LIGO-G000413

Fabry-Perot Cavity Ringdowns

(Theory & Experiment)

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November 16, 2000

Impulse Response of FP Cavity

Let input field = impulse

cavity field:

$$E_n = E_0 (r_a r_b)^n$$

where n is the number of round-trips:

$$n = \text{floor}\left(\frac{t}{2T}\right)$$

for large n:

$$n \approx \frac{t}{2T}$$

continuous approximation:

$$E_n = E_0 (r_a r_b)^{\frac{t}{2T}}$$

exponential decay:

$$E_n = E_0 \, e^{-\frac{t}{\tau}}$$

where

$$\tau = \frac{2T}{\ln\left(\frac{1}{r_a r_b}\right)}$$

is the storage time.

Staircase Exponential Processes

equation for field dynamics:

$$E(t) = t_a E_{\text{in}}(t) + r_a r_b E(t - 2T)$$

abrupt shutdown of incident field:

$$E_{\text{in}}(t) = A\,\theta(-t),$$

where $\theta(t)$ is Heaviside step function.

field in the cavity (sum over round-trips):

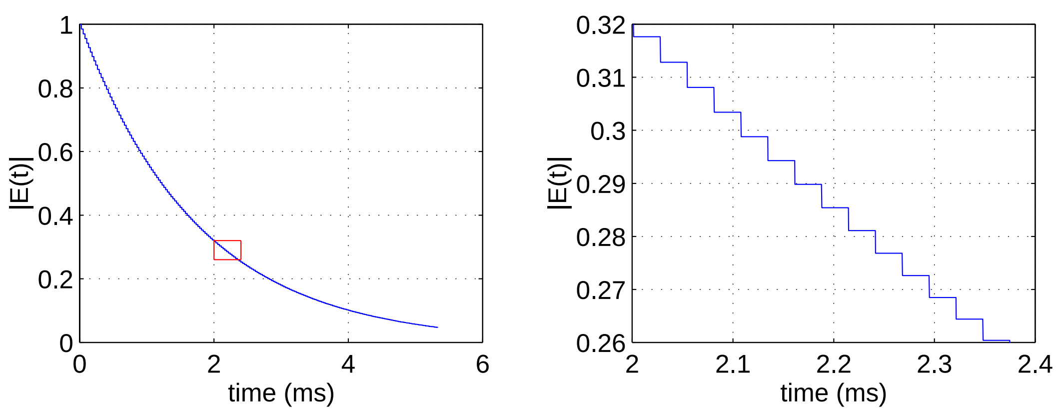
$$E(t) = (r_a r_b)^{n(t)+1} \bar{E},$$

where $\bar{E}$ is the amplitude of the equilibrium field:

$$\bar{E} = \frac{t_a A}{1 - r_a r_b}.$$

for large n

$$E(t) = \bar{E}\, e^{-t/\tau}$$



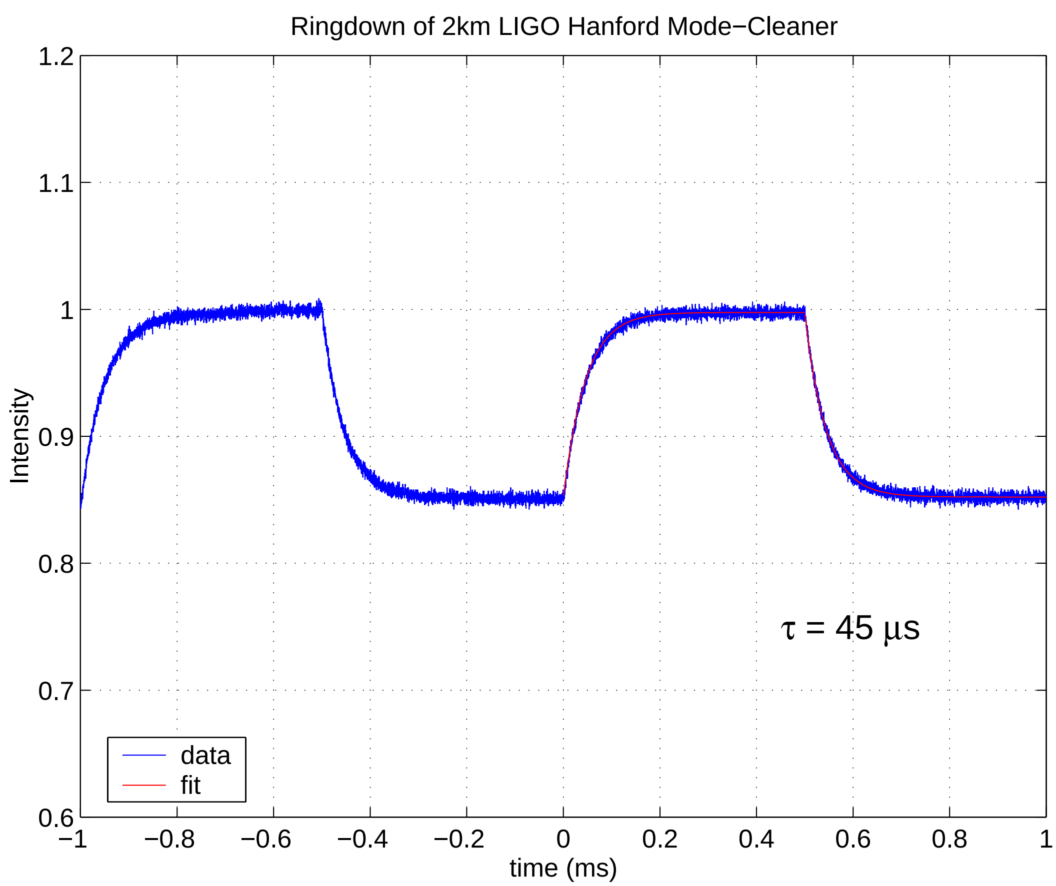
Square-Wave Intensity Modulation

amplitude buildup:

$$E(t) = A - Be^{-t/\tau}$$

power buildup:

$$\begin{aligned} P(t) &\equiv E(t)^2 \\ &= A^2 - 2ABe^{-t/\tau} + B^2 e^{-2t/\tau} \end{aligned}$$



Power Transients in Detuned Cavity

detuning phase: $\phi = k\xi$

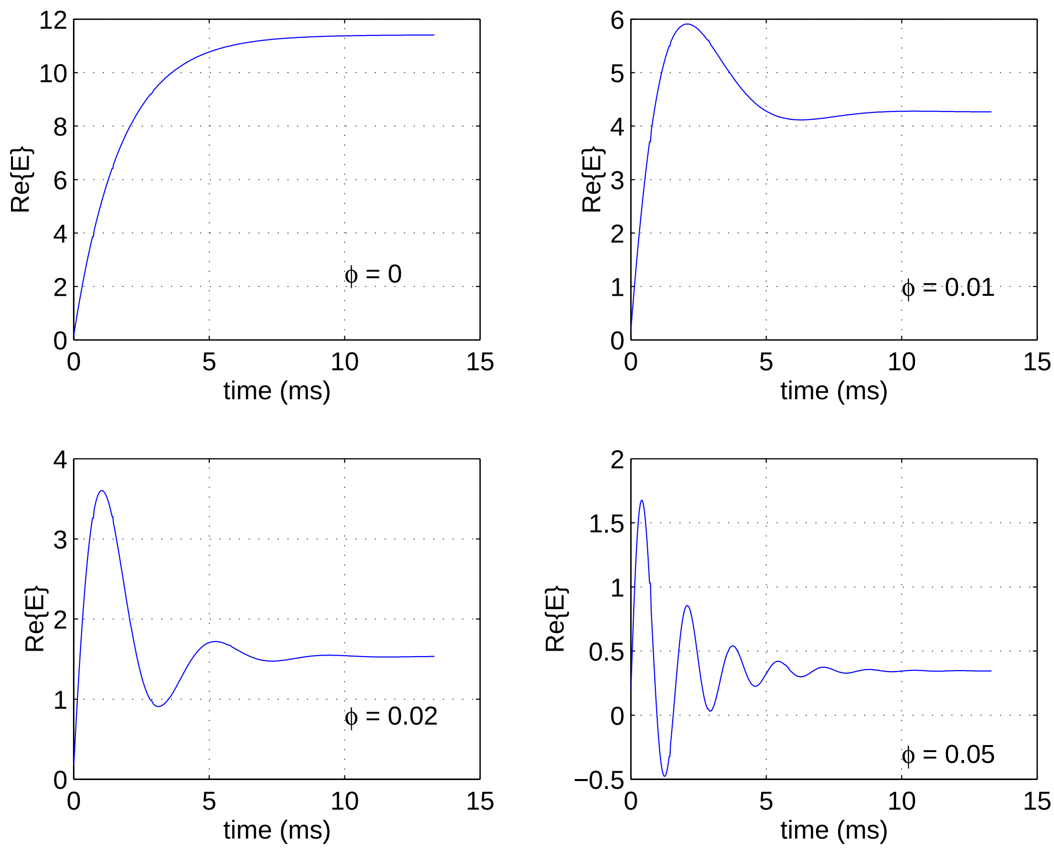
$$q = r_a r_b e^{-2i\phi}$$

field in the cavity:

$$E(t) = \left[1 - q^{n(t)+1}\right] \bar{E}$$

equilibrium amplitude:

$$\bar{E} = \frac{t_a A}{1-q}.$$



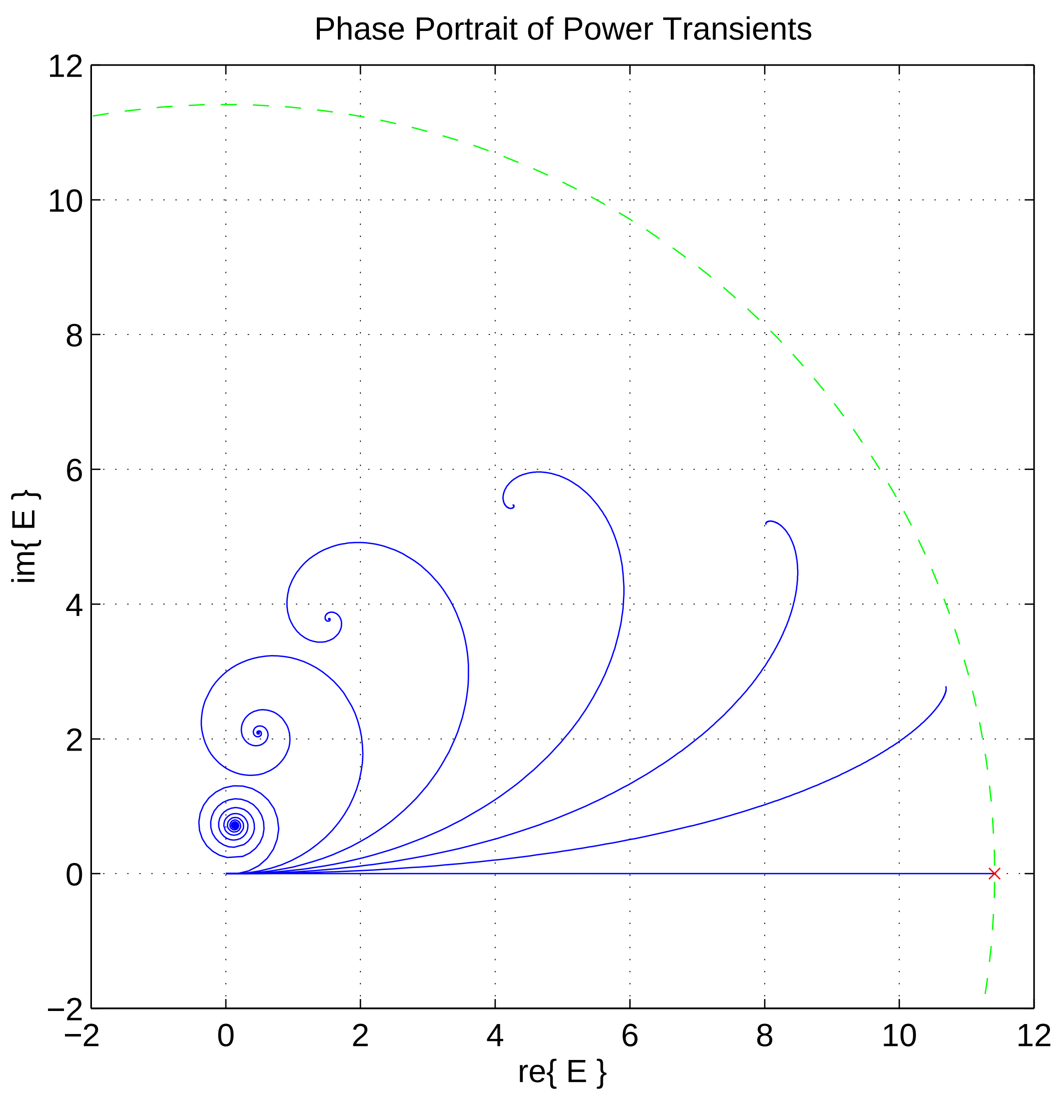
Oscillations as Function of Detuning

frequency of the oscillations:

$$\omega = \phi / T.$$

similar to Cornu spirals

(difference: the spirals are parametric functions of time)



Doppler Transient

Shutdown of power and simultaneous push on one of the mirrors at $t = 0$.

mirror velocity v then the cavity length:

$$\xi(t) = v(t - T).$$

cavity field:

$$E(t) = r_a r_b e^{-2ikv(t-T)} E(t - 2T),$$

The solution is

$$E(t) = (r_a r_b)^{n(t)+1} e^{i\phi(t)} E_0,$$

where $\phi(t)$ is the phase of the field, and $n(t)$ is the number of round-trips, and E_0 is the initial amplitude.

to find the phase:

1) find frequency shift in one reflection:

$$\Delta\omega = -2\frac{v}{c}\omega = -2kv$$

2) find total frequency shift by the time t:

$$\omega_s(t) = n(t + T)\ \Delta\omega$$

3) integrate the frequency shift:

$$\phi(t) = -2kv\,[t - Tn(t + T)]\, n(t + T)$$

continuous approximation (large n):

$$\phi(t) \approx -\frac{kv}{2T} t^2$$

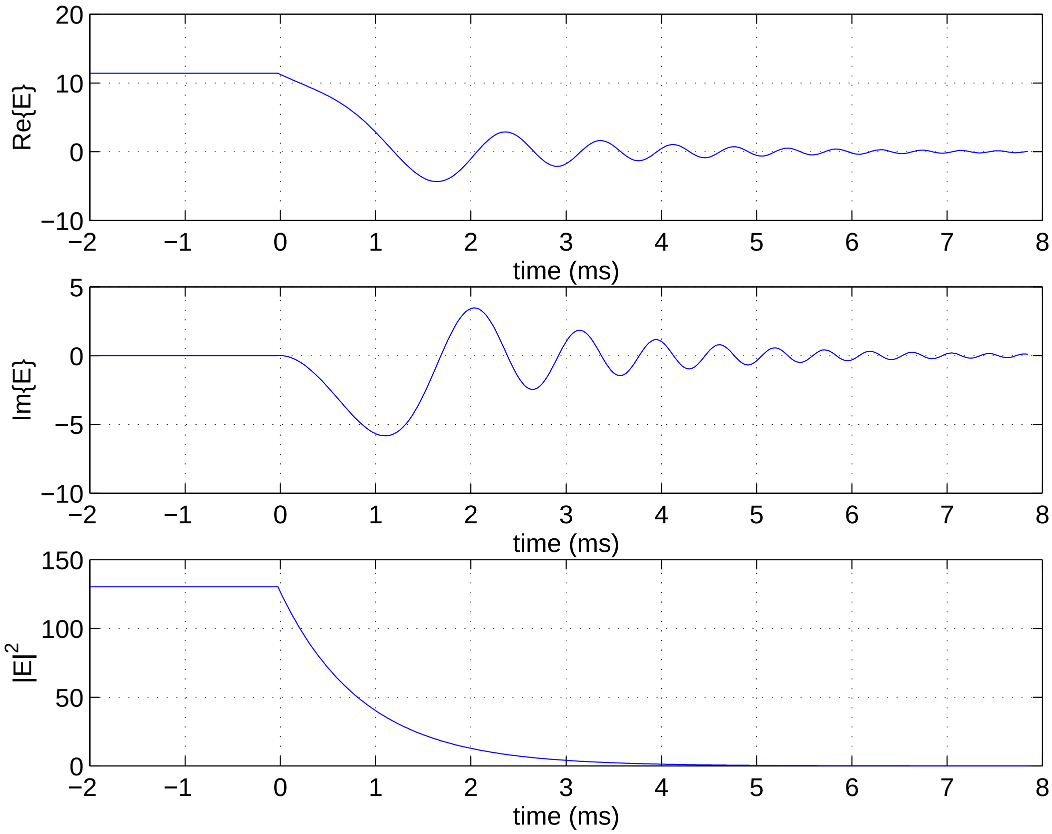
Frequency of Oscillations

the solution is

$$E(t) \approx E_0 \ \exp\left(-\frac{t}{\tau} - i\frac{kv}{2T}t^2\right),$$

frequency of oscillations = accumulated Doppler shift

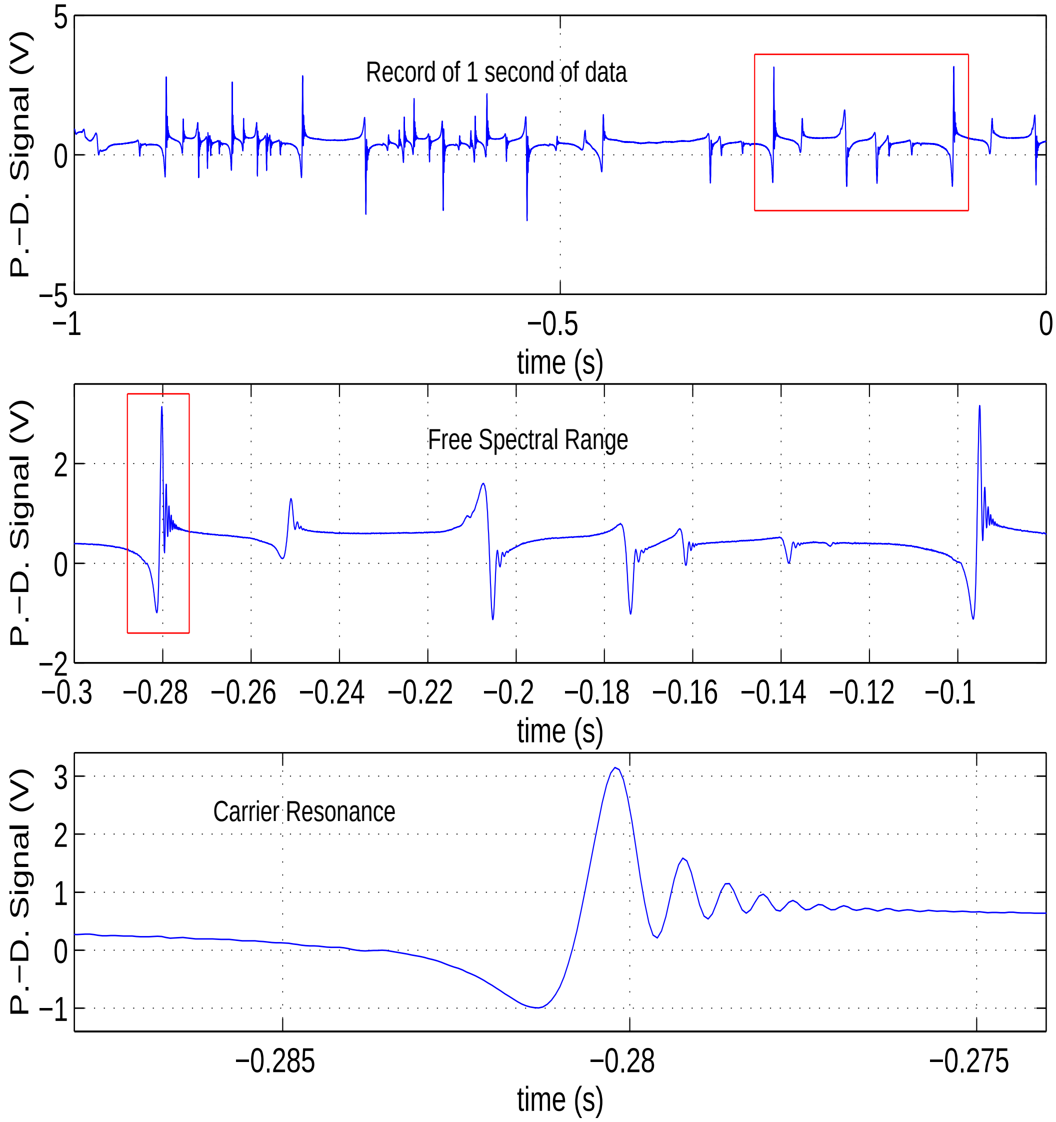
$$|\omega_s(t)| \equiv \left|\frac{d\phi}{dt}\right| = \frac{k|v|}{T}t,$$



$v = 5.0 \times 10^{-6}$ m/s

there is no interference between the incident field and the cavity field.

LIGO Hanford 2km FP Transient



Length Sweep Transient

constant incident field:

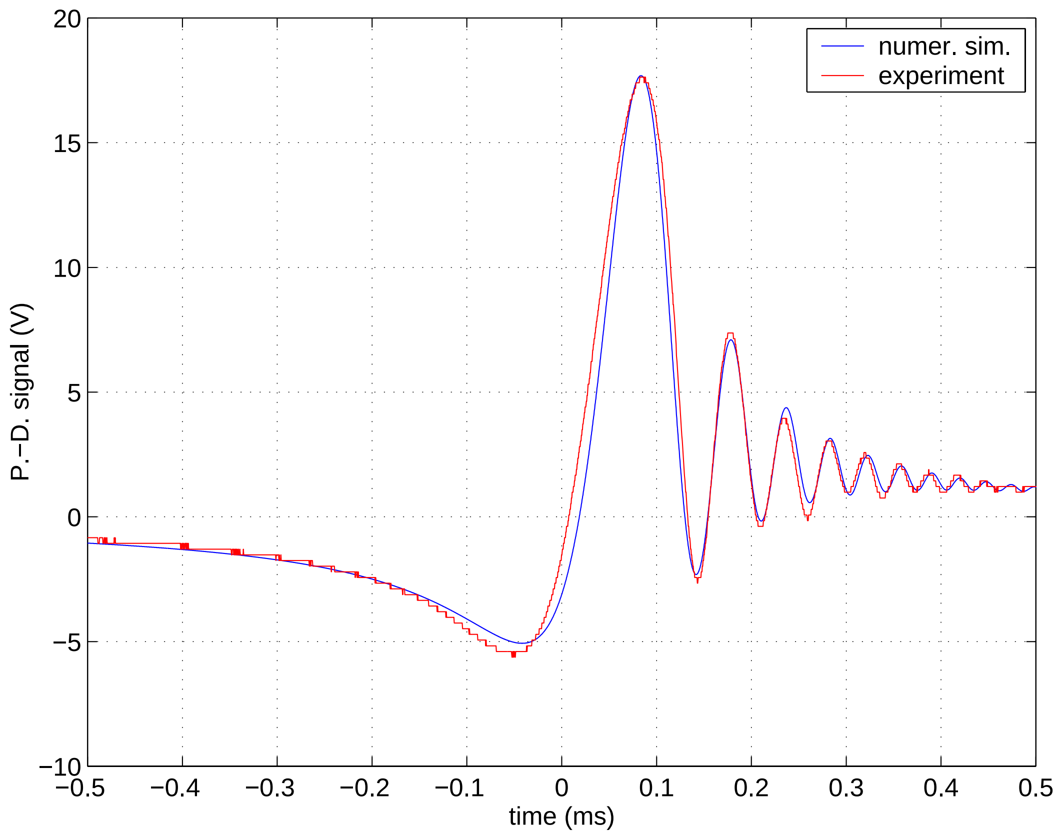
$$E_{\text{in}}(t) = A$$

pendular mirror motion (constant velocity within one resonance):

$$x(t) = vt.$$

field in the cavity:

$$E(t) = t_a A + r_a r_b e^{-2ikx(t-T)} E(t - 2T).$$



Dynamic Regimes and Critical Velocity

adiabatic regime:

$$E(t) = \frac{t_a A}{1 - r_a r_b e^{-2ikvt}}.$$

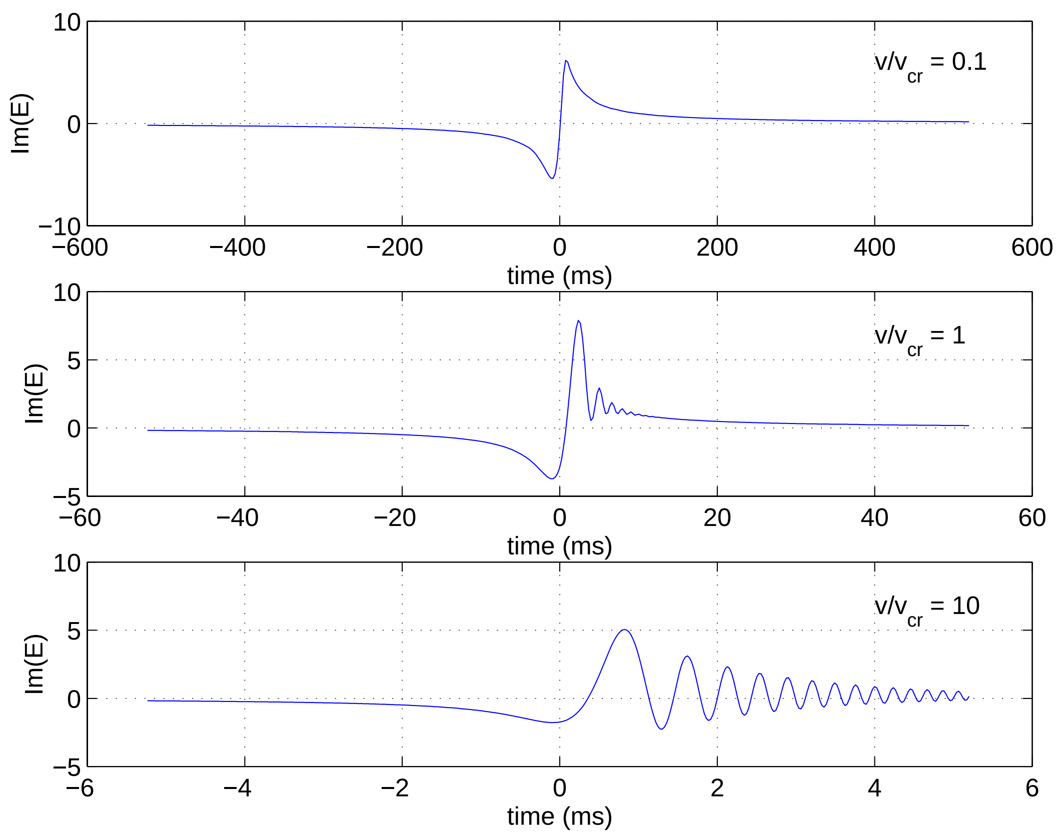
delay regime:

$$E(t) \approx D_0 \ \exp\left(-\frac{t}{\tau} - i\frac{kv}{2T}t^2\right) + \frac{t_a A}{1 - r_a r_b e^{-2ikvt}}.$$

for $t > 0$.

critical velocity:

$$v_{\text{cr}} \equiv \frac{\lambda}{2\tau\mathcal{F}} \approx \frac{\pi c \lambda}{4L\mathcal{F}^2}.$$



Measurement of Cavity Finesse

Adjusted Pound-Drever signal (with adiabatic component removed):

$$V_D(t) = Ae^{-(t-t_0)/\tau} \sin\left[\gamma - \frac{kv}{2T}(t-t_0)^2\right],$$

γ is an oscillator phase.

envelop of oscillations:

$$|V_D(t)| = A\, e^{-(t-t_0)/\tau}$$

exponential fit to the envelope $\rightarrow$ storage time.

storage time $\rightarrow$ coefficient of finesse:

$$F = \frac{1}{\sinh^2 \frac{T}{\tau}},$$

finesse:

$$\mathcal{F} = \frac{2}{\pi}\sqrt{F},$$

result of the fit:

$$\mathcal{F} = 1066 \pm 58$$

for comparison, the measurement of the mirror reflectivities:

$$F = \frac{4 r_a r_b}{(1 - r_a r_b)^2},$$

lead to

$$\mathcal{F} \approx 1050$$

Doppler Shift Accumulation

t_n = peak positions (or zero-crossings):

$$\Delta t_n = t_{n+1} - t_n, \quad \bar{t}_n = (t_n + t_{n+1})/2$$

average frequency of oscillations: $\bar{\nu}_n = \frac{1}{2\Delta t_n}$.

linear function:

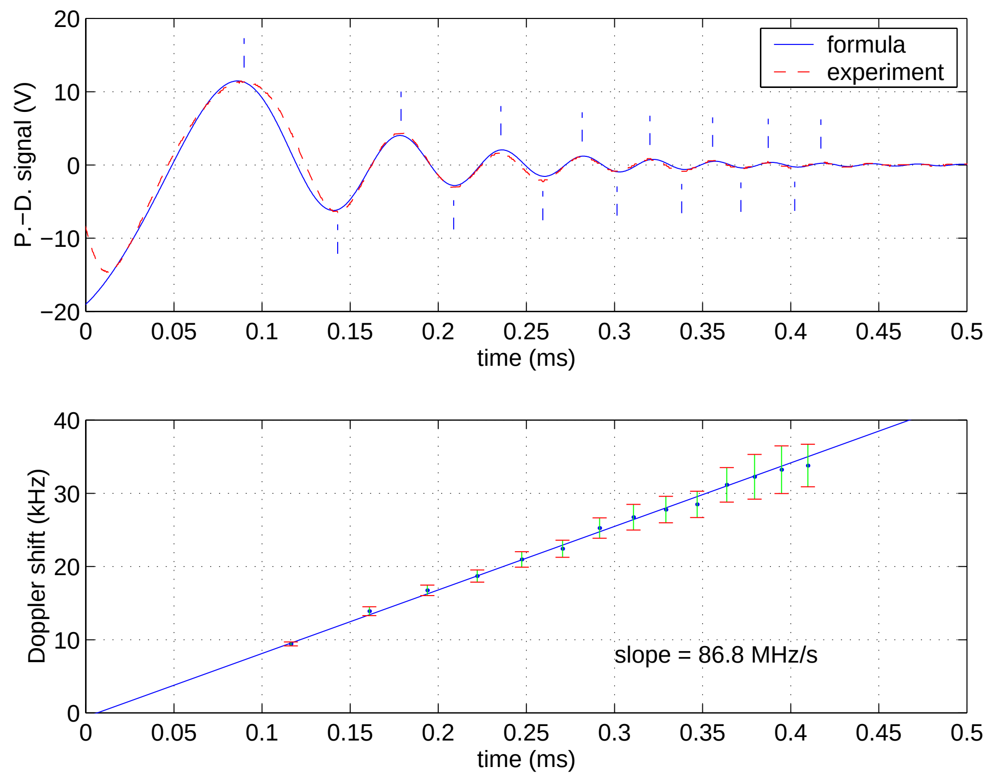
$$\bar{\nu}_n = \frac{v}{\lambda T}(\bar{t}_n - t_0).$$

rate of frequency shift:

$$\text{slope} = 86.8 \pm 0.6 \quad \text{MHz/s}$$

velocity of the mirror:

$$v = (5.7 \pm 0.4) \times 10^{-6} \text{ m/s}$$



Frequency Sweep Transient

the rate of the frequency scan: $u = d\omega/dt$.

critical rate:

$$u_{\text{cr}} = \frac{1}{2}\left(\frac{\pi c}{L\mathcal{F}}\right)^2. \tag{1}$$

the amplitude of the input beam:

$$E_{\text{in}}(t) = Ae^{iut^2/2} \tag{2}$$

the amplitude of the field in the cavity:

$$E(t) = t_a Ae^{iut^2/2} + r_a r_b E(t - 2T) \tag{3}$$

equivalence:

$$kv \to uT$$

the approximate solution:

$$E(t) \approx \frac{t_a A\ e^{iut^2/2}}{1 - r_a r_b e^{-2iuTt}} + D_0 e^{-t/\tau}$$

natural identity:

$$u_{\text{cr}} T = k v_{\text{cr}}$$

(the critical rate does not depend on the laser frequency.)