Demodulation Code

Steven Berukoff

Albert-Einstein-Institut

Golm, Germany

LIGO-G000288-00-Z

Why Demodulate?

- Remove Doppler modulation of source frequency
- Remove effects of neutron star spindown
- Concentrate power of signal in one frequency bin

How? -- Theory

For time series x_a and known phase function Φ, the long time baseline DeFT is

$$\hat{x}_b(\overset{\varpi}{\lambda}) = \sum_{a=0}^{NM-1} x_a e^{-2\pi i \Phi_{ab}(\overset{\varpi}{\lambda})}$$

Convert this to short-time baseline by introducing index j, such that $N\alpha + j = a$:

€

$$\sum_{\alpha=0}^{M-1} \sum_{j=0}^{N-1} x_{\alpha j} e^{-2\pi i \Phi_{\alpha j b}(\overset{\varpi}{\lambda})}$$

If $\tilde{x}_{\alpha k}$ is the matrix formed by the FTs along short time index j, we can write

$$x_{\alpha j} = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{x}_{\alpha k} e^{2\pi i \frac{jk}{N}}$$

Substitution gives

$$\hat{x}_b(\overset{\varpi}{\lambda}) = \sum_{\alpha=0}^{M-1} \sum_{k=0}^{N-1} \tilde{x}_{\alpha k} \left[\frac{1}{N} \sum_{j=0}^{N-1} e^{-2\pi i (\Phi_{\alpha j b}(\overset{\varpi}{\lambda}) - \frac{jk}{N})} \right]$$

Reference:

- Schutz, B.F., Proceedings of GWDAW 1997, Orsay, France. Pp. 133-143.
- Williams, P. and Schutz, B.F. gr/qc 9912029.
- Schutz, B.F., Williams, P., Papa, M. (in prep).

If we now "separate" terms which are dependent on the index k and those that are not, we may write

$$\hat{x}_b(\vec{\lambda}) = \sum_{\alpha=0}^{M-1} Q_\alpha(b,\vec{\lambda}) \sum_{k=0}^{N-1} \tilde{x}_{\alpha k} P_{\alpha k}(b,\vec{\lambda})$$

where $Q_\alpha(b,\vec{\lambda}) P_{\alpha k}(b,\vec{\lambda})$ are our "matched filters".

<u>NOTE: Computational cost still large, so tinker with approximations!</u>

•Taylor expand Φ around midpoint of each short time chunk. This gives

$$P_{\alpha k}(b,\vec{\lambda}) = \frac{\sin x'}{x'} - i\,\frac{1-\cos x'}{x'} \qquad\qquad Q_\alpha(b,\vec{\lambda}) = \cos y + i \sin y$$

$$x' = \sum_s f_s b_{s\alpha} - k \qquad\qquad y = \sum_s f_s a_{s\alpha}$$

where $a_{s\alpha}$ and $b_{s\alpha}$ are from the phase model, and f_s are spindown parameters.
This approximation causes about 5% loss in power.

•Numerical experiments have shown that we can take only a few terms in the k summation which contribute the greatest, on the order of 4. *This gives about 3% power loss.*

How? -- Practice

<u>Stored in RAM</u>: SFT data, sky position, spindown parameters, $a_{s\alpha}$ and $b_{s\alpha}$

Loop over l
    Loop over β → Comprise long time baseline index b=β+Ml, defines DeFT frequency bin.
        Loop over α → SFT data block index
            Compute $k^* = \sum_s f_s b_{s\alpha}$
                k loop, 8 terms around k*
                    Compute $\tilde{x}_{\alpha k} P_{\alpha k}(b, \vec{\lambda})$
                End Loop
            Compute $Q_\alpha(b, \vec{\lambda})$
            Compute DeFT $\hat{x}_b(\vec{\lambda})$
        End Loop
    End Loop
End Loop

The Ins and Outs of Demodulation

Inputs:

FFTdata structure:
- SFT data
- parameters
- # of SFTs

Parameters:

- frequency indices
- timescale of SFTs
- index of parent loop
- spindown parameters (B. Owen)
- sky constants (S. Berukoff)

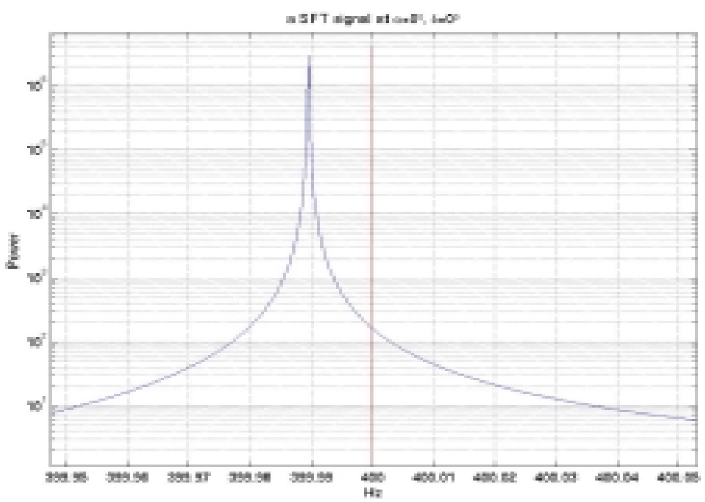
Output:

DeFFT structure:
- DeFT data
- parameters

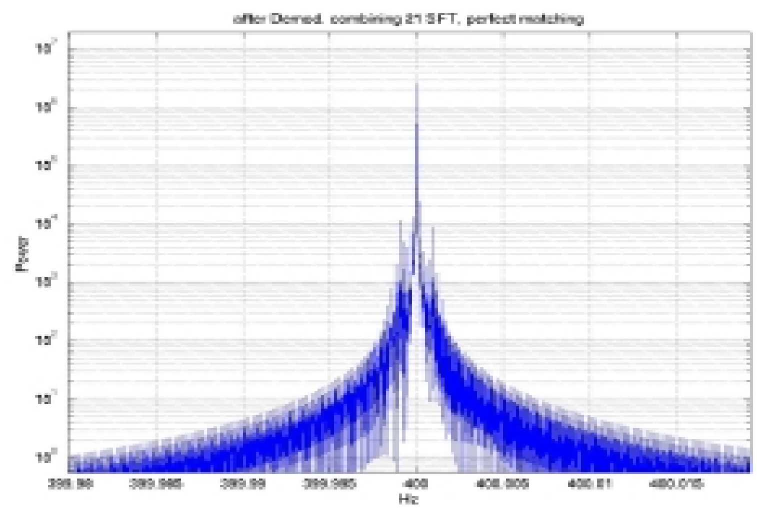
Well, that's nice math, but does it work?

Undemodulated signal, shifted off 400 Hz. This is one of the SFT chunks, with time baseline of approximately **one hour**. Put together, the plot of SFTs which form the DeFT would look like several similar spikes transposed on each other, **each depositing power in a different bin**.

Demodulated signal, centered on 400 Hz. This is the amalgamated DeFT, with time baseline of approximately 21 hours. The SNR is enhanced due to the power being consolidated into one frequency bin



Before
(1 hour baseline)



After
(21 hour baseline)

The path ahead…

- Completion of LAL documentation
- Optimizations of current code
- Tweaking code to function with follow-up stage
- Analysis of optimal strategies for follow-up stages
- Construction and testing of full 3-stage code