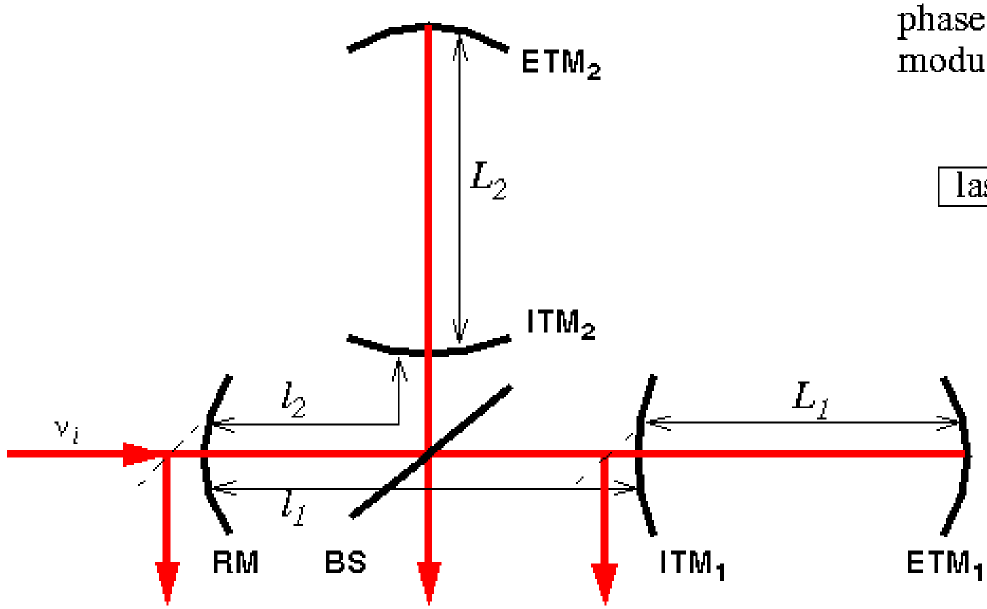
• In absence of GW, dark port is *dark*; carrier power ~ $\sin^2(\Delta\phi)$, quadratic in $\Delta\phi = 2kL_-$ for small signal

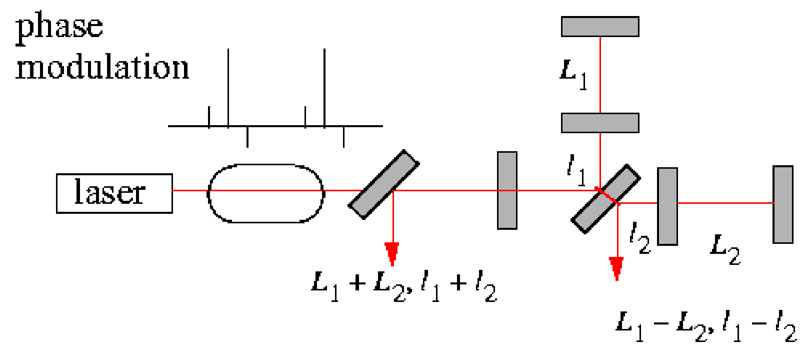
• Add Schnupp (Michelson) asymmetry: $l_1 \neq l_2$; port is still dark for carrier ($l_1 = l_2 \mod \lambda_c$), but sidebands leak out to dark port PD

• Error signal is then *linearly* proportional to amount of carrier light (GW signal)

$$v_Q = 2\,\mathrm{Im}\left(E_0^* E_+ + E_0 E_-^*\right)$$

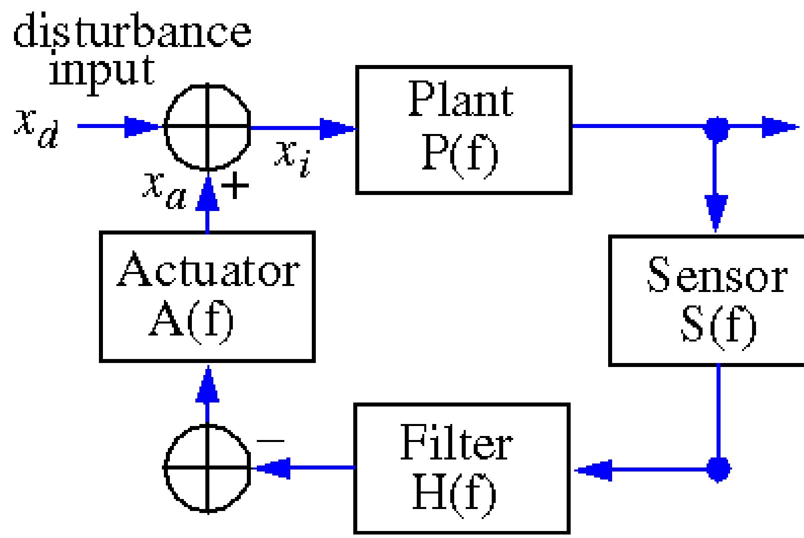
The control problem in LIGO





- **Four interferometer lengths ⇒ four sensors/actuators**
- **Ten mirror angles ⇒ ten sensors/actuators**

Elements of a control system



$$x_a = -PSHAx_i$$

$$x_i = x_d + x_a$$

$$x_i = \frac{x_d}{1 + G(f)}$$

where

$$G(f) = PSHA$$

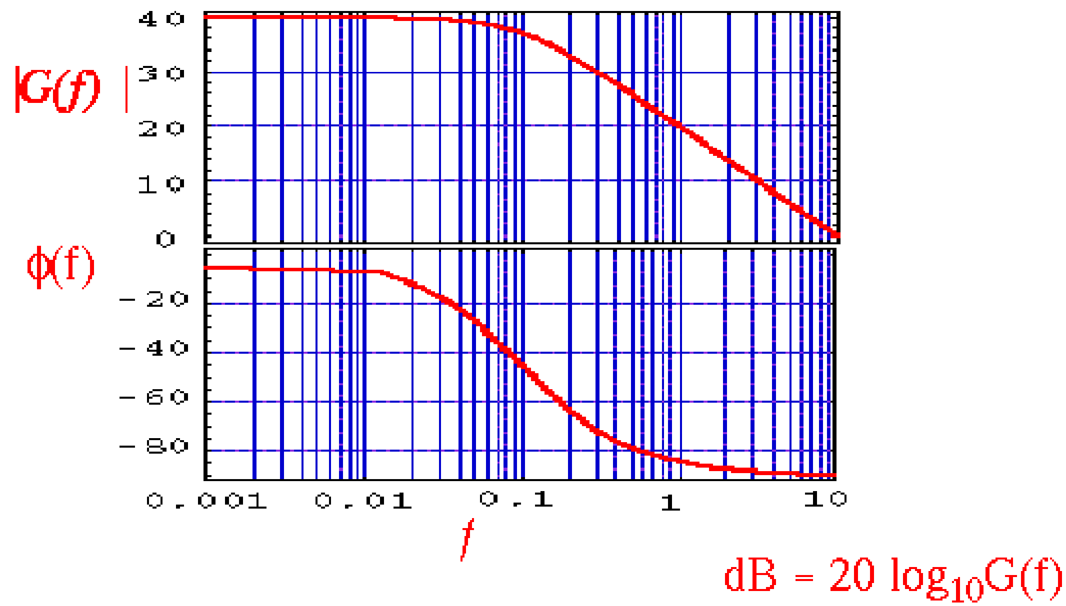
- Time delays are inevitable; Plant, and control system, response is best characterized in *frequency domain* (as in communication engineering)

- When $G(f) >> 1$ then $x_i << x_d$,

  Plant input is much smaller than original disturbance

Controls terminology

- Transfer function (frequency response) magnitude [$|G(f)|$] and phase [$\phi(f)$] of output when input is a sinusoid of unit magnitude at frequency f
- Bode Diagram:
- Pole: magnitude falls off with f ($f>f_o$), phase lags
- Zero: magnitude increases with f ($f<f_o$), phase leads



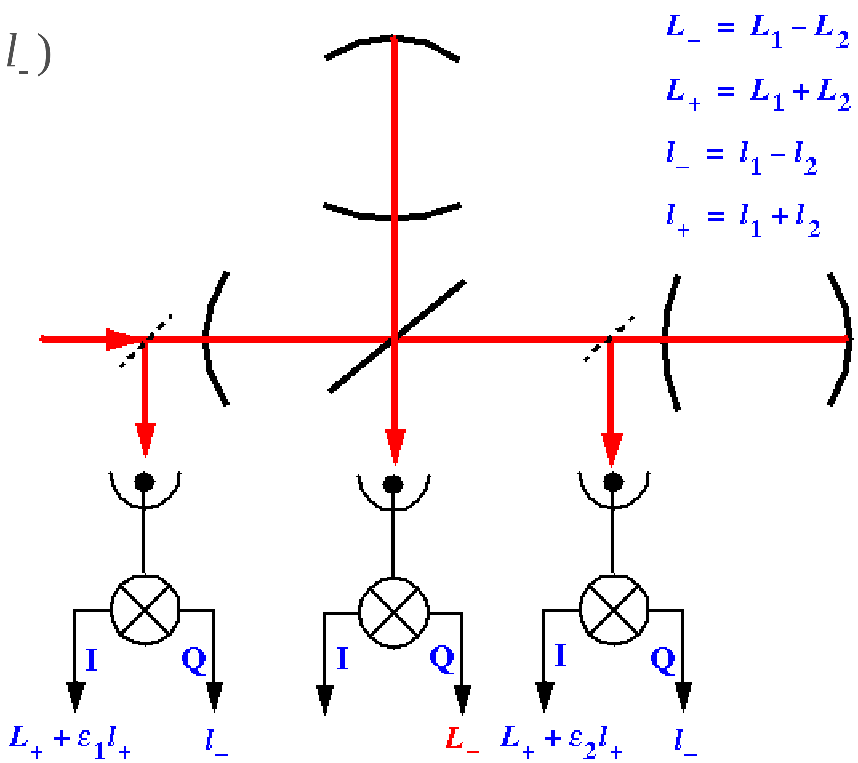
Want high gain at frequencies where control is effective (sensor fidelity, actuator response), and phase < 90°;

Then fast rolloff of gain to avoid injecting noise

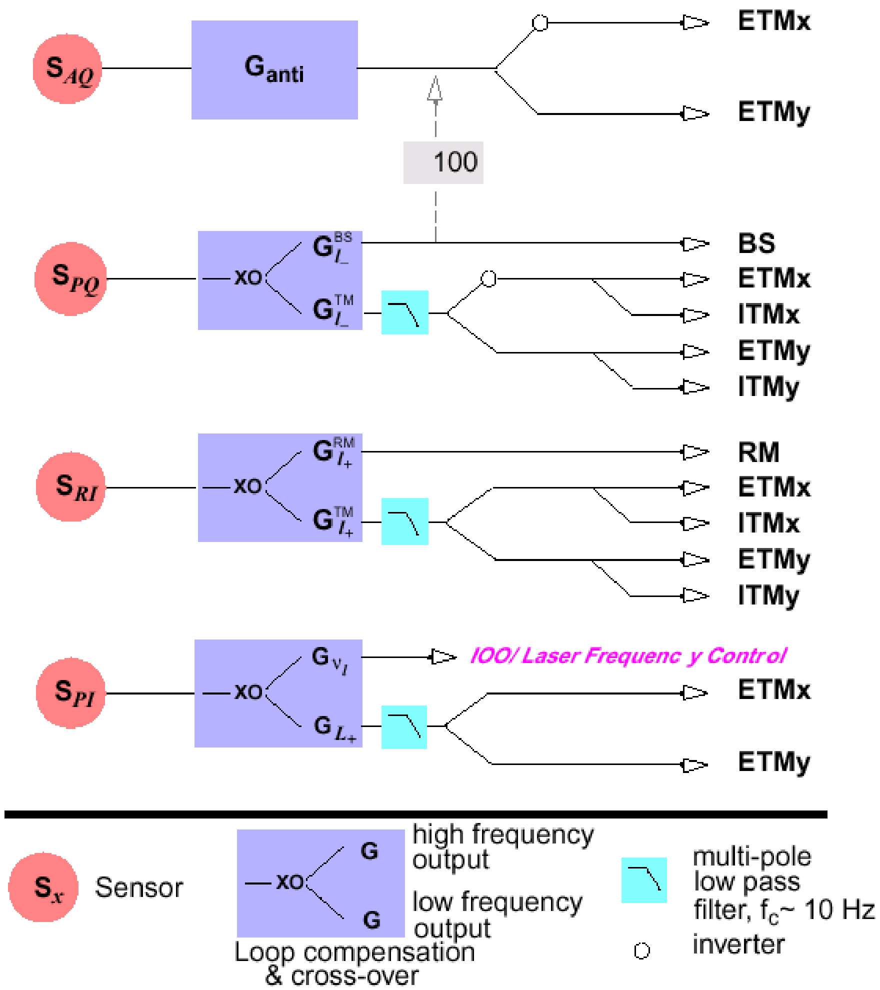
Length Sensing and Control (LSC)

Length control:

- 4 length degrees of freedom (L_+, L_-, l_+, l_-)
- L_- = gravity wave signal
- l_- = Michelson dark fringe (*contrast*)
- Diff mode (L_-, l_-) controlled by quad-phase demod signal
- Common mode (L_+, l_+) controlled by in-phase demod signal
- Need *gain hierarchy* to control l_+
- Hold lengths to 10^{-9} m in presence of 10^{-5} m (seismic) noise



Servo gains and bandwidth



Lock Acquisition

- Servo control loops work (acquire lock) only in the linear regime, when mirrors are very close to resonance.
- When we start, or after any big bump, mirrors are far from resonance, swinging freely.
- If mirrors are moving slow enough, as they pass through resonance, the servo can grab and hold the mirror in place (while keeping it free for $f > 100$ Hz!).
- Each of the 4 length control loops depends on the position of all the mirrors. Must carefully choose the order in which lock is acquired on the length degrees of freedom, especially given the gain hierarchy.
- Lock acquisition must be fast and robust, to keep IFO fully locked as much as possible (lock duty cycle).
- This is one of the biggest challenges in making LIGO work!

Lock Acquisition sequence

- A sequence of steps, each with changes to control loop gains and signs
- Automated lock acq procedures guide the transitions
- MTTL: $\tau_{lock} \sim \dfrac{\lambda/2}{v_{thr} P(v < v_{thr})}$