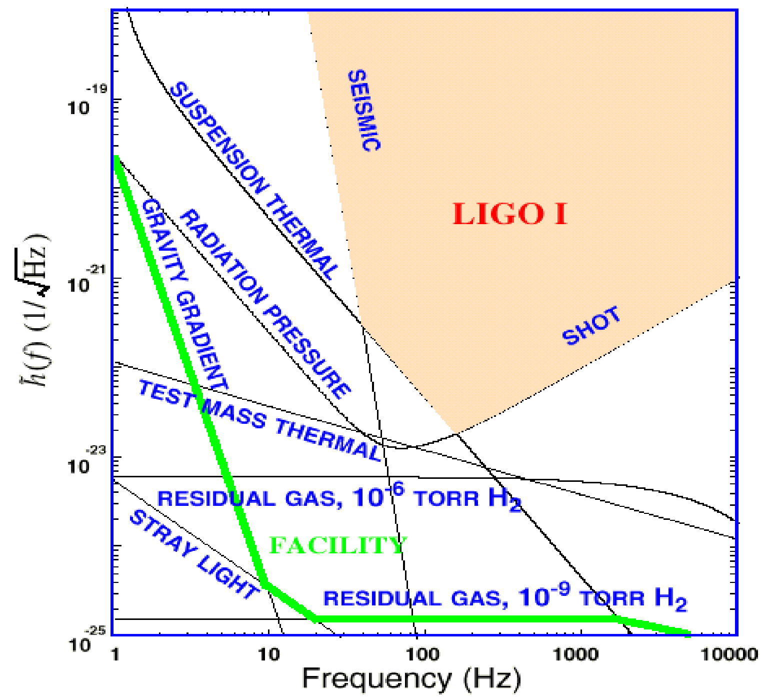
Physics of LIGO
Lecture 3

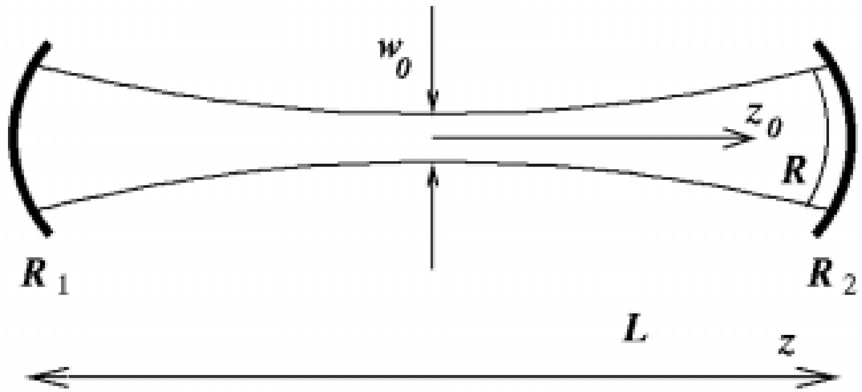
- Transverse modes
- Noise
- LIGO II



Transparencies will be posted at:
http://www.ligo.caltech.edu/~ajw
And in the DCC!

Transverse profile of beam in FP cavity: Hermite-Gaussian modes

The transverse profile of a beam resonant in a FP cavity is completely determined by L, R_1, R_2, λ



Beam waist: $w_0 = \lambda/\pi\ f(L,R_1,R_2)$

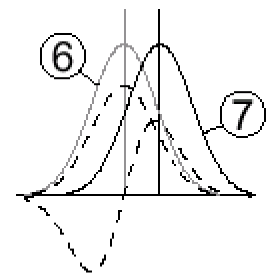
Rayleigh length: $z_0 = \pi w_0^2/\lambda$

- beam waist at position z : $w(z) = w_0\sqrt{1+\left(\frac{z}{z_0}\right)^2}$
- beam ROC at position z : $R(z) = z + \frac{z_0^2}{z}$
- beam *Guoy phase* at position z : $\eta(z) = \tan^{-1}\left(\frac{z}{z_0}\right)$

Hermite Gaussian Modes

$$E(x,y,z) = \sum a_{mn} U_{mn}(x,y,z), \quad U_{mn}(x,y,z) = U_m(x,z) U_n(y,z) e^{-ikz}$$

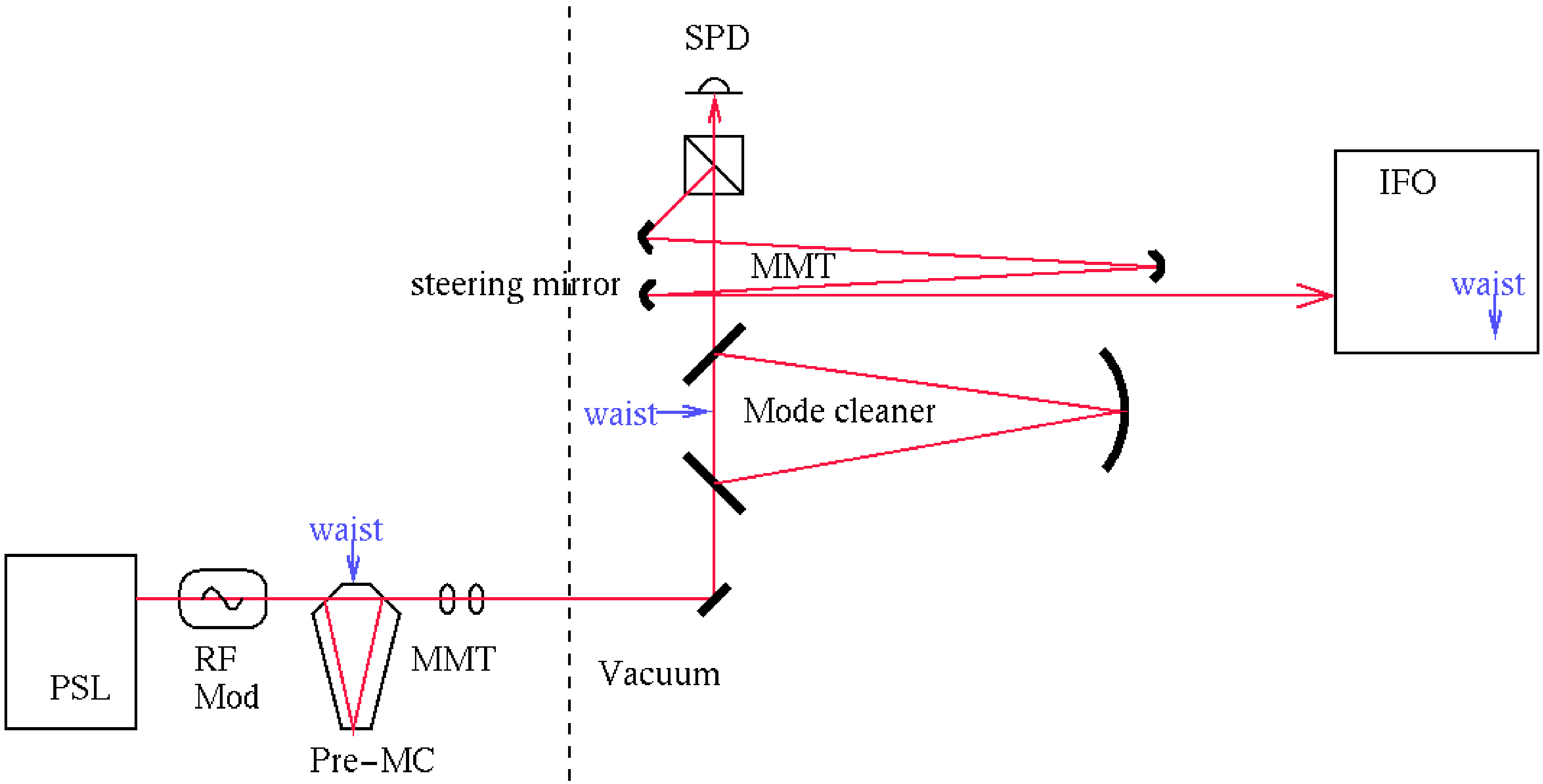
- U_{mn} are *Hermite-Gaussian* or TEM_{mn} transverse modes



$$U_m(x,z) = \sqrt{\frac{\sqrt{2/\pi}}{2^m m! w(z)}} \, H_m\left[\frac{\sqrt{2}x}{w(z)}\right] e^{-x^2\left[\frac{1}{w(z)^2} + \frac{ik}{2R(z)}\right]} e^{i(m+\frac{1}{2})\eta(z)}$$

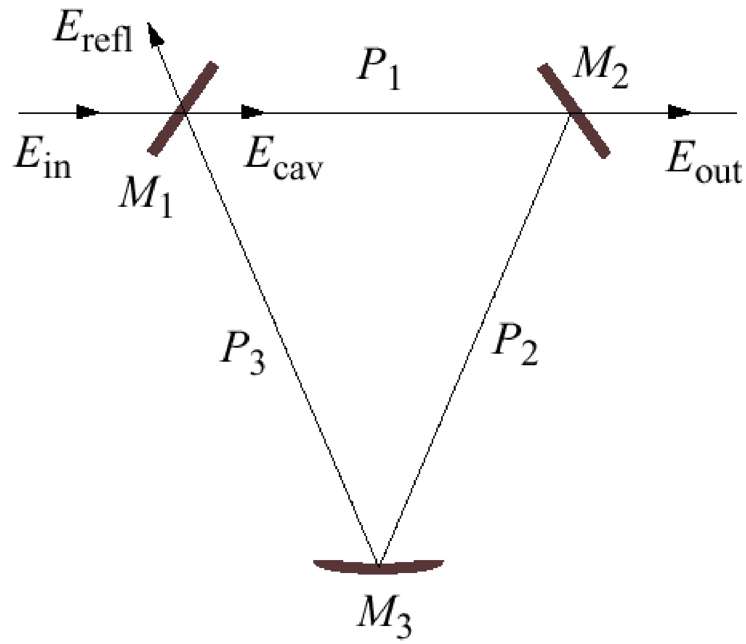
- In a perfect IFO (perfect mirror ROCs, perfect alignment, all cavities *mode matched*), only TEM_{00} mode exists.
- In LIGO cavities, all higher order modes (TEM_{01}, TEM_{10}, etc) represent *beam loss* and *excess noise*;
- Must control mirror imperfections, pitch and yaw, input beam position and direction, mode matching between cavities, *etc*, to minimize this.

Input Optics (IOO)



Mode Cleaner

- Filter out HOMs
- Filter frequency noise from laser
- Triangular MC ensures that reflected light doesn't head back to laser, accessible for reflection locking
- M_3 is very curved, to ensure tight beam (small g-factor)
- Waist is halfway between M_1 and M_2

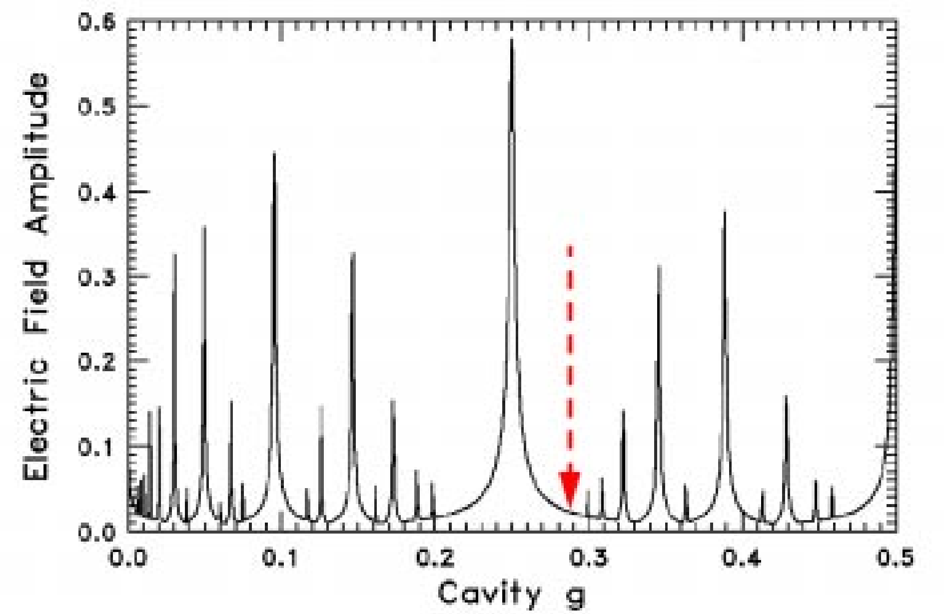


Cavity g-factor

• LIGO Mode Cleaner has two flat and one curved mirror.
• The radius of curvature (ROC) of curved mirror determines g-factor.

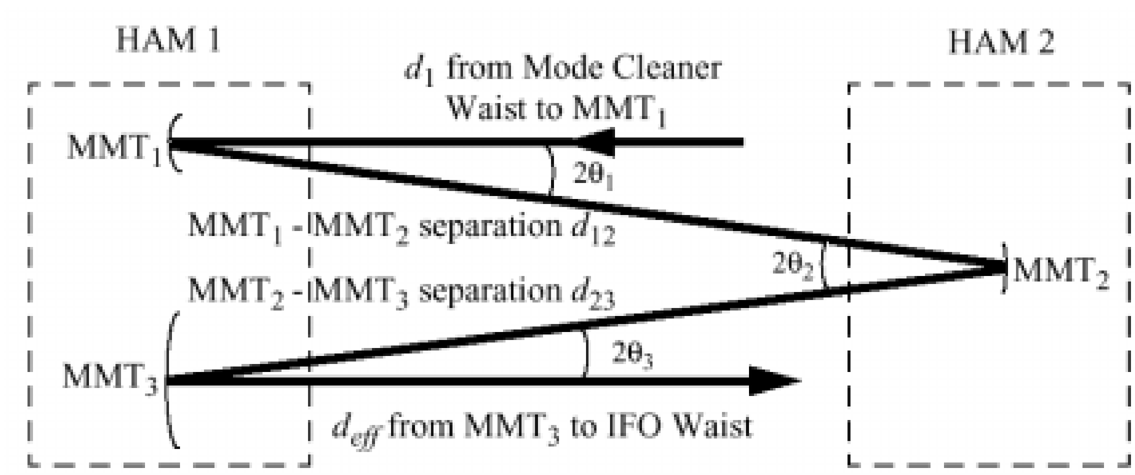
$$g = \left(1 - \frac{L}{R}\right)$$

• $g<1$ gives a stable cavity (beam does not diverge as in $R<0$, $g>1$).
•As g-factor decreases below 1, Guoy phase difference of HOMs gets larger; only one mode resonates in cavity
•g-factor of FP cavity with two curved mirrors is $g = g_1 g_2$, with $g_i=(1-L/R_i)$



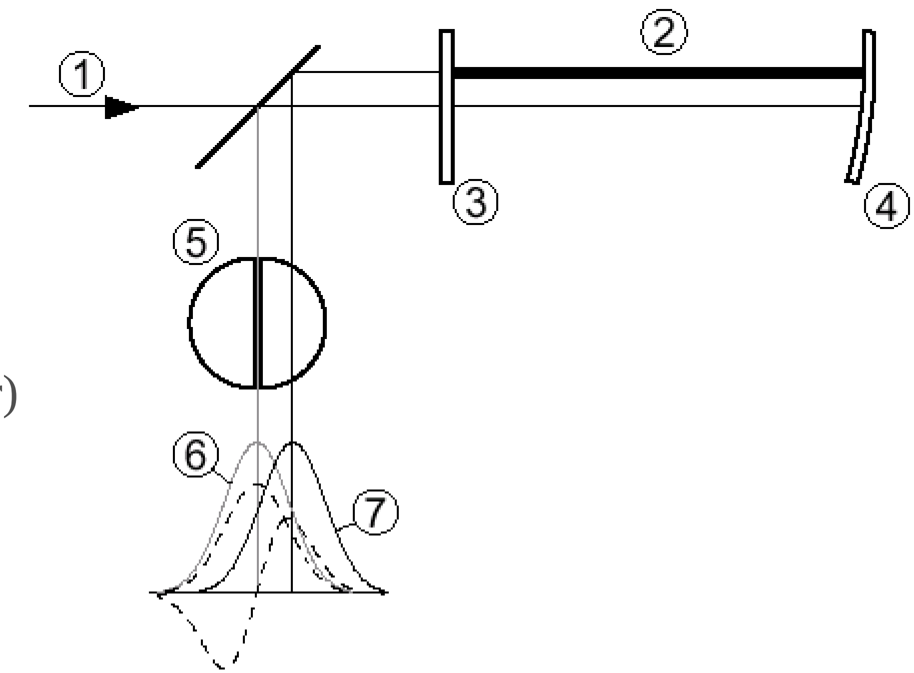
Mode Matching telescope

- Mode Cleaner defines the gaussian beam, with waist in the MC
- The IFO gaussian beam has a waist in the arm cavity
- Need optical telescope to match these beams
- LIGO uses suspended mirrors, rather than transmissive lenses, to minimize noise
- Last MMT mirror steers the beam into IFO



Alignment signals from a misaligned F-P IFO

1) incoming laser beam
2) resonant cavity mode
3) Partially transmitting ITM
4) Tilted ETM
5) Segmented PD (Wavefront Sensor)
6) Reflected SB
7) Reflected carrier light (solid) with modal decomposition (dashed)



Sense and servo out the higher order modes (TEM_{01}, TEM_{10})

Need to control mirror angles to ~ 10^{-8} rad!

NOISE in GW detectors

- After ~ 40 years of effort, no one has detected a GW!
- Why? Noise levels in detectors exceed expected signal; *insufficient sensitivity*
- Want to detect GW strain *h*; can express detector noise in terms of equivalent *h* sensitivity
- Most of the effort in GW detection has gone into *understanding and reducing noise* to the fundamental quantum limit (and beyond!)
- We are the beneficiaries of that pioneering and frustrating work: on the threshold of doing what sounds almost impossibly hard!

NOISE SOURCES IN THE DETECTOR

- Noise $\Rightarrow$ signals which appear in detector as GWs but are imposters
- Three categories:
- Displacement noise $\Rightarrow$ moves mirrors (path length changes)

  $\delta x = L\ \delta h$, so to achieve $h \approx 10^{-21}/\sqrt{Hz}$ with L = 4km,

  $$\Rightarrow \delta x \approx 10^{-18}\ m/\sqrt{Hz}$$

  (*cf*: diameter of proton is 10^{-15} m)

- Phase noise $\Rightarrow$ changes the phase of the light:

  $\delta\phi = 4\pi NL\ \delta h / \lambda$, with $N \approx 100$ and $\lambda \approx 1.064\ \mu m$,

  $$\Rightarrow \delta\varphi \approx 10^{-10}\ rad/\sqrt{Hz}$$

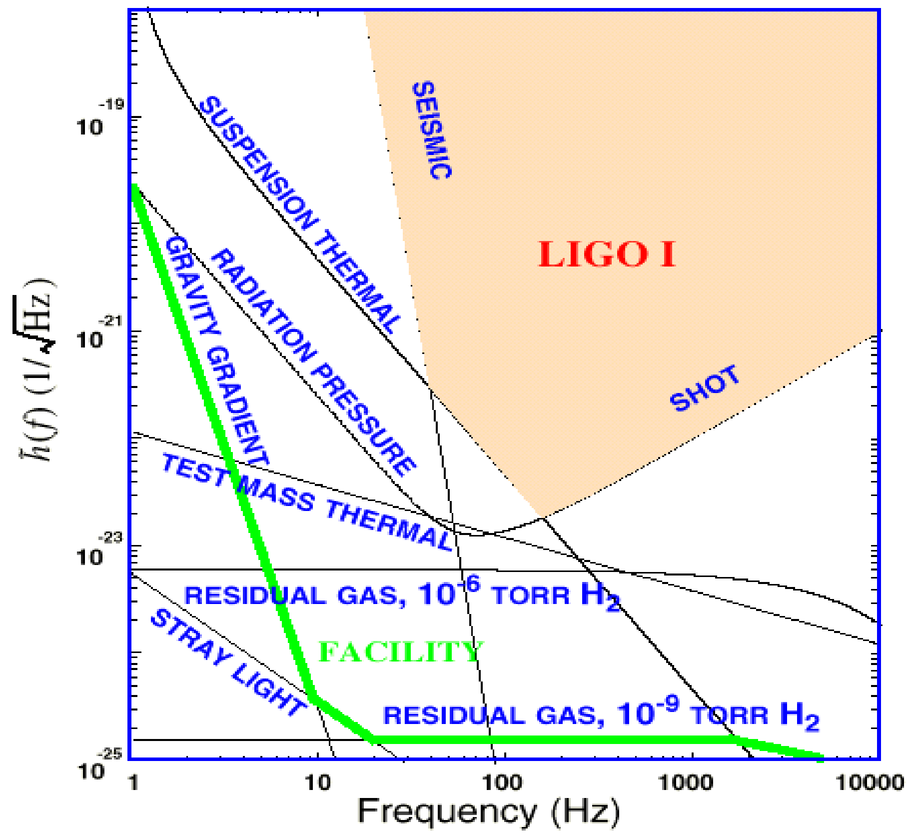
- Technical or instrumental noise (electronics, EMF pickup, *etc*)

  must engineer IFO to keep this *below* the fundamental noise!

LIGO I noise floor

- **Interferometry is limited by three fundamental noise sources**
  - ➢ **seismic noise at the lowest frequencies**
  - ➢ **thermal noise at intermediate frequencies**
  - ➢ **shot noise at high frequencies**

- **Many other noise sources lurk underneath and must be controlled as the instrument is improved**



Displacement noise

Displacement noise in each of the 4 test masses:
- seismic and other environmental disturbances
- suspension thermal noise
- test mass thermal noise

is random and uncorrelated,
resulting in an equivalent strain noise of:

$$h = \Delta L / L = \left[(z_{ETMx} - z_{ITMx}) - (z_{ETMy} - z_{ITMy}) \right] / L$$

$$\Rightarrow \partial h = \left[(\partial z_{ETMx} - \partial z_{ITMx}) - (\partial z_{ETMy} - \partial z_{ITMy}) \right] / L$$

$$\Rightarrow h_{rms} = 2 z_{rms} / L$$

Phase sensing shot noise

- We detect GW strain by its effect on light phase:

$$\Delta\phi = 2k\Delta L = 2k\,L\,\boldsymbol{h}$$

- We detect light phase shift via its "beat" with sidebands
- RF-demodulated power at Asymmetric Port Photodiode (APD)

  $P_{APD} = P_{laser}\, T(f)\, \Delta\phi$, where $T(f) = d(P_{APD}/P_{laser}) / d(\Delta\phi)$

  is (unitless) transfer function of the IFO (proportional to G_{prc}, G_{arm})
- Sensitivity to small $h \Rightarrow$ small power levels at APD:
- $\delta h = \delta P_{APD} /(P_{laser}\, T(f)\, 2\, k\, L)$
- Laser power comes in discrete packets (photons)
- Quantum fluctuations $\Rightarrow$ photon number fluctuations in P_{APD} obeying Poisson statistics: *shot noise*

  (uncertainty in power due to counting statistics)
- Equivalent to "standard" quantum limit on strain sensitivity

Sensing limits

Photon shot noise:

$$E_{APD} = P_{APD}\tau_{\text{int}} = N_{photon}(h_{Pl}\, c/\lambda)$$

uncertainty in intensity due to counting statistics: $\Rightarrow \delta P_{APD} = \sqrt{P_{APD} h_{Pl}\, c/\lambda \tau_{\text{int}}}$

can solve for equivalent strain:

$$h_{shot} = \frac{\delta L}{L} = \frac{1}{L}\sqrt{\frac{h_{Pl} c\lambda}{2\pi T(f) P_{laser}}}$$

Note: scaling with $1/\sqrt{P_{laser}}$; gives requirement for laser power

Radiation Pressure

quantum limited intensity fluctuations anti-correlated in two arms

photons exert a time varying force, spectral density

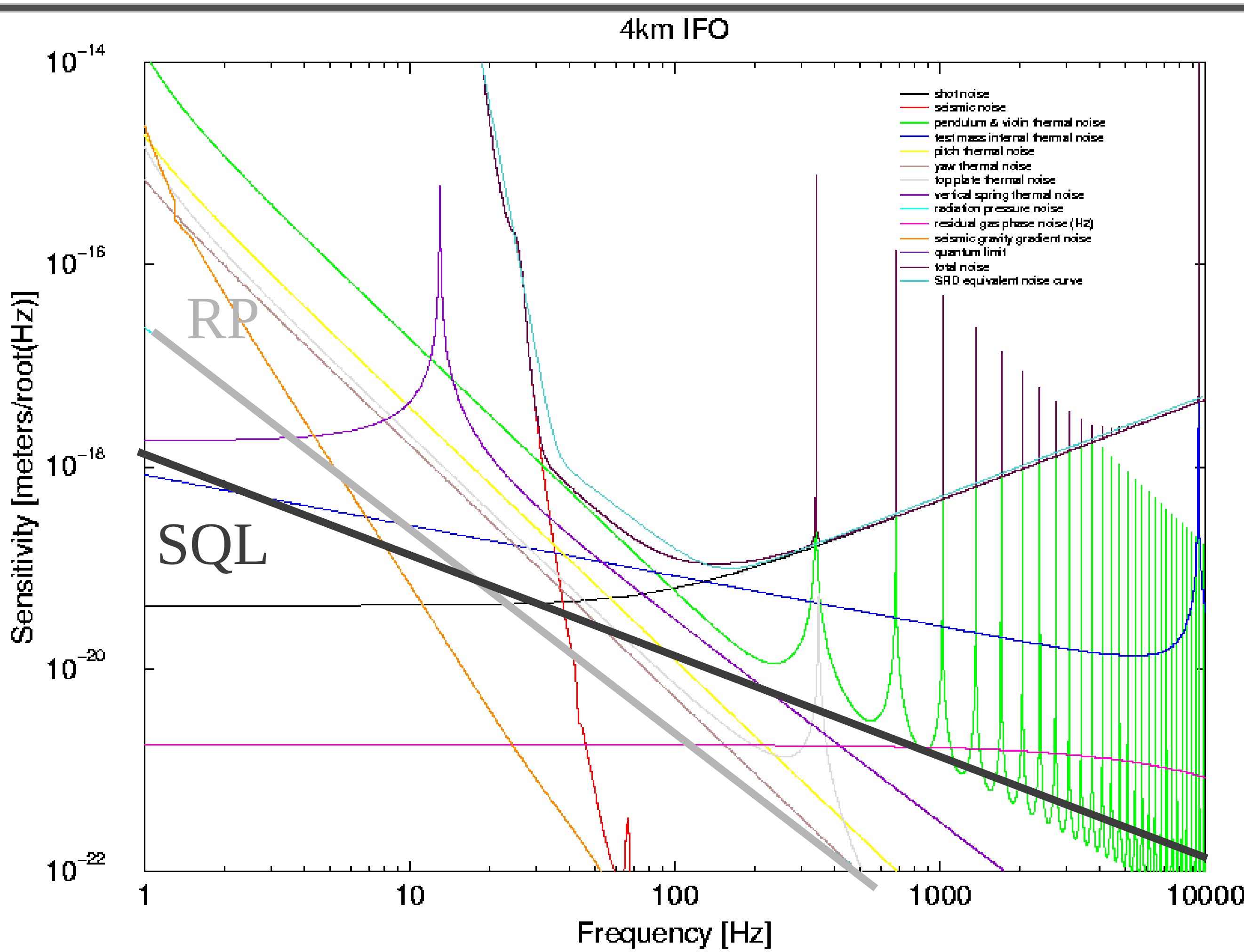
results in opposite displacements of *each* of the masses; strain $h_{rp} = \frac{\delta L}{L} = \frac{2}{L}\frac{1}{mf^2}\sqrt{\frac{h_{Pl} T(f) P_{laser}}{8\pi^3 c\lambda}}$

NOTE: scaling with $\sqrt{P_{laser}}$, scaling with the arm length

Total optical readout, or quantum noise:

quadrature sum $h_q = \left(h_{shot}^2 + h_{rp}^2\right)^{1/2}$; can be optimized

Optical readout noise



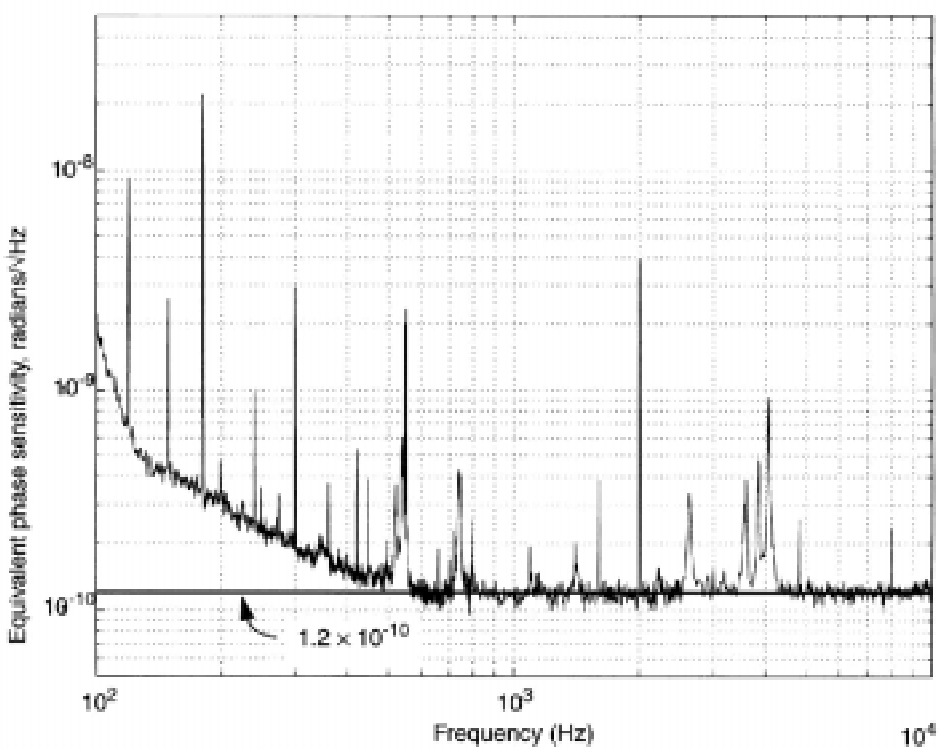
Optical readout noise:

$$h_{ro}(f) = \sqrt{h_{shot}^2(f) + h_{rp}^2(f)}$$

Optimize h_{ro} wrt P_{laser} at each point in f; Locus of points is the Standard Quantum Limit, Obtainable from Heisenberg Uncertainty

$$h_{SQL} = \frac{1}{\pi f L}\sqrt{\frac{\hbar}{m}}$$

Phase Noise
splitting the fringe



- spectral sensitivity of MIT phase noise interferometer (PNI)
- above 500 Hz, shot noise limited near LIGO I goal
- additional features are from 60 Hz powerline harmonics, wire resonances (600 Hz), mount resonances, etc

Thermal displacement noise

Mechanical systems excited by the thermal environment

results in physical motions of the tests masses

Each normal mode of vibration has $k_B T$ of energy; for a SHO, $x_{rms} = \sqrt{\left\langle (\delta x)^2 \right\rangle} = \sqrt{k_B T / k_{spring}}$

An extended object has many normal modes at discrete frequencies;

each will experience thermal excitation.

Dissipation causes the energy, and fluctuations in position, to spread over a range of frequencies, according to Fluctuation-Dissipation theorem:

$\tilde{x}(f) = \frac{1}{\pi f} \sqrt{\frac{k_B T}{\Re(Z(f))}}$, $\Re(Z)$ is the real (lossy) impedance

e.g., damping term in an oscillator: $m\ddot{x} = F_{ext} - \Re(Z)\dot{x} - k_{spring} x$

•viscous damping: $\Re(Z) = b$ = constant. Recall, at a definite f, $\dot{x} = i\,2\pi f\; x$

•internal friction: $F = -kx \quad \Rightarrow \quad F = -k(1 + i\phi(f))x$

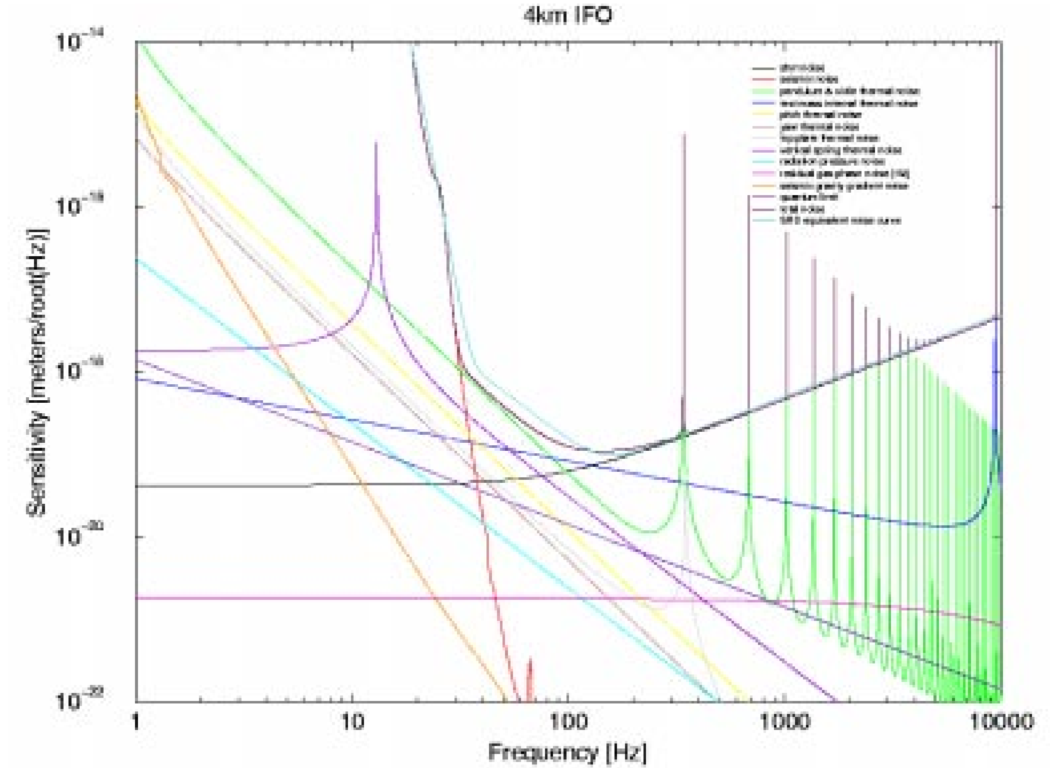
$\phi(f)$ is often a constant, = 1/Q

Minimize thermal motion $\Rightarrow$ materials and techniques for very low loss (high Q)

Thermal displacement noise

Sum of many normal modes, $x^2_{thermal} = \frac{4kT}{2\pi f} \sum_n \frac{\phi_n(f)}{m_n(2\pi f_n)^2} \left\{ \frac{1}{\left(1-(f/f_n)^2\right)^2 + \phi_n{}^2(f)} \right\}$

Each with loss $\phi_n(f)$:



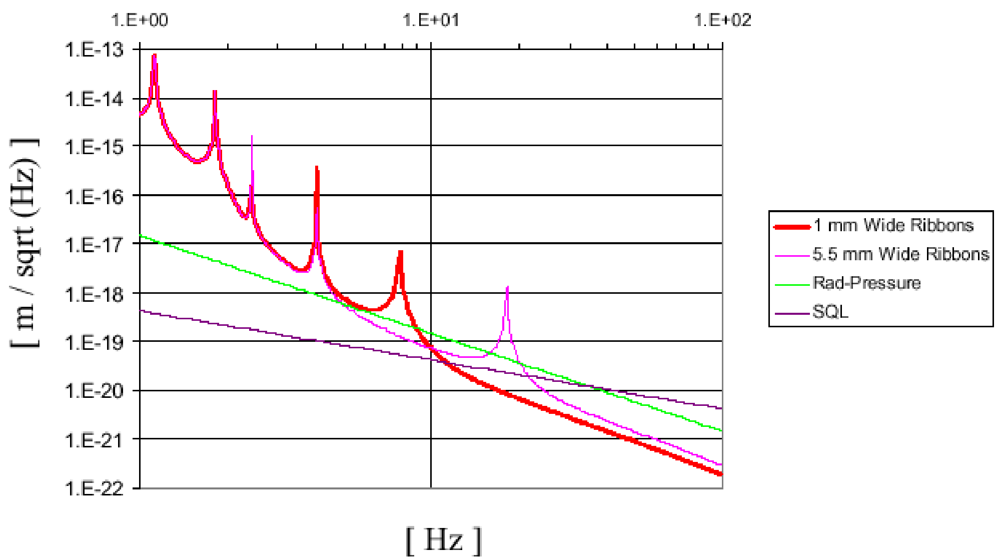
Equivalent strain (noise):

$$h_{thermal}(f) = \frac{2}{L}\sqrt{x^2_{thermal}}$$

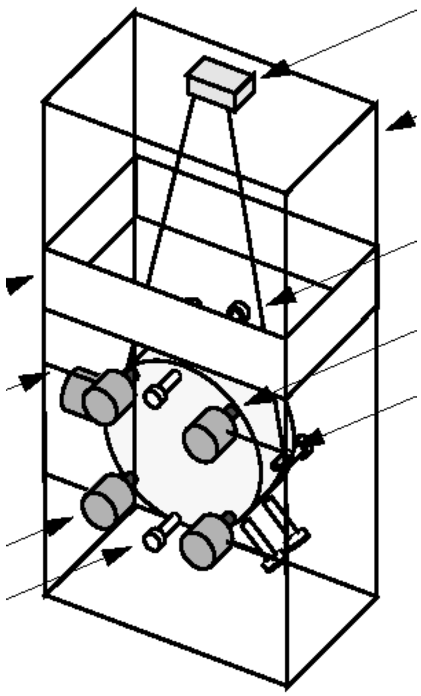
Suspension thermal noise

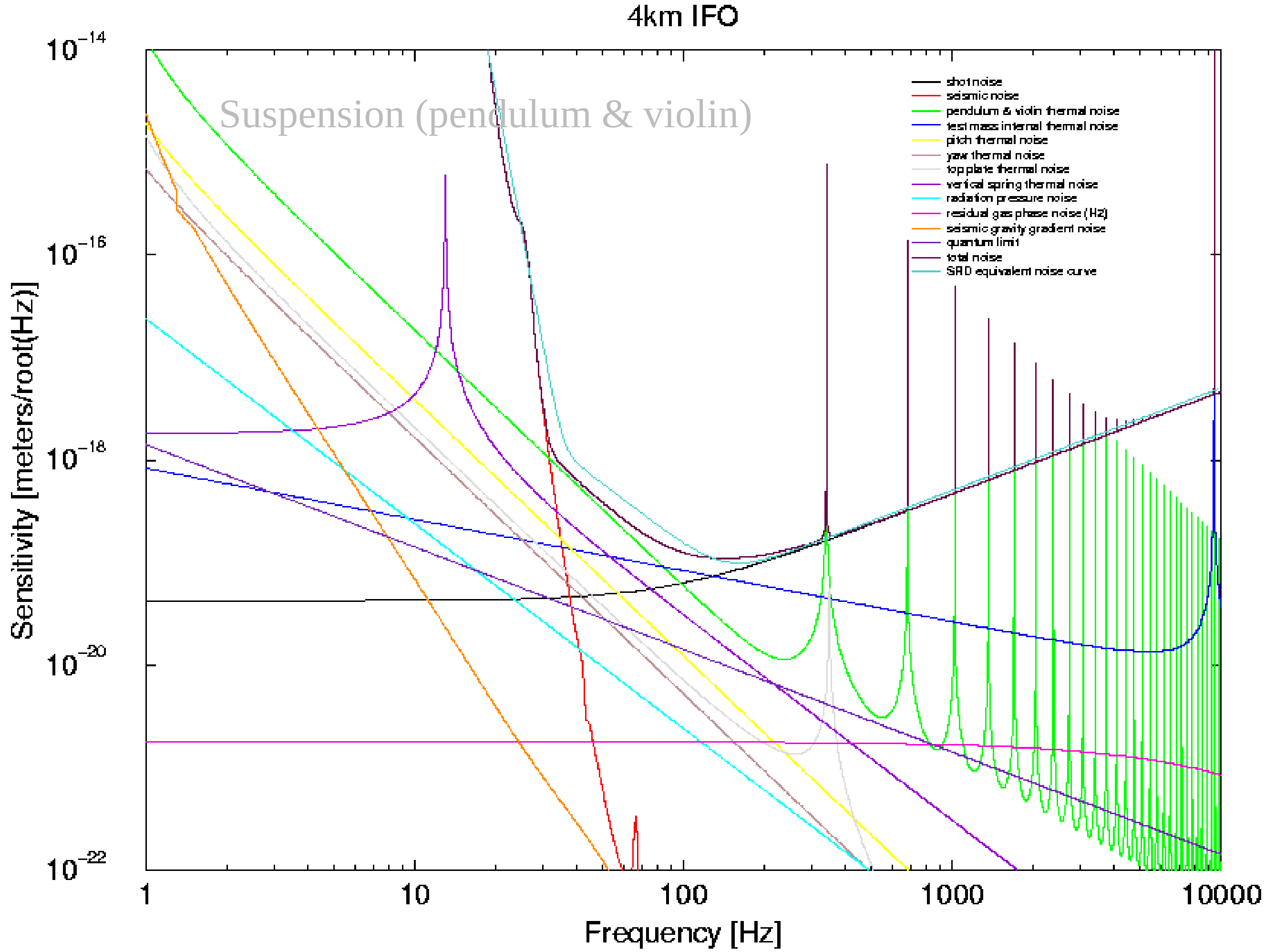
Suspension wires vibrate (violin modes, stretch/bounce modes), kick the test mass around, introducing an harmonic series of noise lines

- Incoherent motion of the 4 test masses produces noise in GW channel, at fundamental and harmonics
- Most severe just after lock acquisition; then they ring down

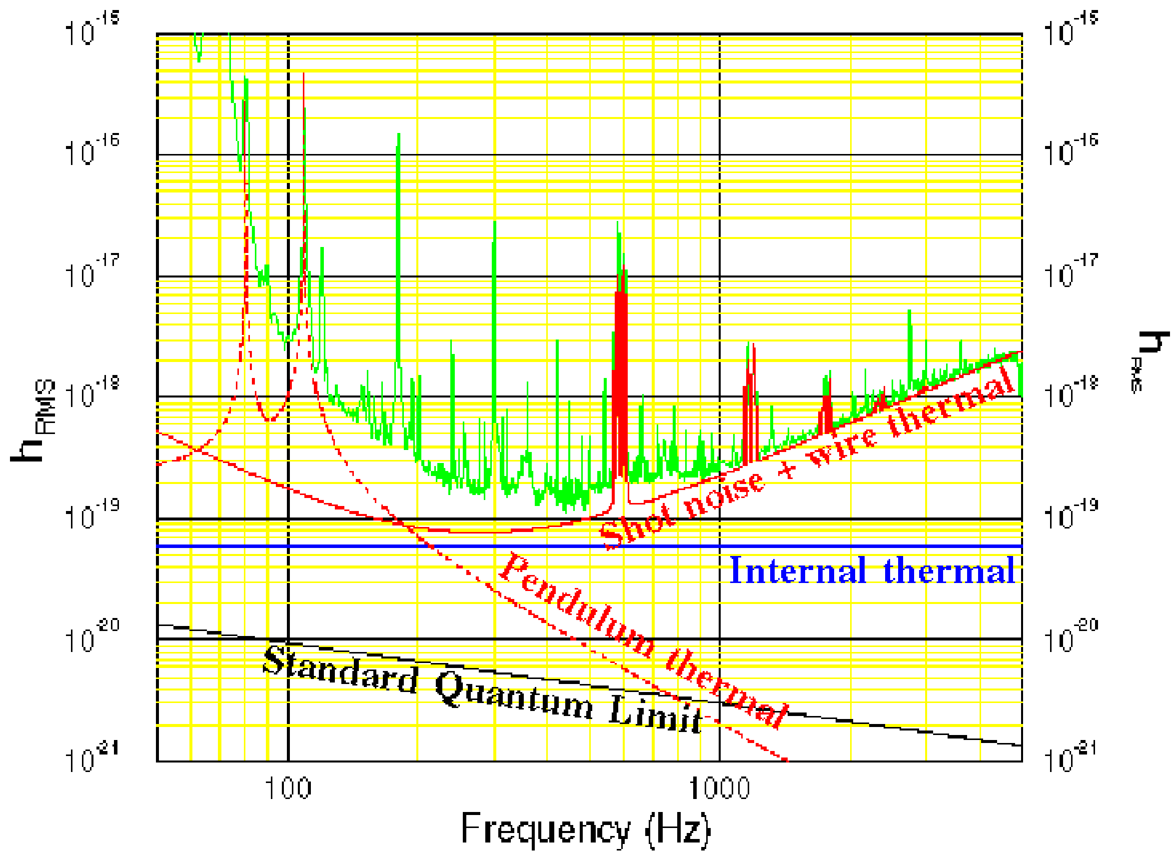


Suspension thermal noise





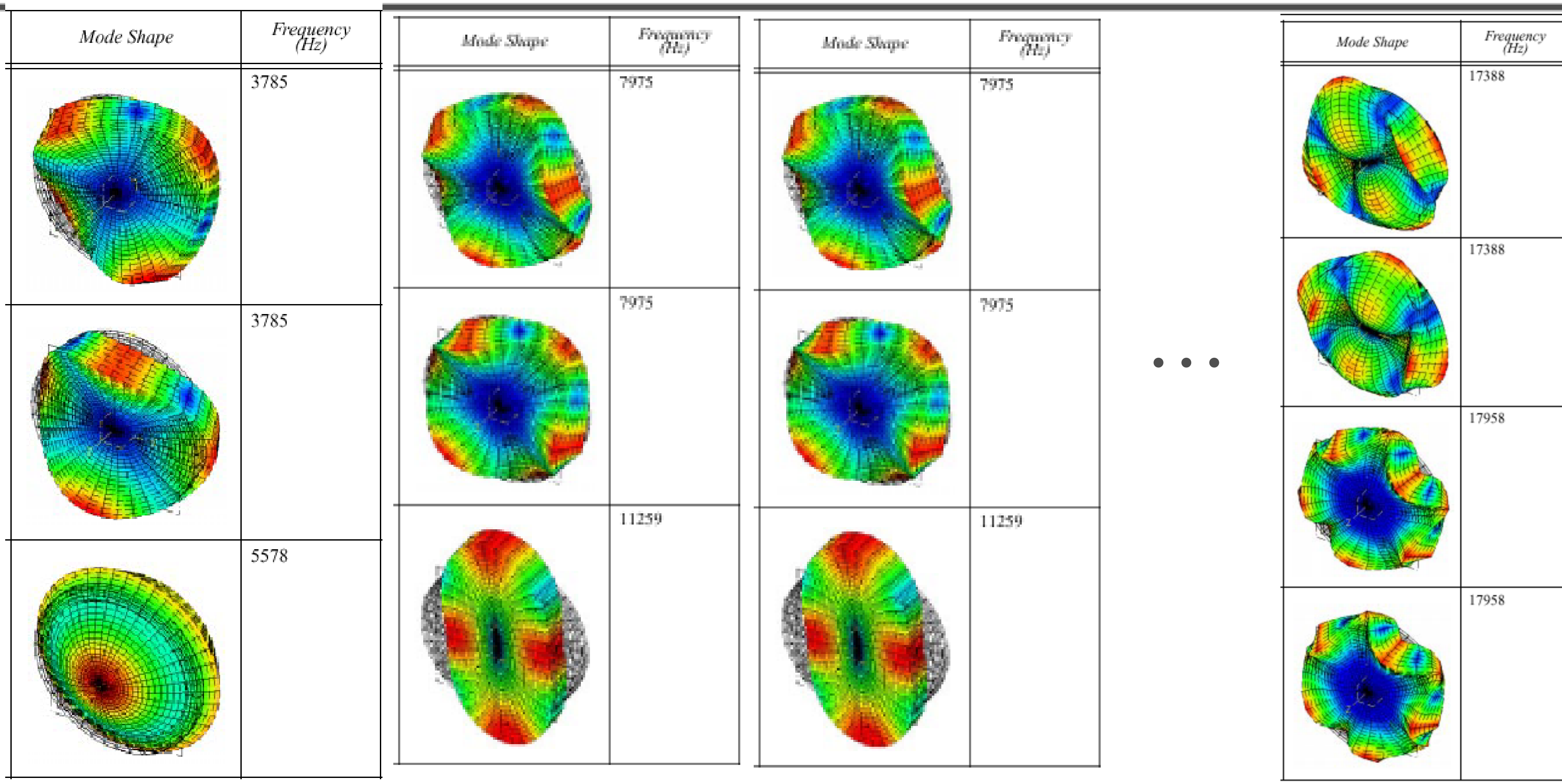
40 meter noise spectrum, 1994



Internal test mass thermal noise

- LIGO test masses have internal normal modes at ~several kHz and up (outside of LIGO sensitivity band)
- Dissipation causes thermal energy to leak into LIGO band *f* < few kHz
- Test mass vibrates about its center of mass; but the reflective mirror is on the surface, *not* the COM, so it introduces displacement noise
- Minimize dissipation: high Q materials (fused Si, sapphire). BUT, suspension wires, magnets for actuation, cause dissipation, reducing Q dramatically
- Solutions for LIGO II: replace suspension wire with silica ribbons and welds; eliminate magnets (use electrostatic force via capacitive coupling, or photon pressure)

Vibrational modes of test masses

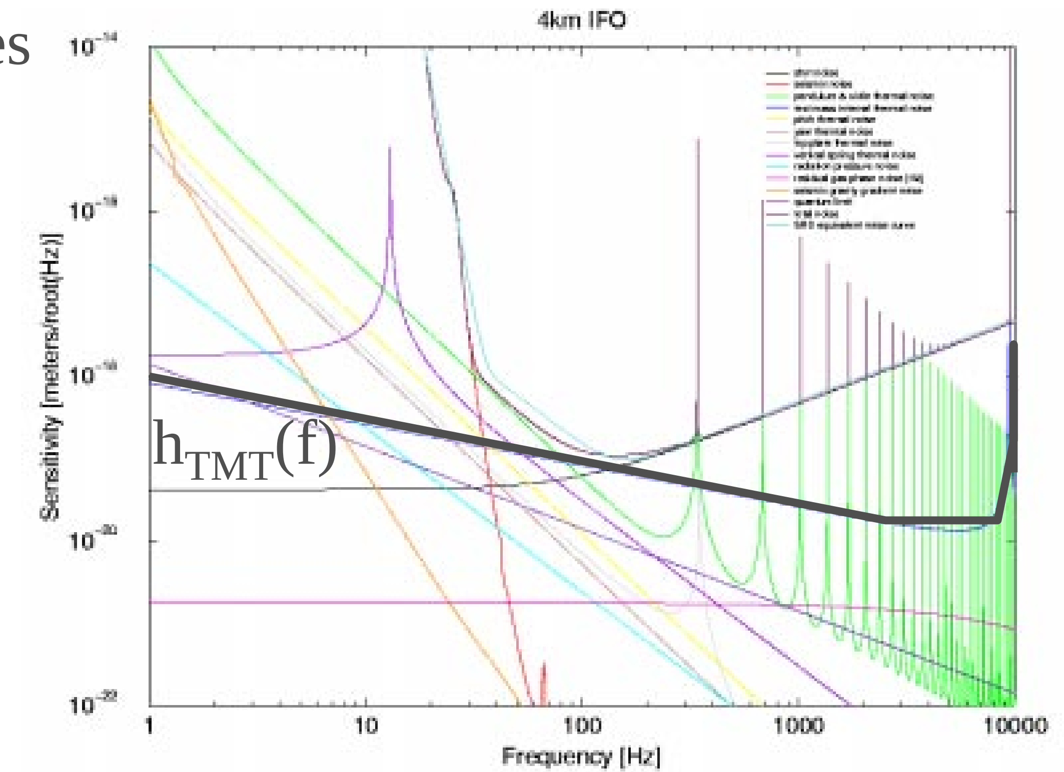


This is for beam splitter. Test masses have no resonances below ~8KHz (?).

Test mass internal thermal noise

$$x_{TMT}^2 = \frac{4kT}{2\pi f}\sum_n \frac{\phi_n(f)}{m_n(2\pi f_n)^2}\left\{\frac{1}{\left(1-(f/f_n)^2\right)^2+\phi_n^{\,2}(f)}\right\}$$

Test masses have normal modes
Above the LIGO band



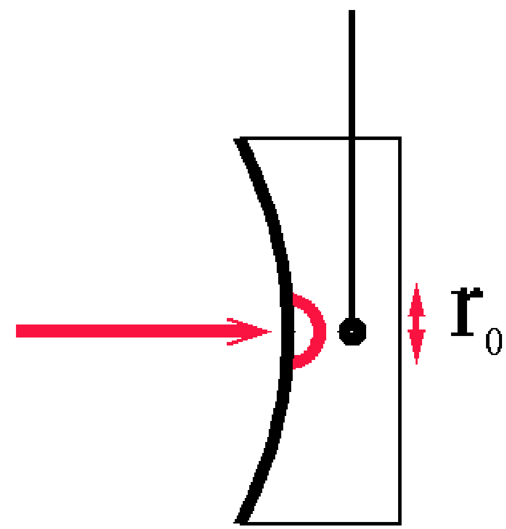
Equivalent strain:

$$h_{TMT}(f) = \frac{2}{L}\sqrt{x_{TMT}^2}$$

Thermoelastic noise

- Mirror is at finite temperature, and any small volume in the mirror experiences fluctuations in temperature (the smaller the volume, the greater the fluctuation, and the beam samples only a small volume)
- The material expands thermoelastically, so fluctuations in temperature cause fluctuations in the expansion
- Since the COM of the suspended mirror is not at the mirror reflective surface, this induces a fluctuation in the mirror position, with spectral density
- Coefficient of thermal expansion α is 10x larger for sapphire than for fused silica, and thermal conductivity λ^* is 30x larger, (Braginsky, 2000).
- So for LIGO II , sapphire (much higher Q) will have much worse thermoelastic noise! (We can try to increase the beam size r_0.)

$$\left\langle \delta T^2 \right\rangle = \frac{k_B T^2}{\rho C V}$$



$$x_{TD}^2 = \frac{8}{\sqrt{2\pi}} \alpha^2 (1+\sigma^2) \frac{k_B T^2}{(\rho C)^2} \frac{\lambda^*}{r_0^3} \frac{1}{(2\pi f)^2}$$

Seismic displacement noise

Motion of the earth

•driven by wind, volcanic/seismic activity, ocean tides, humans

•requires *e.g.*, roughly 10^9 attenuation at 100 Hz

•~300 micron tidal motion, microseismic peak at 0.16 Hz.

• At low frequencies, motion is correlated over two mirrors

Approaches to limiting seismic noise

•careful site selection

   •far from ocean, significant human activity, seismic activity

•active control systems (only microseismic peak for now)

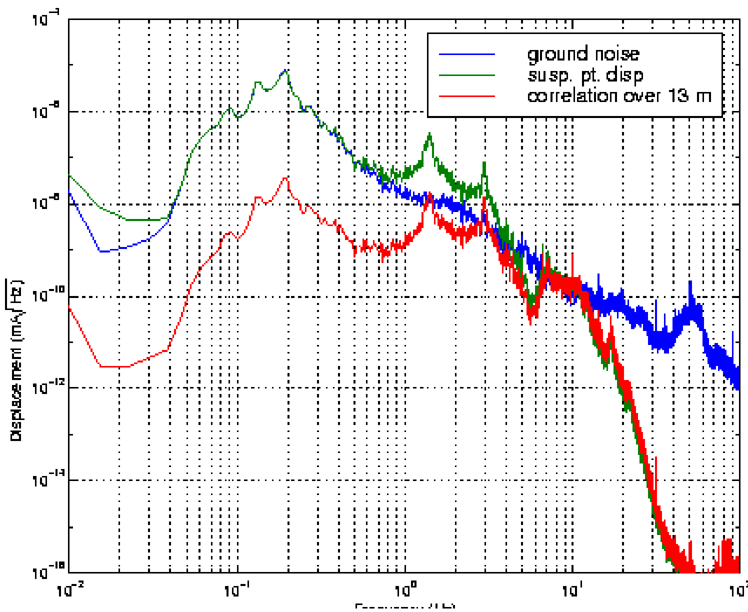
   •seismometers, regression, feedback to test masses

•simple damped harmonic oscillators in series

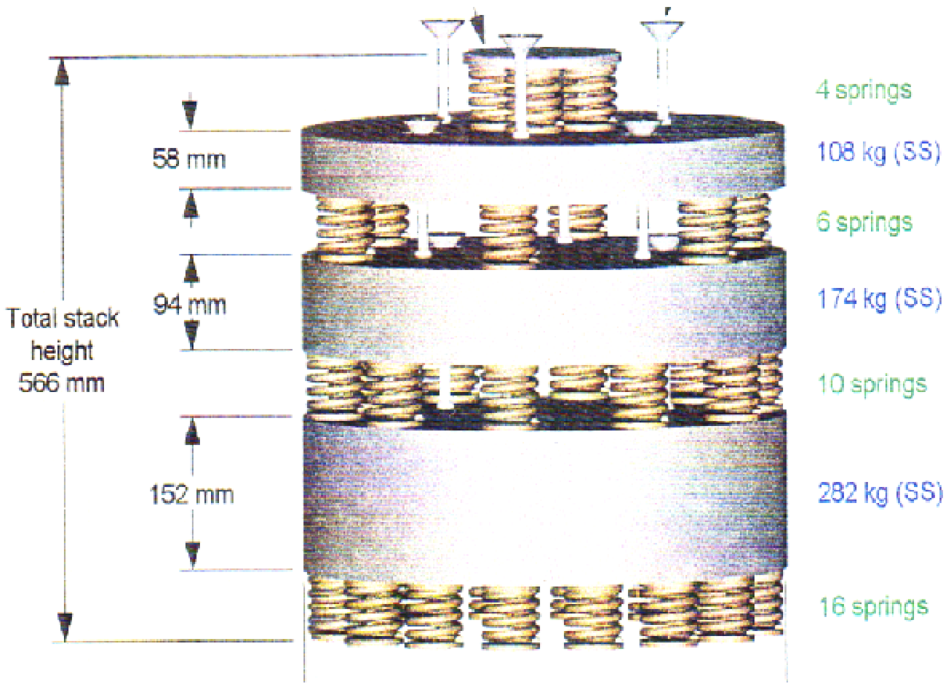
   •`stacks', constrained layer springs and SS masses

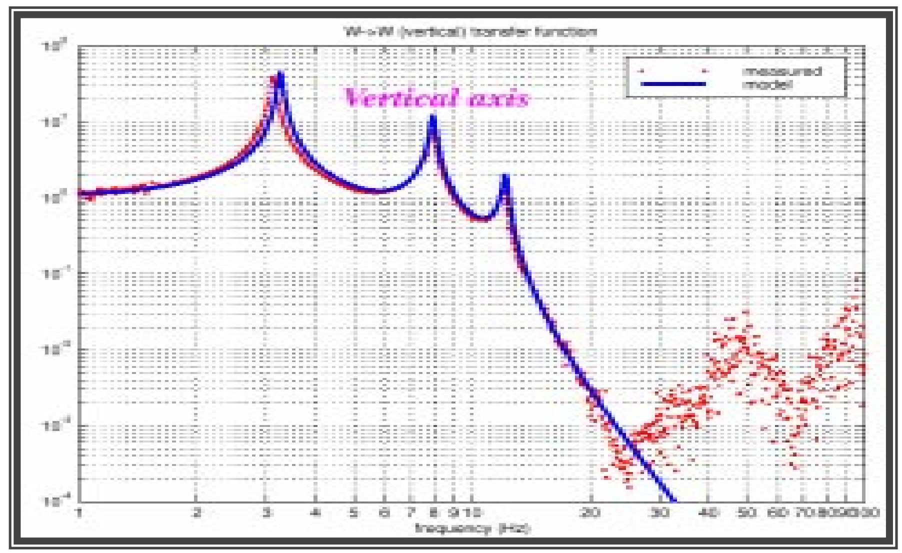
•one or more low-loss pendulums for final suspension

   •gives *1/f²* for each pendulum



Seismic isolation stacks





Seismic Isolation Systems

Support Tube Installation

Stack Installation

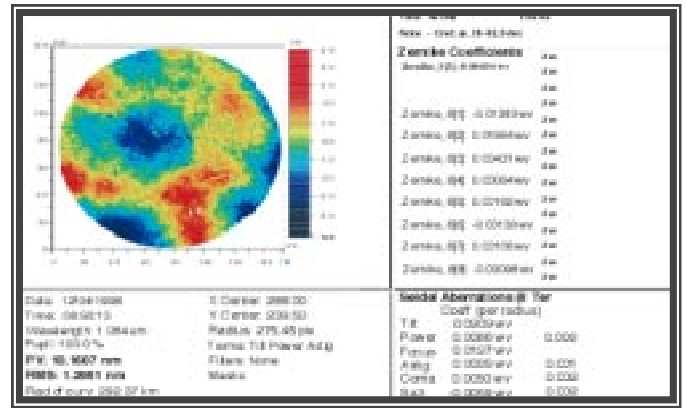
Coarse Actuation System

Noise from imperfect Optics

Highly efficient optical system:

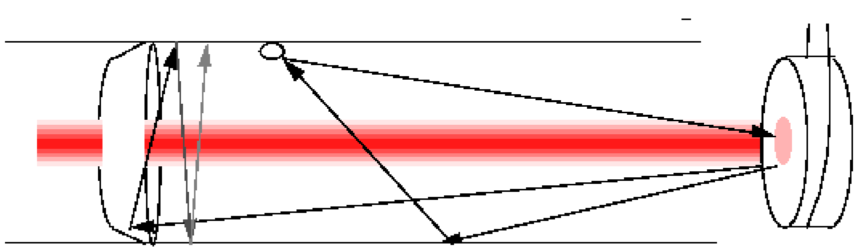
~50 ppm lost per round-trip

- optics are 25 cm diameter, 10 cm thick fused silica cylinders
- light beam ~10 cm diameter; 1ppm scattered, ~1ppm absorbed



Constraints on optical surface due to noise requirements:

- minimize scatter (power loss ⇒ phase noise)
- minimize absorption (thermal distortions, lensing ⇒ phase noise)
- minimize scattering out of beam, onto tube, back into beam (phase noise)
- minimize wavefront distortions (*contrast defect* at dark port ⇒ phase noise)

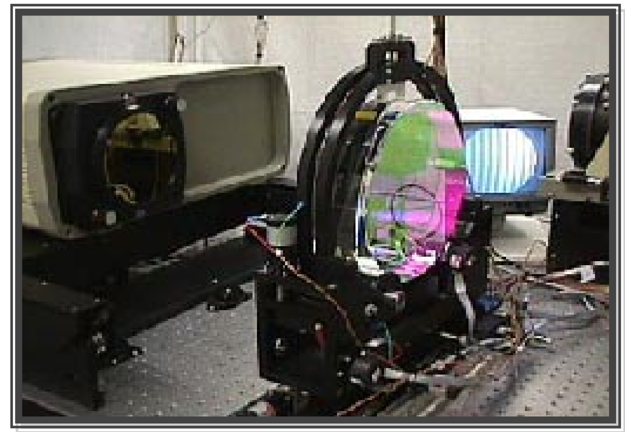


Results

- λ/**800** over central 10 cm (~1 nm rms); fine scale 'superpolish'
- Sophisticated *baffling*

LIGO Optics

mirrors, coating and polishing

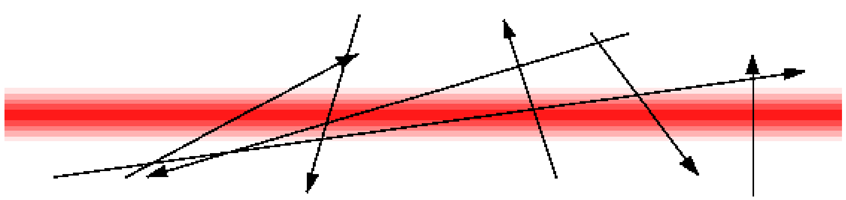


- SUPERmirrors:
  - » High uniformity fused silica quartz
  - » reflectivity as high as 99.999%
  - » losses < 1 ppm in coating, 10 ppm in substrate
  - » polished with mircoroughness < $\lambda/1800 \approx 0.5$ nm
  - » and ROC within spec.
  - ≈ ($\delta R/R < 5\%$, except for BS)
- Suspensions: hang 10kg optic by a single loop of wire, and hold it steady with feedback system

Residual gas in beam tube

Light must travel 4 km without attenuation or degradation

- refractive index fluctuations in gas cause variations in optical path, phase noise
- residual gas scatters light out of, then back into, beam; phase noise
- Residual gas pressure fluctuations buffet mirror; displacement noise
- Contamination: low-loss optics can not tolerate surface 'dirt';
  High circulating powers of ~10-50 kW burns dirt onto optic surface



requirement for vacuum in 4 km tubes:

- H_2 at 10^{-6} torr initial, 10^{-9} torr ultimate
- H_2O at 10^{-7} torr initial, 10^{-10} ultimate
- Hydro-, flourocarbons < 10^{-10} torr
- vacuum system, 1.22 m diameter, ~10,000 m^3
- strict control on in-vacuum components, cleaning

LIGO beam tubes

LIGO Livingston Observatory
LLO

LIGO Hanford Observatory
LHO

LIGO *Beam Tube*

Beam light path must be high vacuum, to minimize "phase noise"

- LIGO beam tube under construction in January 1998
- 65 ft spiral welded sections
- girth welded in portable clean room in the field

LIGO vacuum equipment

All optical components must be in high vacuum, so mirrors are not "knocked around" by gas pressure

LIGO Vacuum Chambers

HAM Chambers

BSC Chambers

Input Optics

Hanford 2 km

Control System Racks

Input Optics Section

Gravity gradient noise

Local "static" gravitational force from sum of mass distributions (Newtonian Background)

• dominated by unchanging attraction of earth

• additional time-varying contributions from other sources:

- seismic compression (surface seismic compression waves)
- weather (variations in atmospheric pressure changing air density)
- moving massive objects (humans, machines)

Places limit on lowest frequencies detectable by ground-based interferometers

• Most of these sources are irreducible for a given site

•everyone talks about the weather, but no one does anything about it!

• practical limit: down to roughly 10 Hz

• lower frequencies are domain for space-based interferometers

Another crucial reason to make interferometers long:

these motions must be small compared with GW strain

LIGO

Excess (technical) phase noise

Many sources of imperfections!

laser source

•intensity fluctuations greater than shot noise (10^{-8} δP/P)

•frequency noise (10-7 Hz/√Hz)

•angular or translational beam pointing fluctuations

sensing and control systems

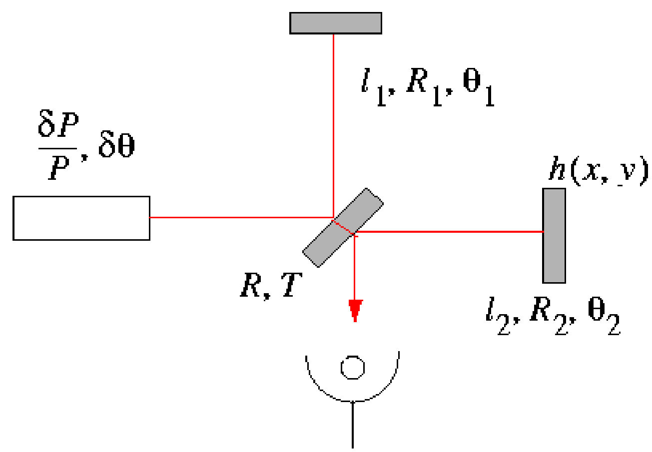
• linearity (microns at 1 Hz, 10-19 m /√Hz at 100 Hz)

• Electronics noise at sensors, actuators, in between

• EMF pickup, spectral lines (60 Hz and harmonics)

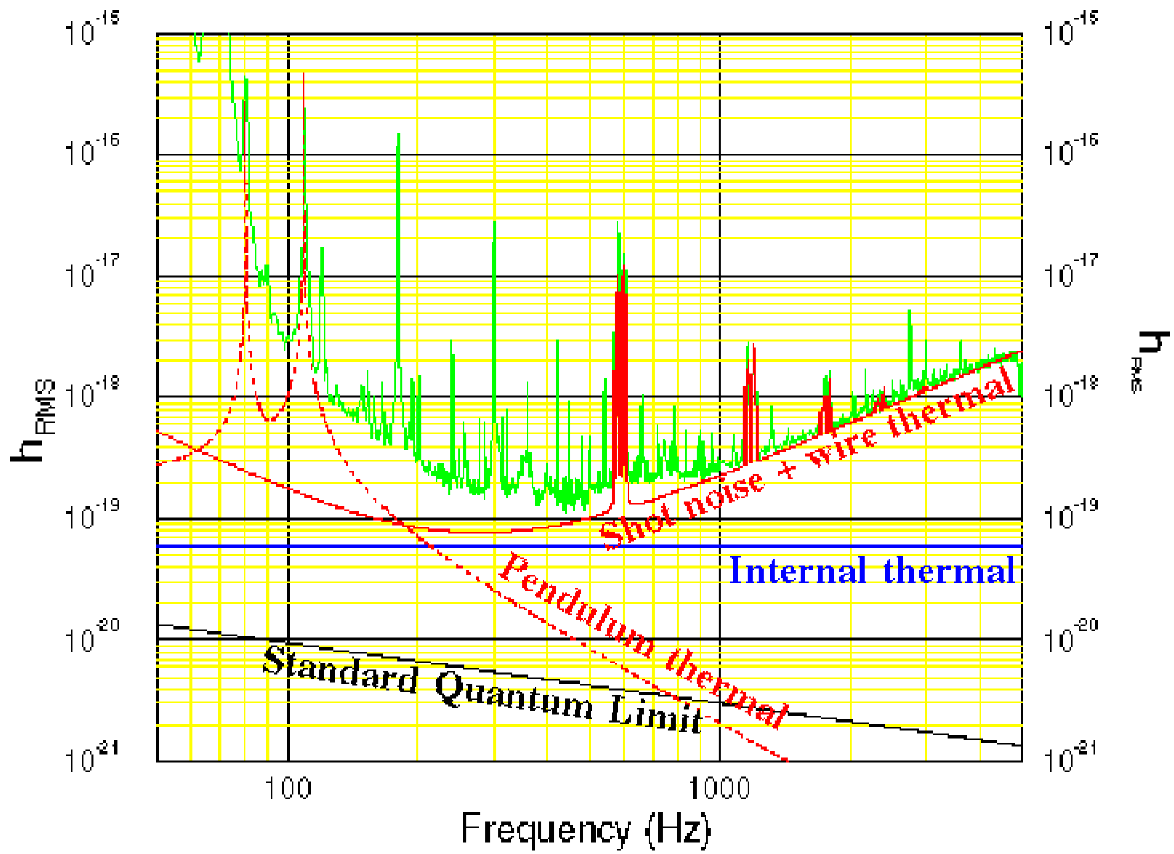
Imperfect optics, misalignments, losses

The UNKNOWN!



Much of the technical effort goes into controlling these noise sources

40 meter noise spectrum, 1994



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