Chaos in Spinning Compact Binaries?

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Outline

- Motivation[1]
- Characterizing chaotic systems
- Test-particle motion in Hamiltonian system
- Compact binary orbits
  - Post-Newtonian (PN) equations of motion
  - Spin effects
  - Phase-space trajectories
  - Waveforms
- Conclusions

[1] J. Levin, *Phys. Rev. Lett.* **84**, 3515 (2000); gr-qc/0010100
comments: S. A. Hughes, gr-qc/0101024; N. J. Cornish, gr-qc/0101041

Nearby trajectories in chaotic systems diverge exponentially

- The Poincare surface-of-section method for identifying chaos generally only works for systems with two degrees of freedom (4-dimensional phase space)
- For systems with many degrees of freedom, the most definitive method for identifying chaos is by measuring Lyapunov exponents [2]
- If the distance in phase-space d grows like:

$$d = d_o \exp(\gamma t)$$

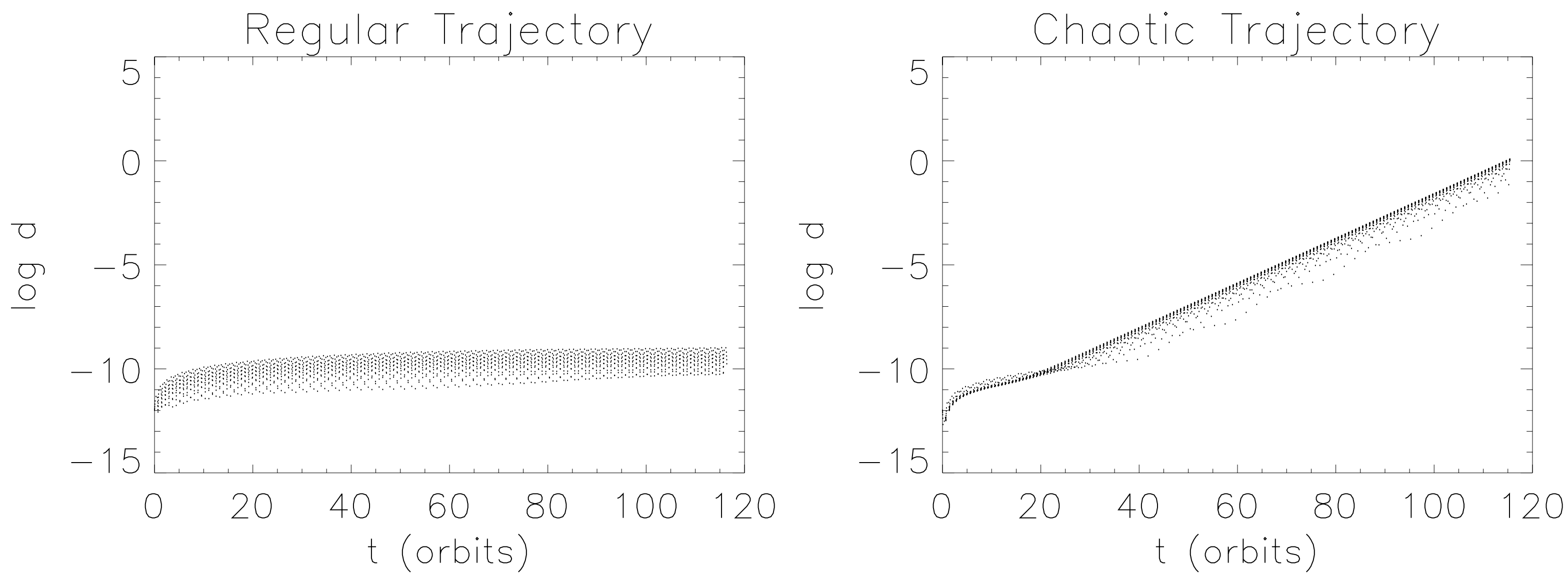
then we can define the characteristic Lyapunov exponent γ as

$$\gamma \equiv \lim_{t \to \infty} \frac{ln(d/d_o)}{t - t_o}$$

- Chaotic systems diverge on a time-scale of the *Lyapunov time* $\equiv \frac{1}{\gamma}$
- For regular (quasi-periodic) systems, $\gamma \to 0$

[2] G. J. Sussman and J. Wisdom. *Science* **241**, 433 (1988).

Test particles orbiting around two fixed point masses exhibit quasi-periodic as well as chaotic behavior [3]



[3] G. Contopoulos, *Proc. R. Soc. Lond. A* **435**, 551 (1991).

We use Post-Newtonian equations of motion to calculate trajectories with spin effects [4]

$$\vec{\mathbf{a}}_{\mathbf{N}} = -\frac{m}{r^2}\hat{\mathbf{n}}$$

$$\vec{\mathbf{a}}_{\mathbf{1PN}} = -\frac{m}{r^2}\left\{\hat{\mathbf{n}}\left[(1+3\eta)v^2 - 2(2+\eta)\frac{m}{r} - \frac{3}{2}\eta\dot{r}^2\right] - 2(2-\eta)\dot{r}\vec{\mathbf{v}}\right\}$$

$$\begin{aligned}\vec{\mathbf{a}}_{\mathbf{2PN}} = & -\frac{m}{r^2}\left\{\hat{\mathbf{n}}\left[\frac{3}{4}(12+29\eta)\left(\frac{m}{r}\right)^2 + \eta(3-4\eta)v^4 + \frac{15}{8}\eta(1-3\eta)\dot{r}^4\right.\right.\\ & \left. -\frac{3}{2}\eta(3-4\eta)v^2\dot{r}^2 - \frac{1}{2}\eta(13-4\eta)\frac{m}{r}v^2 - (2+25\eta+2\eta^2)\frac{m}{r}\dot{r}^2\right]\\ & \left. -\frac{1}{2}\dot{r}\vec{\mathbf{v}}\left[\eta(15+4\eta)v^2 - (4+41\eta+8\eta^2)\frac{m}{r} - 3\eta(3+2\eta)\dot{r}^2\right]\right\}\end{aligned}$$

$$\vec{\mathbf{a}}_{\mathbf{SO}} = \frac{1}{r^3}\left\{6\hat{\mathbf{n}}[(\hat{\mathbf{n}}\times\vec{\mathbf{v}})\cdot(2\vec{\mathbf{S}} + \frac{\delta m}{m}\vec{\boldsymbol{\Delta}})] - [\vec{\mathbf{v}}\times(7\vec{\mathbf{S}} + 3\frac{\delta m}{m}\vec{\boldsymbol{\Delta}})] + 3\dot{r}[\hat{\mathbf{n}}\times(3\vec{\mathbf{S}} + \frac{\delta m}{m}\vec{\boldsymbol{\Delta}})]\right\}$$

$$\vec{\mathbf{a}}_{\mathbf{SS}} = -\frac{3}{\mu r^4}\left\{\hat{\mathbf{n}}(\vec{\mathbf{S}}_{\mathbf{1}}\cdot\vec{\mathbf{S}}_{\mathbf{2}}) + \vec{\mathbf{S}}_{\mathbf{1}}(\hat{\mathbf{n}}\cdot\vec{\mathbf{S}}_{\mathbf{2}}) + \vec{\mathbf{S}}_{\mathbf{2}}(\hat{\mathbf{n}}\cdot\vec{\mathbf{S}}_{\mathbf{1}}) - 5\hat{\mathbf{n}}(\hat{\mathbf{n}}\cdot\vec{\mathbf{S}}_{\mathbf{1}})(\hat{\mathbf{n}}\cdot\vec{\mathbf{S}}_{\mathbf{2}})\right\}$$

The spins also precess due to frame-dragging and the Lens-Thirring effect.

$$\dot{\vec{\mathbf{S}}}_{\mathbf{1}} = \vec{\boldsymbol{\Omega}}_{\mathbf{1}}\times\vec{\mathbf{S}}_{\mathbf{1}} \qquad \dot{\vec{\mathbf{S}}}_{\mathbf{2}} = \vec{\boldsymbol{\Omega}}_{\mathbf{2}}\times\vec{\mathbf{S}}_{\mathbf{2}}$$

where

$$\vec{\boldsymbol{\Omega}}_{\mathbf{1}} \equiv \frac{1}{r^3}\left[\left(2+\frac{3m_2}{2m_1}\right)\vec{\mathbf{L}}_{\mathbf{N}} - \vec{\mathbf{S}}_{\mathbf{2}} + 3(\hat{\mathbf{n}}\cdot\vec{\mathbf{S}}_{\mathbf{2}})\hat{\mathbf{n}}\right] \quad \text{and} \quad \vec{\boldsymbol{\Omega}}_{\mathbf{2}} \equiv \frac{1}{r^3}\left[\left(2+\frac{3m_1}{2m_2}\right)\vec{\mathbf{L}}_{\mathbf{N}} - \vec{\mathbf{S}}_{\mathbf{1}} + 3(\hat{\mathbf{n}}\cdot\vec{\mathbf{S}}_{\mathbf{1}})\hat{\mathbf{n}}\right]$$

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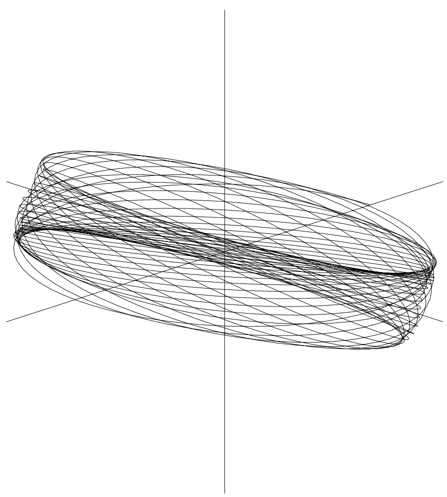
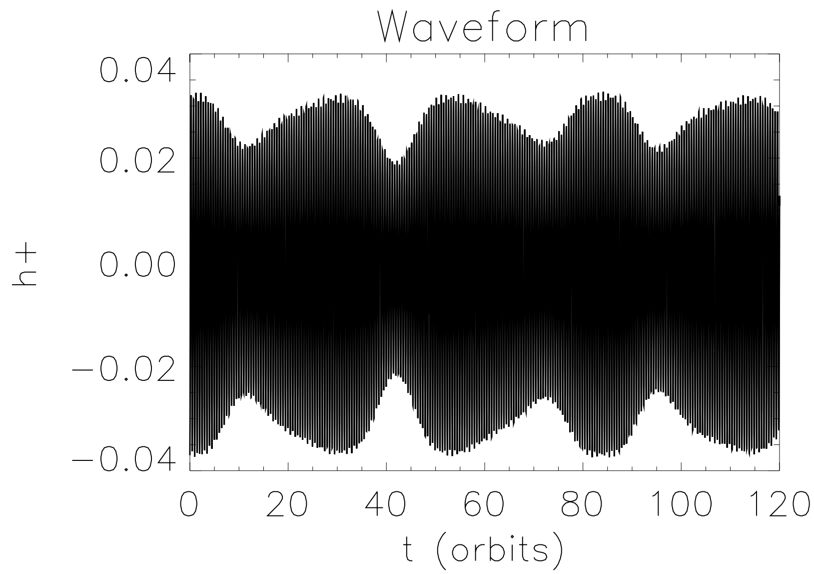
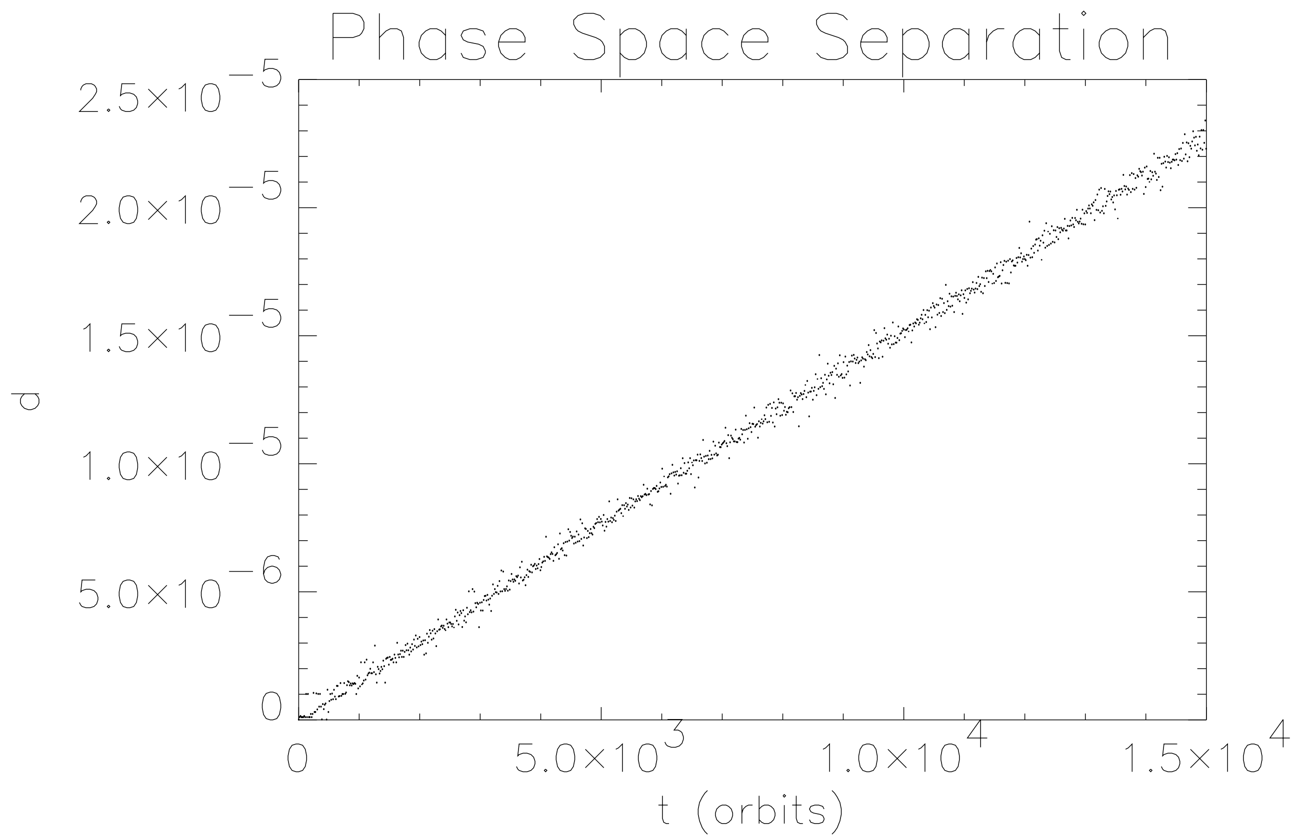
[4] L. E. Kidder, C. M. Will, and A. G. Wiseman, *Phys. Rev. D* **47**, R4183 (1993).

With both objects spinning maximally, the orbits appear to be irregular and perhaps even "chaotic"...





...but NO CHAOS IS OBSERVED, even on a time scale much longer than the typical in-spiral time



Conclusions and future work

- We can identify chaotic and quasi-periodic orbits for test-particle motion in a Hamiltonian system by measuring Lyapunov exponents
- Using PN equations of motion, we have calculated compact binary trajectories including spin effects
- NO CHAOTIC behavior has been observed in the in-spiral region of the LIGO frequency band
- Future work includes modifying existing templates to account for spin effects
- Look for precessional resonance signals as system sweeps through orbital frequency band