

LIGO Scientific Collaboration meeting  
Hanford Aug 16th, 2000

**A Highly Sensitive Accelerometer  
(HSA)  
for the LIGO-II SEI and SUS,  
a Preliminary Study.**

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# Motivations for HSA

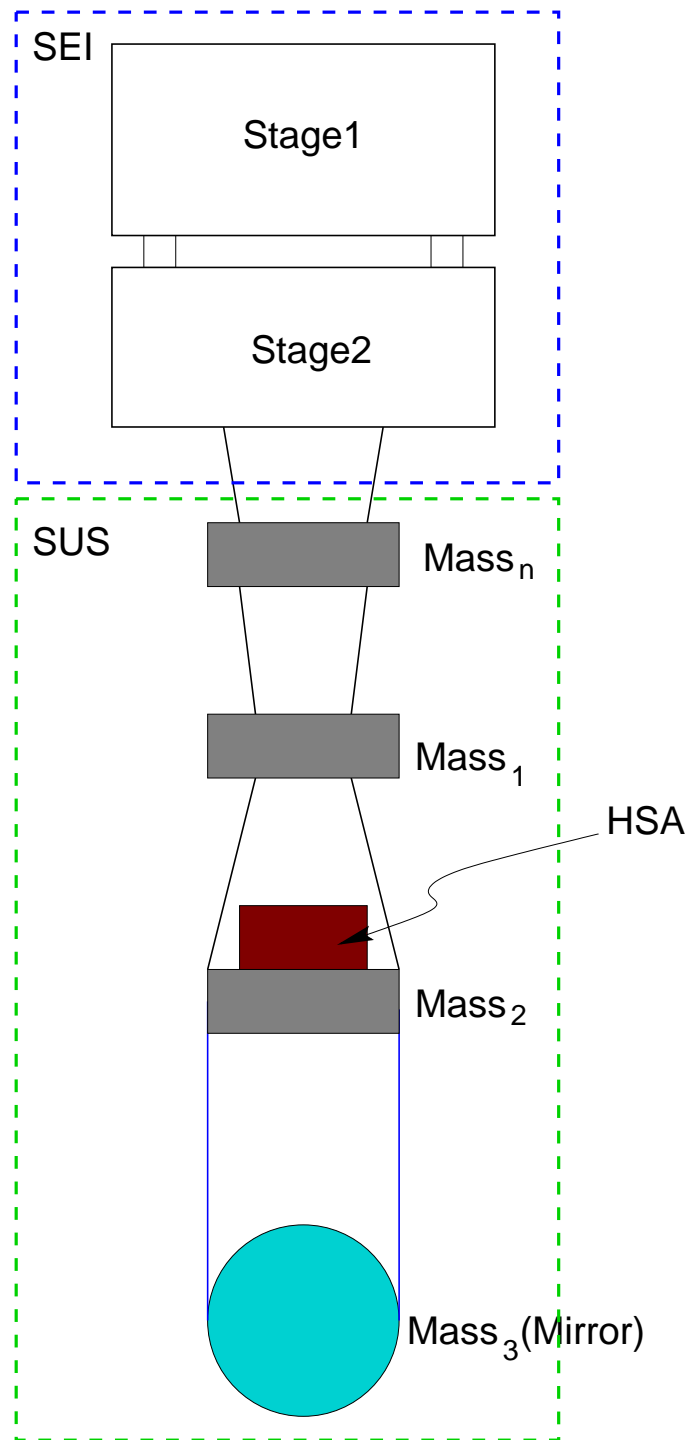
The increasing performance of the seismic attenuation system and the mirror suspension raises the probability of excess local noise that can limit the theoretical LIGO-II sensitivity noise curve.

(Issue frequently pointed out by Braginsky)

It becomes necessary to characterize the excess noise of SEI-SUS, "locally" to study and validate the system for LIGO-II.

# Location of the Sensor inside SEI-SUS

Actual SEI-SUS system for LIGO-II sketch



(Modification required on the stage above the Mirror  
=> "tollerable" on the prototype stage)

# Required Sensitivity

- $\omega^* \sim 1000$  rad/s (Typical Frequency of interest)
- $\tau \sim 10^{-2}$  (Integration Time)
- $G \sim 10^{-3} - 10^{-4}$  (Typical Transmissibility at  $\omega^*$ )
- $\delta\tilde{x}(\omega^*) \sim 10^{-20} \frac{\text{m}}{\sqrt{\text{Hz}}}$  (expected LIGO-II sensitivity)
- $S/N \sim 10$  (Aimed Signal to Noise ratio)

## Transducer Required Sensitivity

$$\delta\tilde{x}(\omega^*) \sim 10^{-16} - 10^{-17} \frac{\text{m}}{\sqrt{\text{Hz}}}$$

Possible solution (Braginsky's Idea):

A Mechanical resonator with an interferometric readout with an enhancement factor on sensitivity using Fabry-Perot cavities

# Noise Sources Estimation:

## Laser Shot-Noise

$$\delta x = \frac{\lambda}{8F} \sqrt{\frac{h\omega}{\langle P \rangle \tau}}$$

$$P = \frac{\pi}{32} \sqrt{\frac{hc\lambda}{F^2 \delta x^2 \tau}}$$

- $\delta \tilde{x} = 1 \cdot 10^{-17} \text{ m}/\sqrt{\text{Hz}}$
- $F = 10,000$
- $\tau = 10^{-2} \text{ s}$

$$\delta P_{min} \simeq 4 \text{ uW}$$

# Noise Sources Estimation:

## Fabry-Perot Finesse limit

The dynamic range of the actuator needs to be greater than the Brownian oscillation of the Mechanical oscillator

$$F < \alpha \lambda \sqrt{\frac{M \omega_0^2}{k_b T}}$$

- $M = 539 \text{ g}$
- $\omega_0 = 100 \text{ rad/s}$
- $\lambda = 1.064 \text{ um}$
- $\alpha = 10\%$

$$F < 1.09 \cdot 10^5$$

# Noise Sources Estimation:

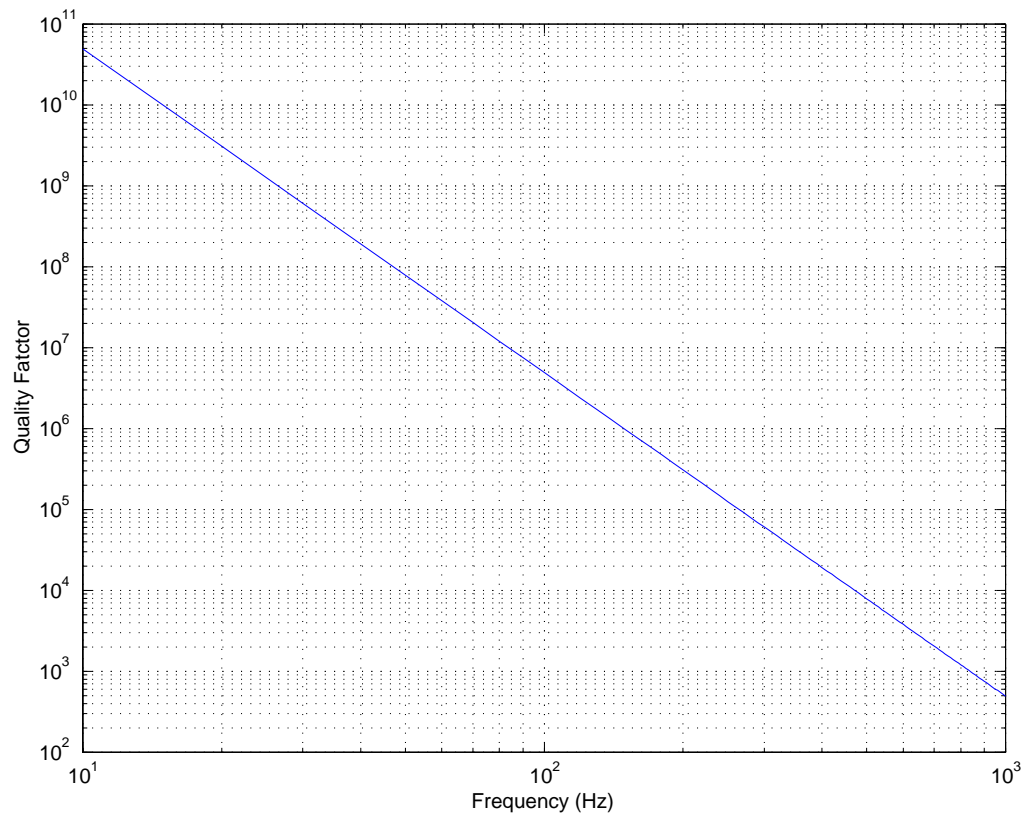
## Mechanical Oscillator Thermal Noise

$$\delta\tilde{x}(\omega) \simeq \sqrt{\frac{4k_b T \omega_0^2}{M Q \omega^5}}$$

$$Q_{min} = \frac{k_b T \omega_0}{M \delta x^2 \omega^4}$$

- $\omega_0 = 100$  rad/s
- $M = 0.539$  kg
- $\delta x = 1 \cdot 10^{-17}$  m/ $\sqrt{\text{Hz}}$
- $\omega^* = 1000$ ; rad/s

$$Q_{min} \simeq 8 \cdot 10^5 \quad \text{at} \quad 1000 \text{ rad/s}$$



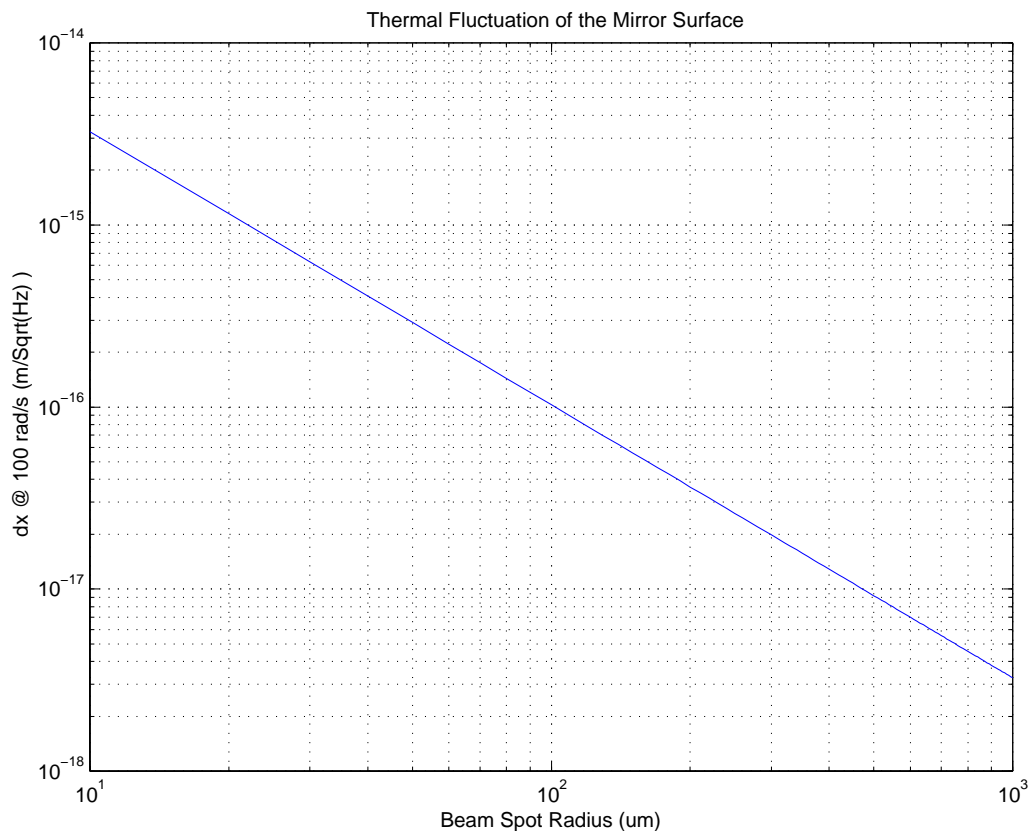
# Noise Sources Estimation:

## Photo-Thermal Shot-Noise of Mirror Surfaces

$$\delta\tilde{x}(\omega) = 2 \sqrt{\sqrt{2}\alpha(1 + \sigma^2) \frac{k_b T \lambda^*}{\sqrt{\pi} C_v^2 \rho^2 r^3 \omega^2}}$$

- $r = 150 \mu\text{m}$  (Spot Radius on the mirror)
- $\omega = 1000 \text{ rad/s}$  (pulsation)

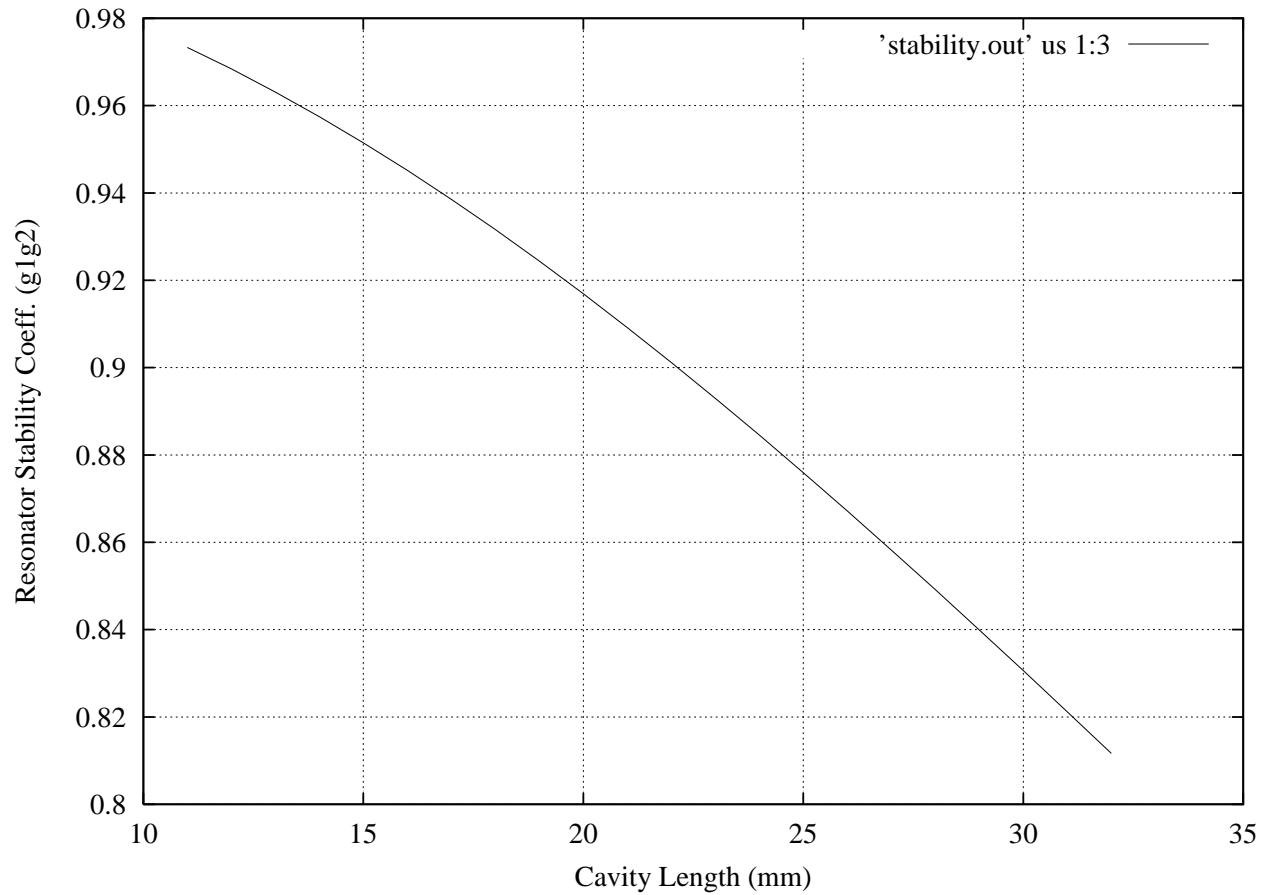
$$\delta x_{rms} \simeq 1.8 \cdot 10^{-17} \text{ m}/\sqrt{\text{Hz}} \quad \text{at} \quad 1000 \text{ rad/s}$$





# Fabry-Perot Cavity

## Photo-Thermal Shot-Noise of Mirror Surfaces vs Spot-Size

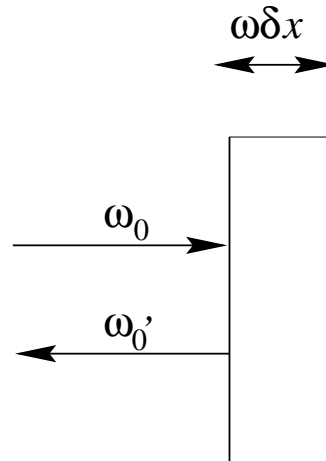


Stability Condition

$$0 < \left(1 - \frac{d}{R_1}\right) \left(1 - \frac{d}{R_2}\right) < 1.$$

# Noise Sources Estimation:

## Doppler Frequency Noise



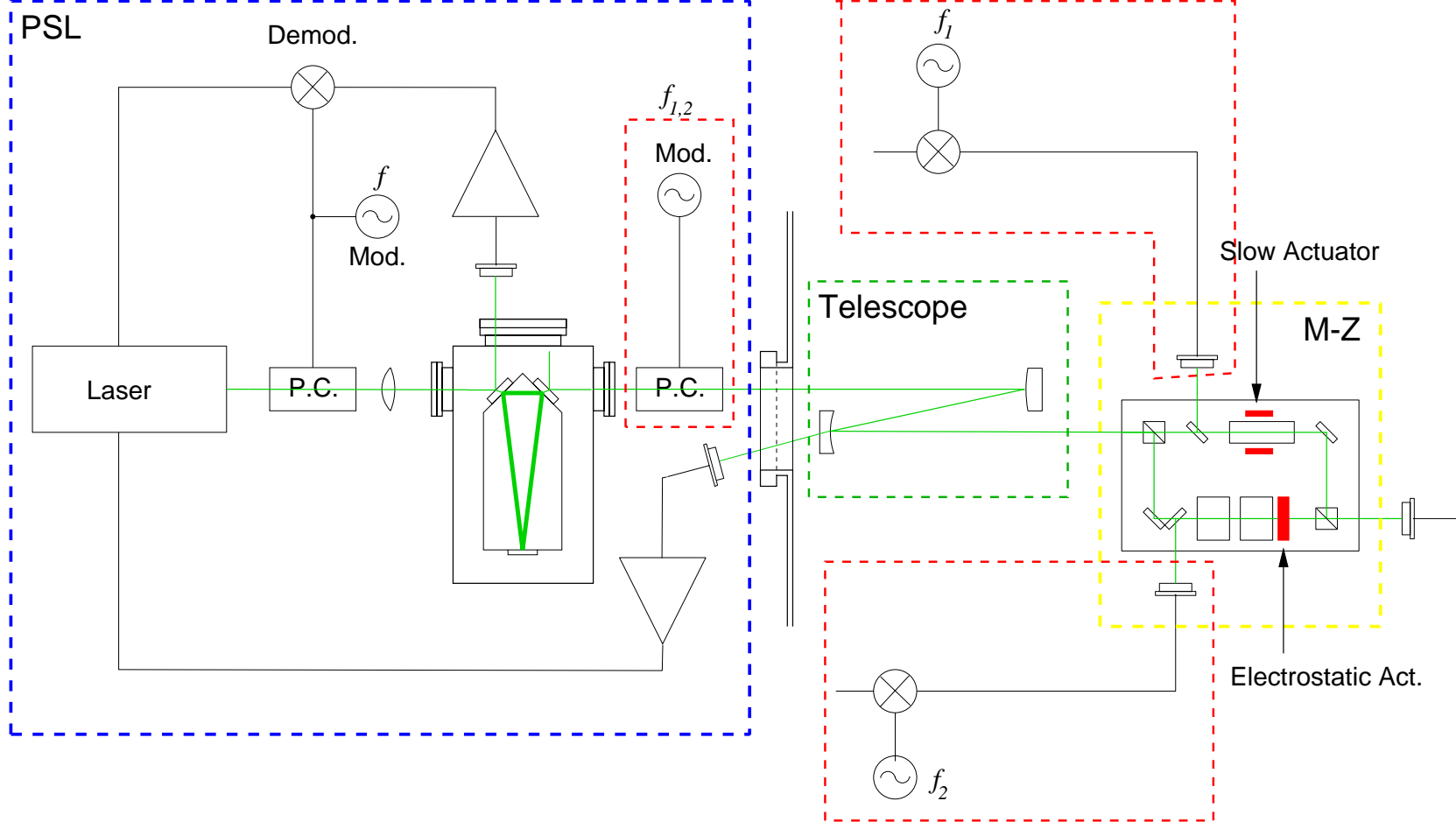
$$\delta \tilde{x}_{doppler} = 2 \frac{\omega}{c} l \delta \tilde{x}_{th}(\omega)$$

- $\omega = 1000 \text{ rad/s}$
- $\delta \tilde{x} = 1 \cdot 10^{-17} \text{ m}/\sqrt{\text{Hz}}$
- $c = 3 \cdot 10^8 \text{ m/s}$

$$\delta \tilde{x}_{th} < 1 \cdot 10^{-11} \text{ m}/\sqrt{\text{Hz}}$$

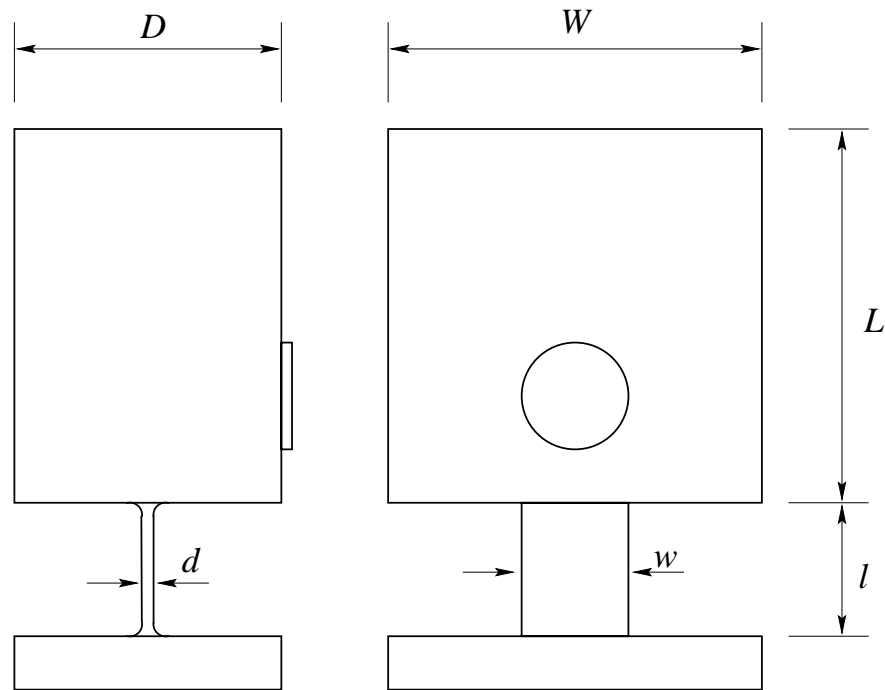
(Noise not rejected from an interferometer common mode)

# Optical Layout



# Mechanical Oscillator

(Very Preliminary Design)

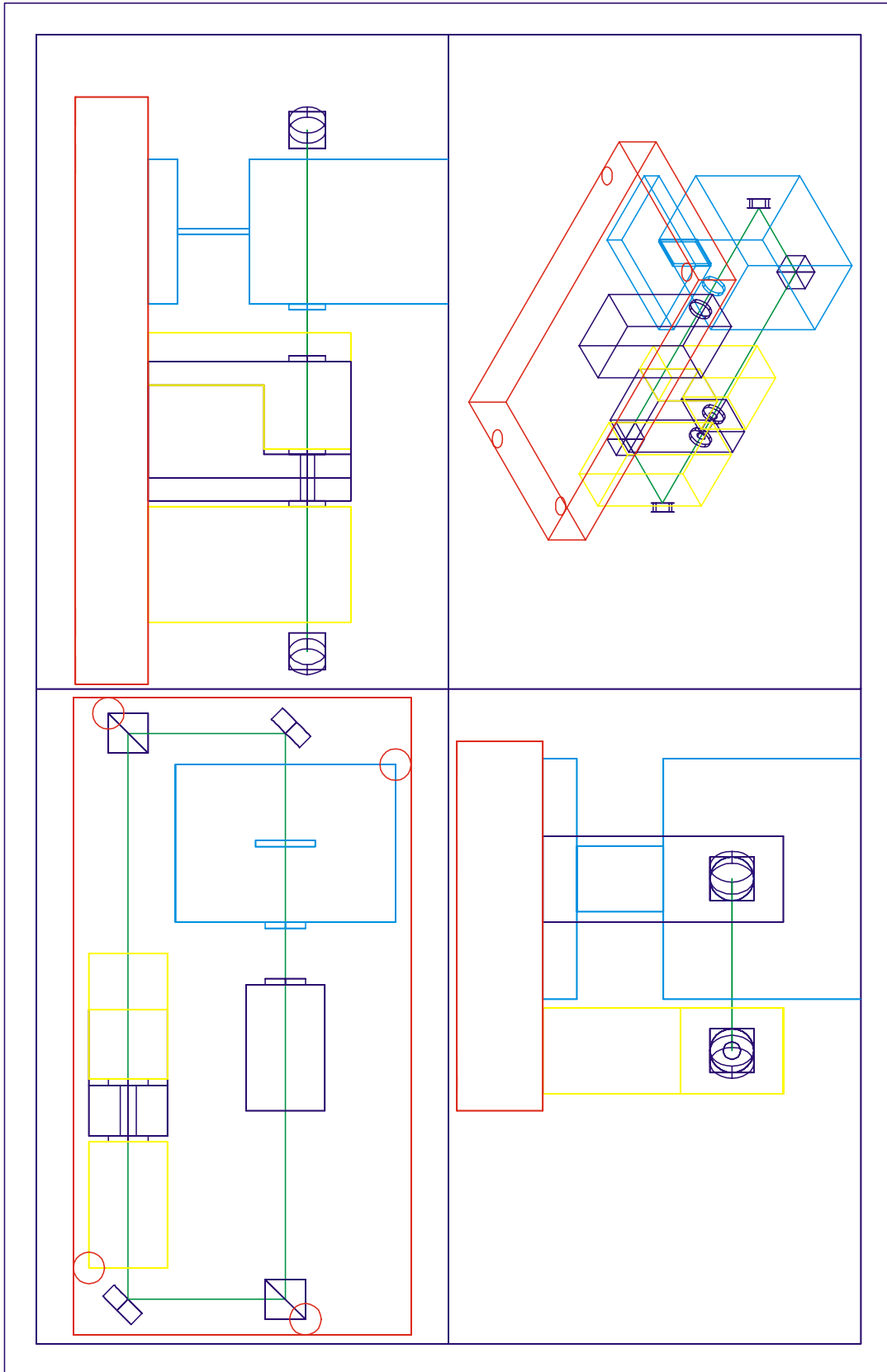


Main Characteristics

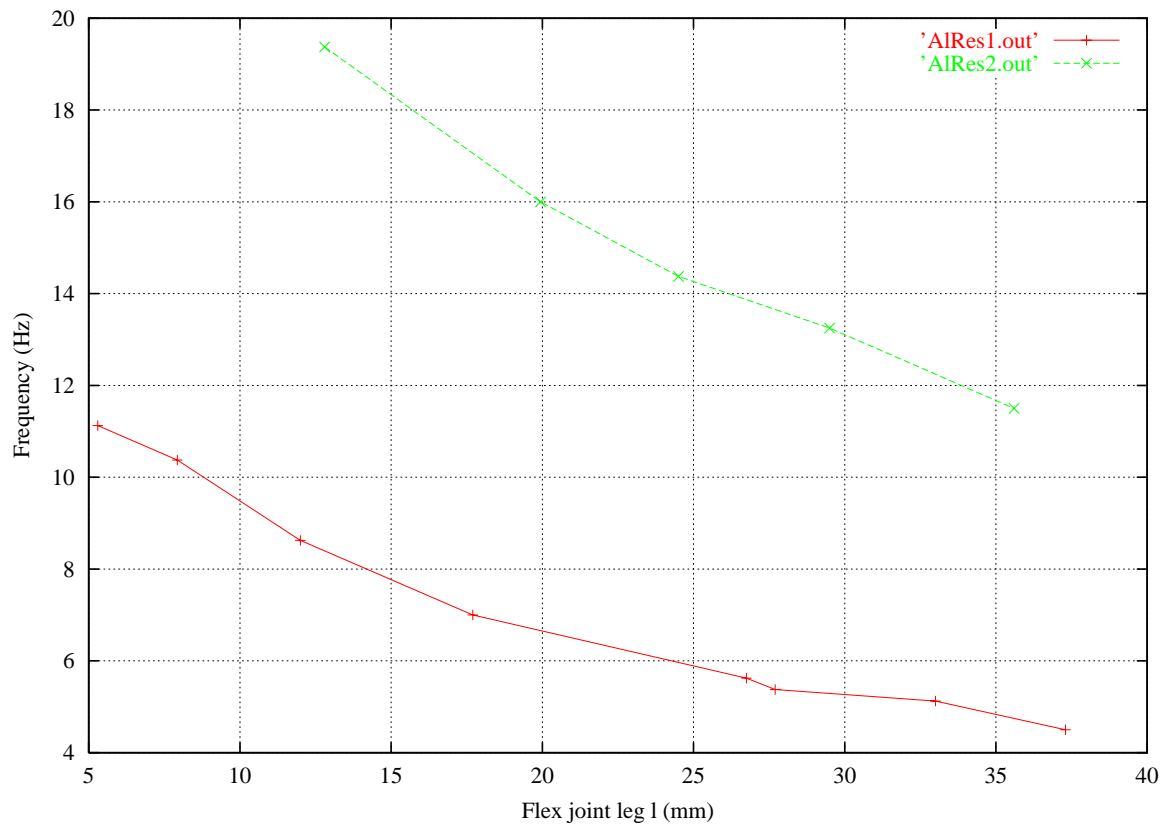
- Material: Fused Silica
- Design: Monolithic
- Longitudinal Rigid Body mode  $\omega_0 = 100$  rad/s
- Quality factor  $Q \sim 10^6$
- Mass Dimension  $D \times W \times L = 50 \times 70 \times 70$  mm<sup>3</sup>
- Mass  $M = 539$  g
- Flex Joint Dimension  $d \times w \times l = 2 \times 19 \times 25$  mm<sup>3</sup>

# Mach-Zender Design

(Very Preliminary Design)



# Mechanical Oscillator Prototype



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# R&D Plan Guideline for HSA

- Design and Realization and Test of a Mechanical Resonator Prototype in Aluminium
- Design and Realization and Test of the Mechanical Resonator Prototype in Fused Silica
- Design and Realization and Test Fabry Perot Cavity.
- Design and Realization of the Fixed Cavity with the Fused Silica.
- Design and Realization and Test of the Mach-Zender ITF in air.
- Test of the Mach-Zender ITF in vacuum.
- Test of HSA suspended from the TAMA-SAS prototype/LASTI.
- Test of HSA in LASTI.

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## Conclusion

- From a brief evaluation of noise sources and from a preliminary conceptual design, it seems feasible to build a inertial sensor with a transducer sensitivity :

$$\delta\tilde{x}(\omega = 1000) \simeq 10^{-16} \frac{\text{m}}{\sqrt{\text{Hz}}}$$

- This sensitivity is sufficient to investigate the performance of a SEI-SUS system (mainly excess noise characterization).
- No over-killing on the specs. has been considered.
- The preliminary design shows that minor modification are needed to host the sensor in the actual LIGO-II suspension design.
- The preliminary design shows also, that it is un-probable to to introduce this modifications not only on the prototype stage but also into the LIGO-II final suspension.