Signal Processing Schemes for LISA

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References:

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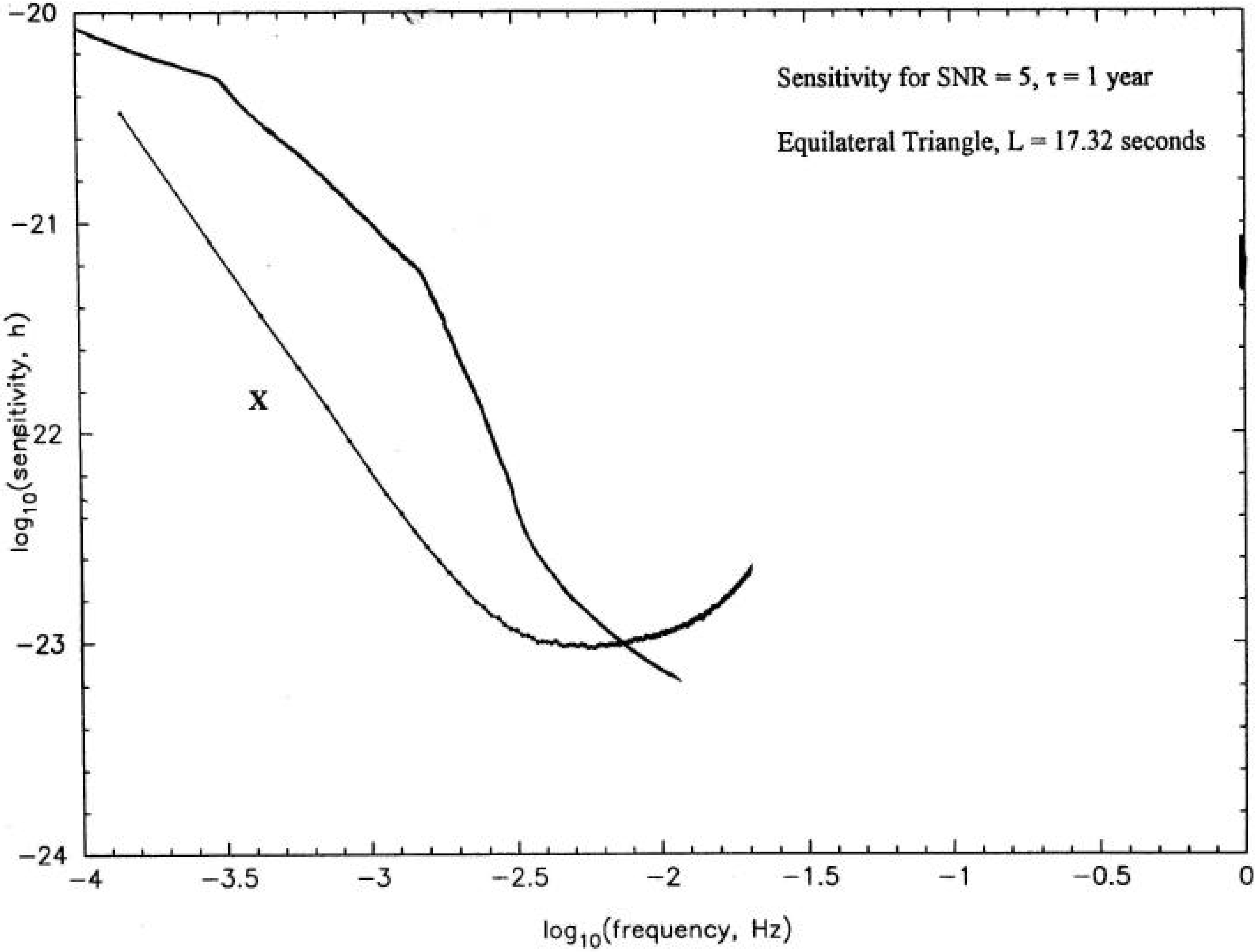
CONFUSION NOISE LEVEL AND LISA SENSITIVITY

- The first measurements that LISA will take can be used to assess its performance.

- The existence of a gravitational wave stochastic background, from many close binary systems in our Galaxy, appears to prevent us from identifying the noise level of the interferometer in the band 0.1 - 8 mHz [1, 2].

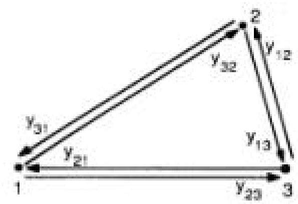
- In other words, we may not be able to reliably detect such a strong background because we will not know how to distinguish it from instrumental noise.



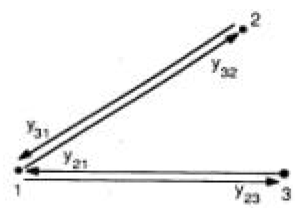
Sensitivity for SNR = 5, τ = 1 year
Equilateral Triangle, L = 17.32 seconds
X
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0

CONFUSION NOISE LEVEL AND LISA SENSITIVITY (Cont.)

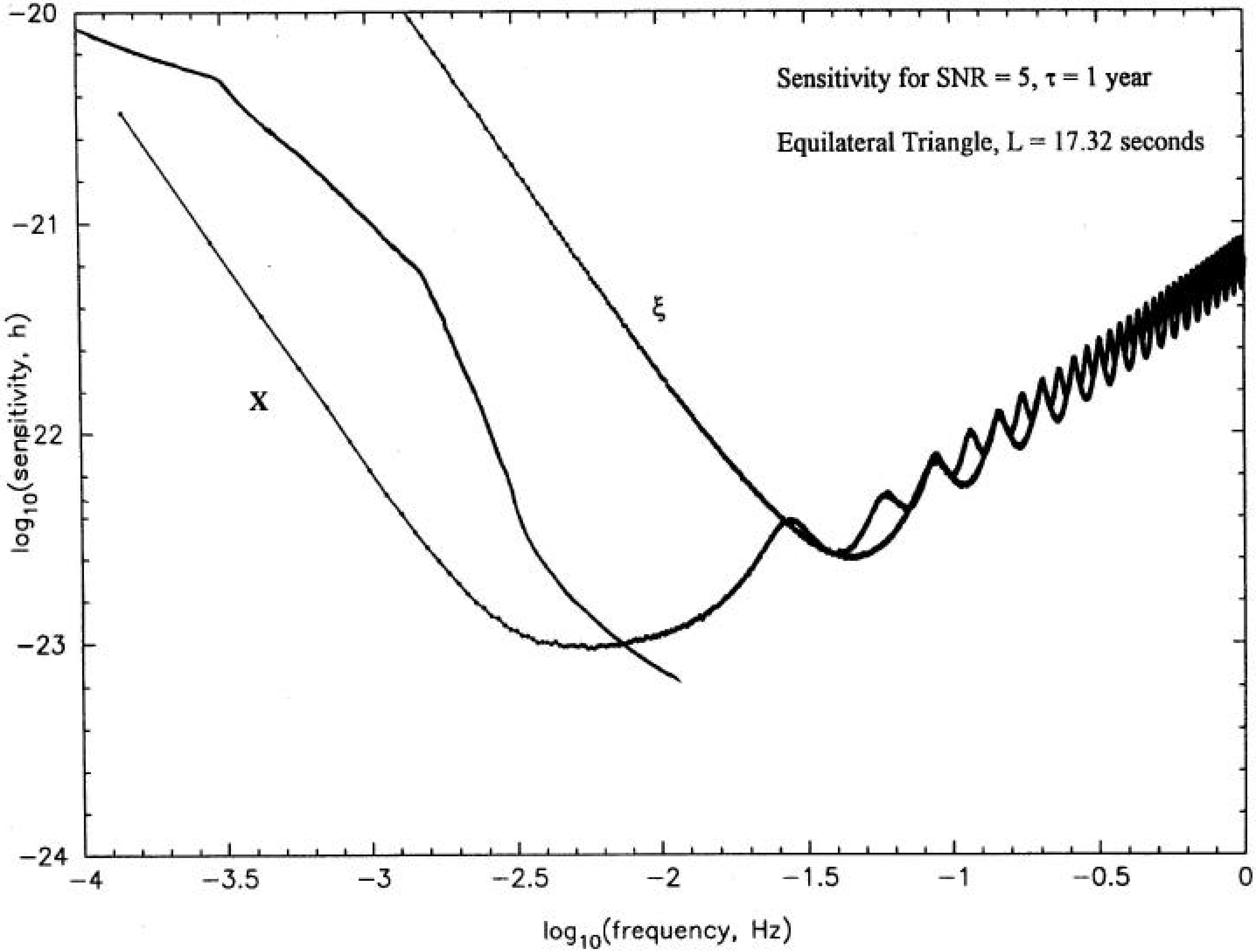
- Can we build a "gravitational wave shield" for LISA?
- Can we find a combination of the data that has zero-response to a gravitational wave (and non-zero response to instrumental noise), and allows us to estimate the LISA sensitivity on-orbit?
- Time-Delay Interferometry with multiple readouts provides such a capability.
- From [3, 4, 5] we have seen that there exists a 3-dimensional manifold of combinations of one-way measurements that are laser & bench noise free.
- Among the elements of this manifold we have considered, the Sagnac combination, $\mathbf{x}$, reduces the signal level by several orders of magnitude in the confusion-limited band, with respect to the regular interferometric combination X.

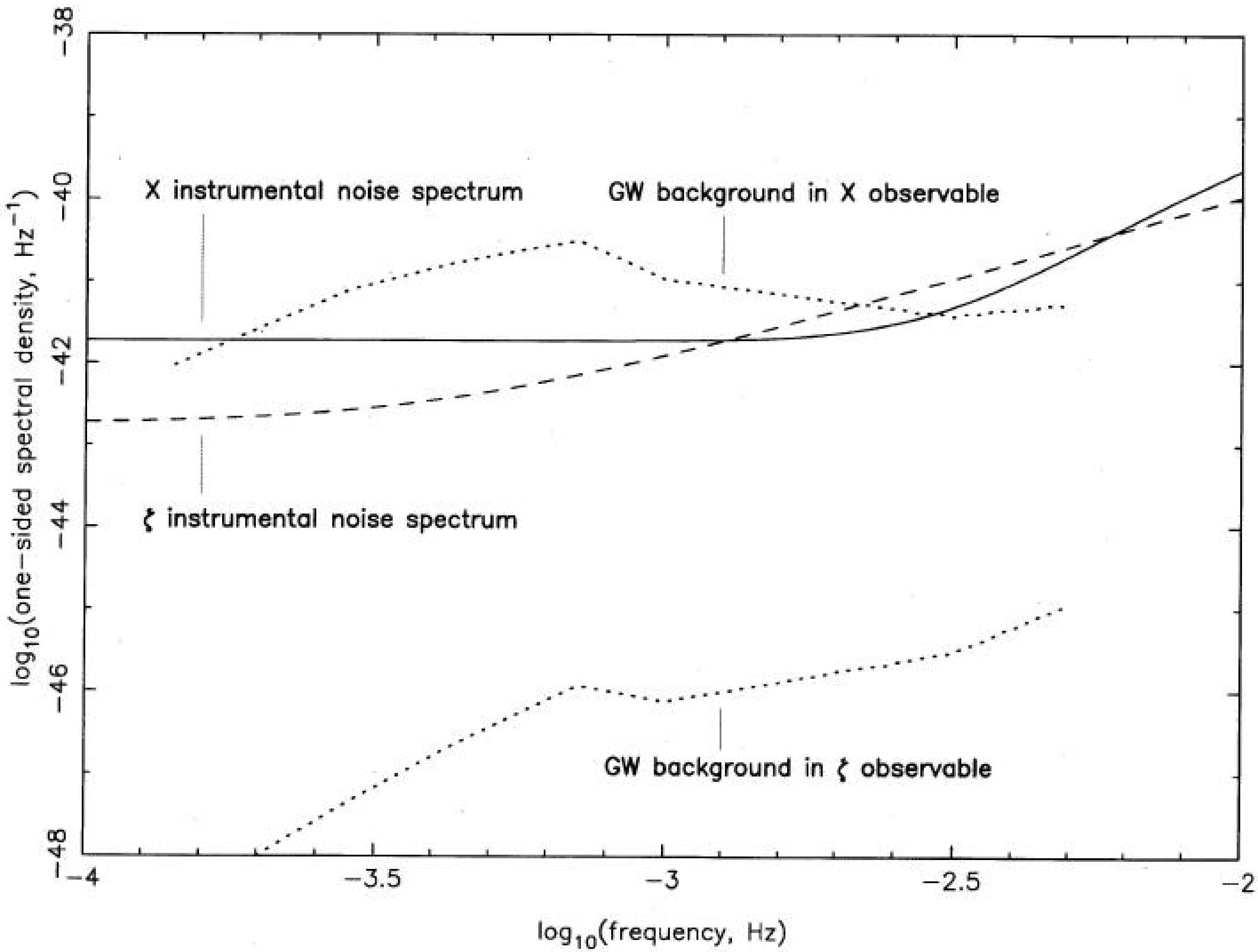


ζ (t)



X (t)


Sensitivity for SNR = 5, τ = 1 year
Equilateral Triangle, L = 17.32 seconds
ξ
X
log10(sensitivity, h)
log10(frequency, Hz)
−20
−21
−22
−23
−24
−4
−3.5
−3
−2.5
−2
−1.5
−1
−0.5
0


X instrumental noise spectrum
GW background in X observable
ζ instrumental noise spectrum
GW background in ζ observable
log10(one-sided spectral density, Hz⁻¹)
log10(frequency, Hz)
−38
−40
−42
−44
−46
−48
−4
−3.5
−3
−2.5
−2

NOISE CALIBRATION and DETECTION OF A STOCHASTIC BACKGROUND

- The gravitational wave background will be below the anticipated sensitivity curve of **x** by several orders of magnitude.

- The Sagnac combination provides a way for estimating the instrumental noise sources: Sagnac greatly attenuates the gravitational wave signal, but instrumental noise persists.

- This allows us to estimate the LISA instrumental noise in the Michelson interferometer mode X, and in turn to detect the stochastic background.

$$\widetilde{X}^{gw}(f) \simeq 2\ (2\pi i f L)^2\ \left[\hat{n}_3 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_3 - \hat{n}_2 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_2\right]\ ,$$

$$\widetilde{\zeta}^{gw}(f) \simeq \frac{1}{12}\ (2\pi i f L)^3\ \left[(\hat{k} \cdot \hat{n}_1)(\hat{n}_1 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_1) + (\hat{k} \cdot \hat{n}_2)(\hat{n}_2 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_2) + (\hat{k} \cdot \hat{n}_3)(\hat{n}_3 \cdot \widetilde{\mathbf{h}}(f) \cdot \hat{n}_3)\right]\ ,$$

$$\begin{aligned} S_{X^{noise}}(f) &\equiv S_{X^{proofmass}}(f) + S_{X^{opticalpath}}(f) \\ &\simeq 16\,[S_1(f) + S_{1^*}(f) + S_3(f) + S_{2^*}(f)]\ (2\pi f L)^2 \\ &\quad +4\,[S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)]\ (2\pi f L)^2 \end{aligned}$$

$$\begin{aligned} S_{\zeta^{noise}}(f) &\simeq [S_1(f) + S_2(f) + S_3(f) + S_{1^*}(f) + S_{2^*}(f) + S_{3^*}(f)]\ (2\pi f L)^2 \\ &\quad + [S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f) + S_{13}(f) + S_{12}(f)]\ , \end{aligned}$$

where S_i (f) and S_{ij} (f) are the power spectral densities associated with the proof mass and optical path Doppler noises

$$S_X^{obs}(f) = S_{X^{gw}}(f) + S_{X^{proofmass}}(f) + S_{X^{opticalpath}}(f)$$

$$\begin{aligned} S_\zeta^{obs}(f) &= \frac{1}{16}\left[S_{X^{proofmass}}(f) + \frac{S_{X^{opticalpath}}(f)}{(\pi f L)^2}\right] + [S_2(f) + S_{3^*}(f)]\ (2\pi f L)^2 \\ &\quad + [S_{13}(f) + S_{12}(f)]\ , \end{aligned}$$

If the spectrum of ζ is at the anticipated level, we may conclude that the noise spectrum of X is known.

$$S_{X_{gw}}(f) = S_X^{obs}(f) - 64\ S^0(f)(2\pi f L)^2 - 16\ S^1(f)(2\pi f L)^2\ ,$$

The noise contributed by any one of the proof masses and optical-path noise sources is unlikely to be smaller than their design values, S^0 (f) and S^1 (f) respectively.

$$\begin{aligned} S_X^{obs}(f) - 16\ S_\zeta^{obs}(f) = S_{X_{gw}} &- 16\,[S_2(f) + S_{3^*}(f)]\,(2\pi f L)^2 \\ &-16\,[S_{13}(f) + S_{12}(f)] \\ &-16\,[S_{32}(f) + S_{23}(f) + S_{31}(f) + S_{21}(f)]\,\left[1 - (\pi f L)^2\right]. \end{aligned}$$

The noise terms on the right-hand-side are all negative-definite, and can be bound from above by their design, or nominal, values

The equation below provides a lower bound for experimental discrimination of the gravitational wave background spectrum.

$$S_{X_{gw}}(f) \geq S_X^{obs}(f) - 16\ S_\zeta^{obs}(f) + 32\ (2\pi f L)^2 S^0(f) + 16\ [6 - (2\pi f L)^2] S^1(f).$$

IS x THE "OPTIMAL COMBINATION?

- In the Fourier domain, we have been able to derive the following equations

$$X(f) + Y(f) + Z(f) = \delta(f L)\, \xi(f)$$

$$\alpha(f) + \beta(f) + \gamma(f) = \varepsilon(f L)\, \xi(f)$$

$$P(f) + Q(f) + R(f) = \kappa(f L)\, \xi(f)$$

$$E(f) + F(f) + G(f) = \eta(f L)\, \xi(f)$$

$$U(f) + V(f) + W(f) = \rho(f L)\, \xi(f)$$

where $\delta(f L)$, $\varepsilon(f L)$, $\kappa(f L)$, $\eta(f L)$, $\rho(f L)$ are known analytic functions of $(f L)$

- The above equations tell us that, in the low-frequency part of the band, the combinations on the left-hand sides, and ξ, have identical signal-to-noise ratios.

Conclusions

- We have shown that there exist several linear combinations of the one-way data that minimize the gravitational wave signal.
- This additional data requires readouts at all three spacecraft.

- In the frequency region of interest they all display the same sensitivity as the Sagnac combination, **x**.

- By using **x** we can estimate the magnitude of the noise sources affecting the LISA response X in the low-frequency band.
- This allows discrimination between a confusion-limited gravitational wave background and instrumental noise.