A Comparison of Spherical, Resonant Mass Gravitational Wave Antennas with Narrowband Interferometers

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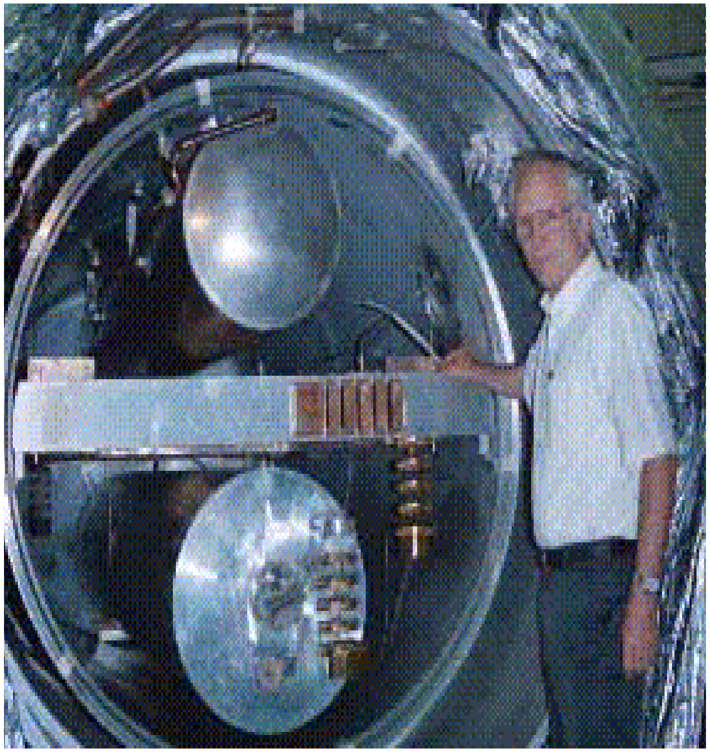
Outline

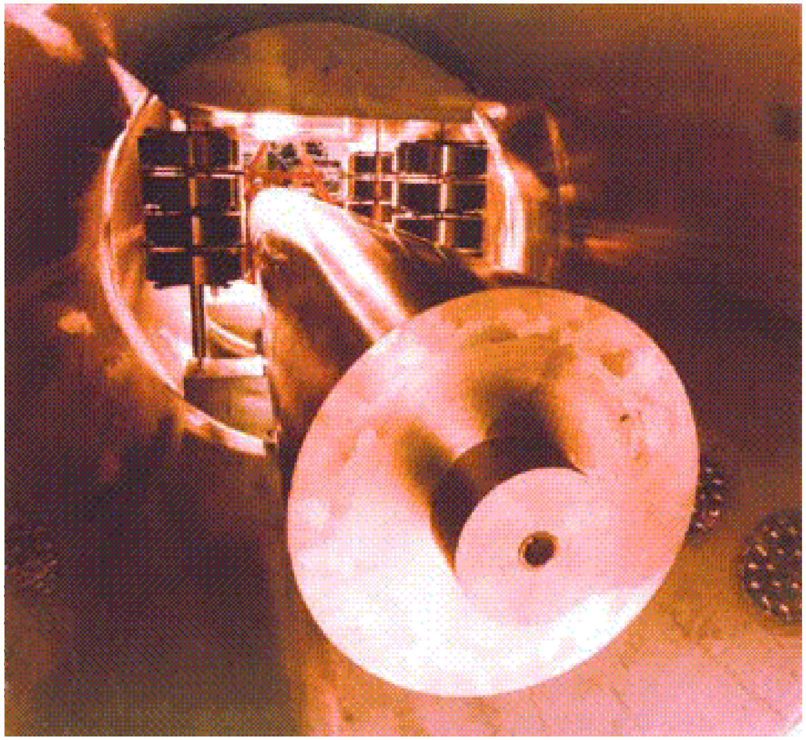
- Goal : A comparison of two different technologies for detecting gravitational waves
- Description of resonant mass antennas
  - Bar antennas
  - Spherical antennas
  - Noise
- Spheres compared with initial LIGO
  - Sources
  - Signal-to-noise ratio calculations
- Spheres compared with advanced LIGO
  - Description of resonant sideband extraction (RSE)
  - Sources
  - Signal-to-noise ratio calculations
- Conclusions

Resonant Mass Antennas

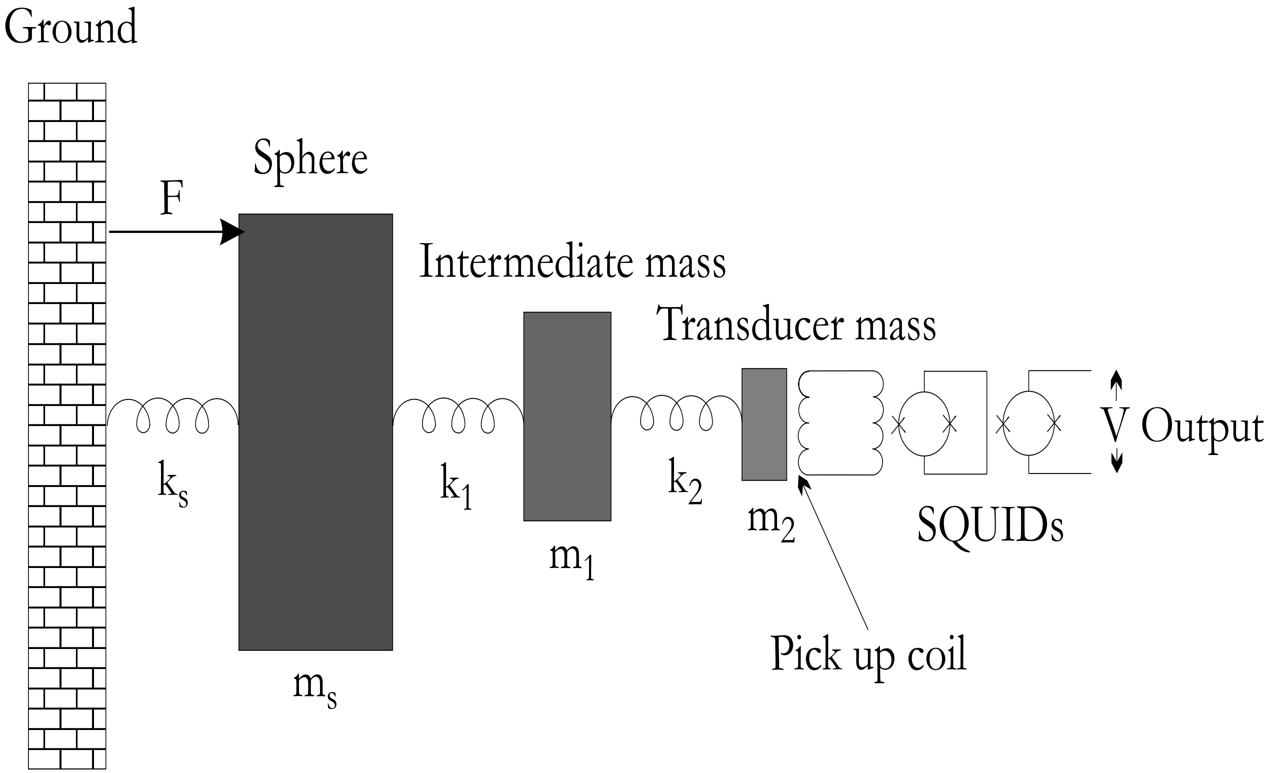
Allegro



NIOBE



Schematic Model of Resonant Mass Antenna



Operating Bar Antennas

- Three antennas in Italy
  - Nautilus outside of Rome
  - Explorer at CERN
  - AURIGA near Padua
- One antenna in the United States
  - Allegro in Baton Rouge, Louisiana
- One antenna in Australia
  - Niobe in Perth, Western Australia

Sensitivity of Resonant Mass Antennas

- Cross Section for Gravitational Waves
  - Treat gravitational wave as force
  - Calculate energy deposited
- Noise
  - Seismic
  - Thermal noise
  - Forward-action noise from transducer
  - Back-action noise from transducer

Cross Section of Bar

$$\Sigma = \frac{G}{c^3}\frac{\rho V_s^{\,5}}{f_0^{\,3}}\frac{4r^2}{L^2}\sin\Theta^4\cos(2\phi)^2$$

- High speed of sound and density gives higher cross section (aluminum, niobium, copper-beryllium, sapphire, etc.)
- Small fundamental constant out front 2.5 10^{-36}
- Angular dependence on both propagation direction (Θ) and polarization (ϕ)

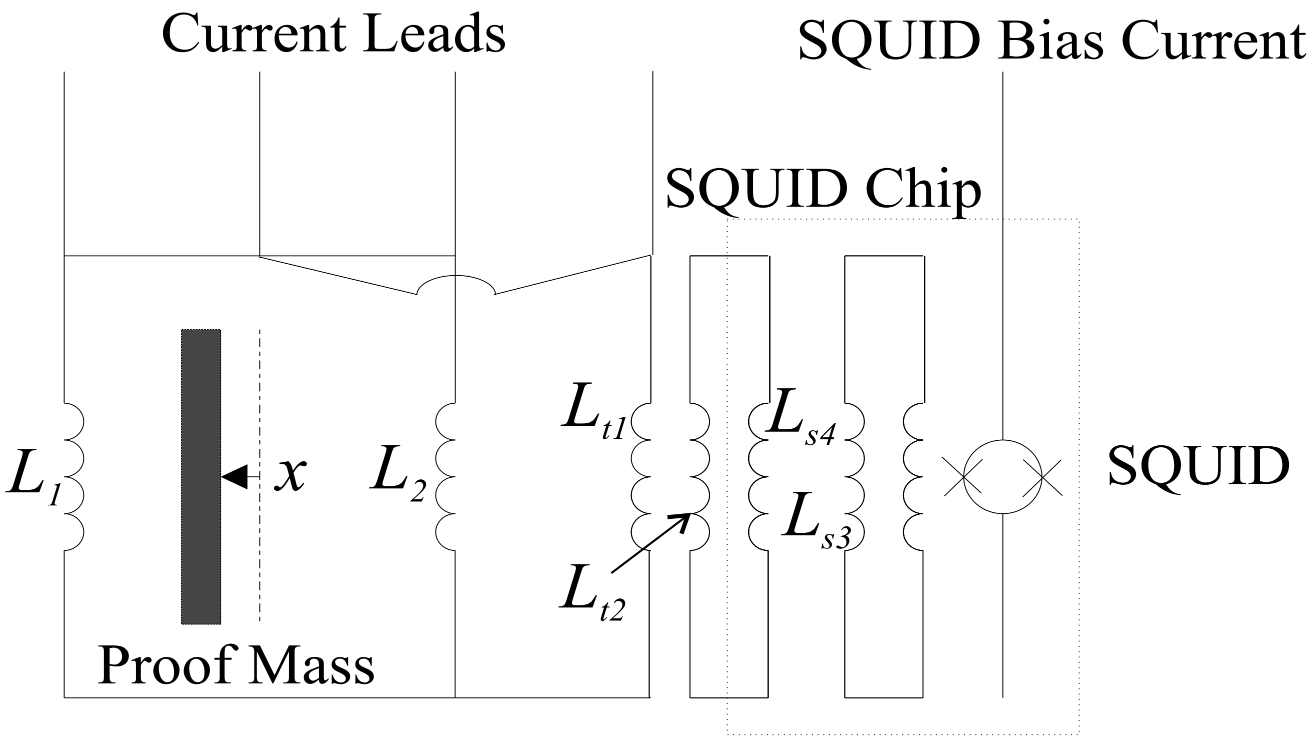
SQUID-based Transducers

- Capacitive
  - Final mass forms a capacitor with bar
  - Requires large impedance matching circuit
  - Used on AURIGA
- Inductive
  - Pick-up coils on final mass form inductor
  - Better impedance matched to bar
  - Most mature of transducer technologies
  - Used on most operating antennas

Other Transducers

- Optical transducer
  - One mirror of resonant cavity is final mass
  - High finesse gives lower noise
- Piezoelectric
  - Original transducer used by Weber
  - No longer in use
- Microwave cavity
  - Used on NIOBE in Western Australia

Transducer and SQUID Circuit



Cross Section of Spherical Resonant Mass Antennas

- Cross section different from bars:

$$\Sigma = \frac{G}{c^3}\frac{\rho V_s^5}{f_0^3}\Pi$$

  - $\Pi = 0.215$ for lowest quadrupole mode of sphere
  - No dependence on source direction or polarization
  - At same frequency, 56 times more sensitive than bar to angle averaged source

Multiple Modes of Spherical Resonant Mass Antennas

- Multiple modes at same frequency
  - Five l=2 modes
  - Five separate values from a single event
  - Source direction (2)
  - Polarization (2)
  - Tracelessness check
- Can instrument at more than one mode
  - l=0 mode can search for scalar waves
  - Higher order quadruple modes at higher frequency

Proposed Spherical Antennas

- Mario Shenberg
  - Sao Paulo Brazil
  - 65 cm diameter, Cu-Al 6%
  - 3 kHz resonant frequency
- miniGRAIL
  - Leiden, Netherlands
  - 65 cm, Cu-Al
- Sfera
  - Italy

Equations of Motion for Spherical Antenna with Transducers

- A sphere with transducers can be treated the same as a bar with the addition of a few factors of order 1 (T. Stevenson, *PRD*, 1997).
  - Factors depend on the number of transducers

- The full machinery used to analyze bars can be used on spheres, provided the number of transducers is known

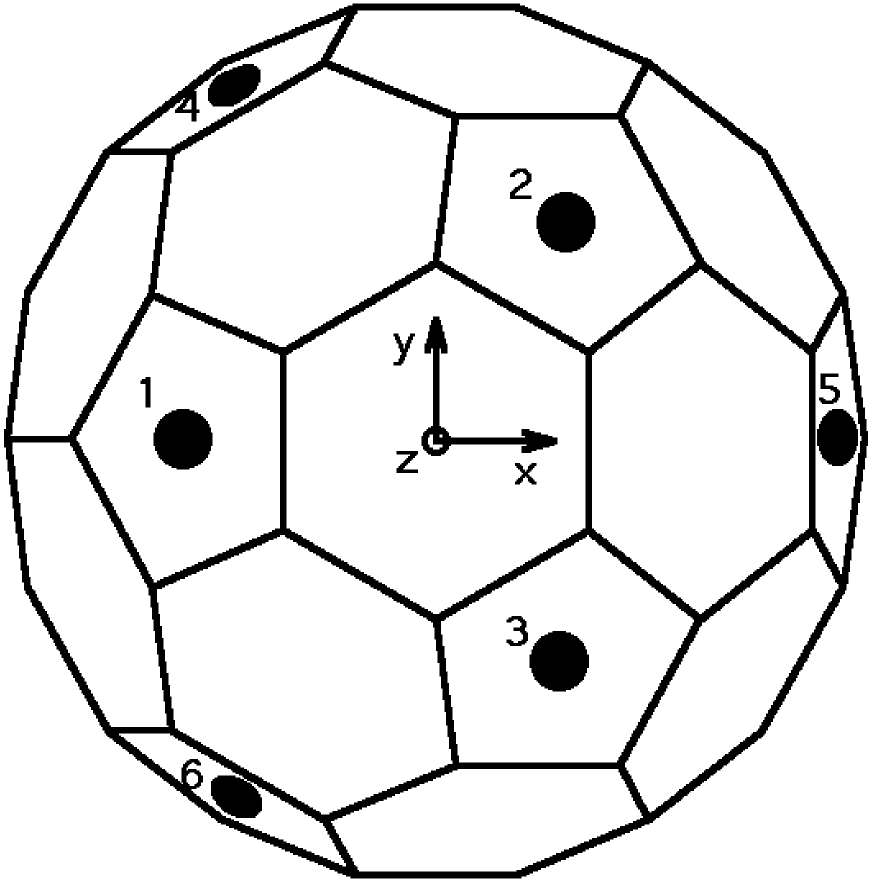
Traditional Problems with Spheres

- Spherical antennas first proposed by Forward in early 1970s
- Analyzed in 1970s by Wagoner & Paik, Ashby & Dreitlein, and others
- Much of the value of spherical antennas depends on symmetry
- How to add transducers without breaking the symmetry?
- How to add a support structure, cooling and electrical leads, etc. without breaking symmetry?

Truncated Icosahedral Gravitational-Wave Antennas (TIGA)

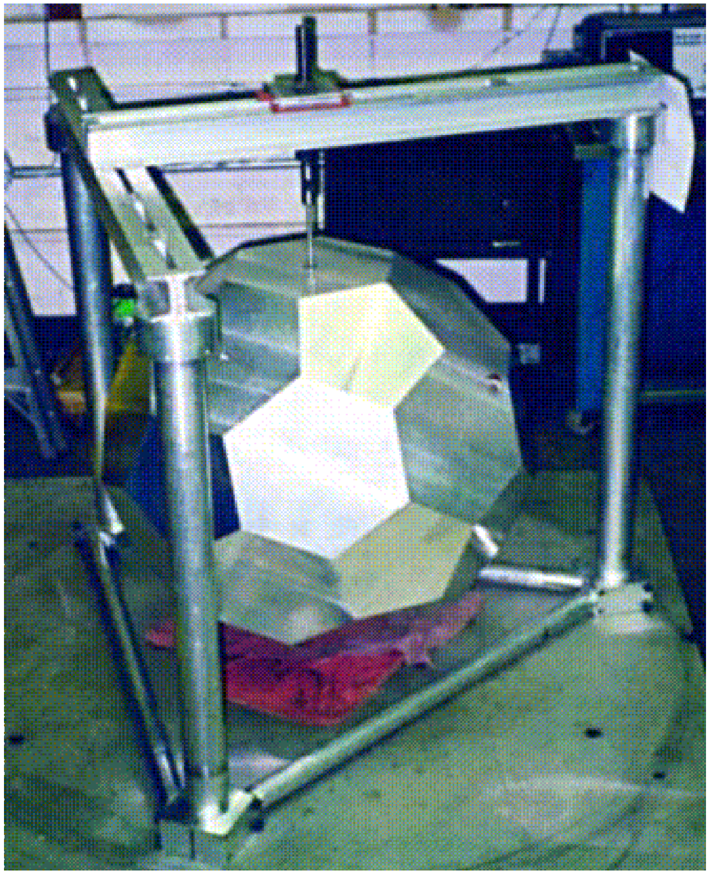
- Proposed by Merkowitz and Johnson (*PRD*, 1995)
- Same point group symmetry as dodecahedron but better approximates sphere
- 32 flat surfaces for transducers, suspension points, etc.
- Six transducers preserves symmetry
  - Allows measurement of five quadrupole modes
  - Sixth transducer can be used as check

TIGA Geometry



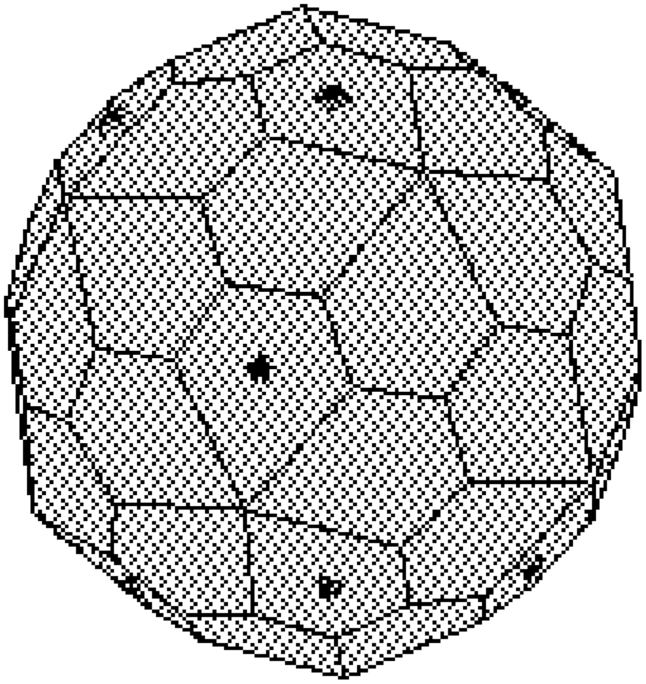
TIGA Prototype

- First prototype built and studied at Louisiana State University
- Made from aluminum 6063
- 33 inch diameter
- Able to calculate position of hammer blows from 6 piezo-transducers
- Second prototype in Rome, Italy



Pentagonal Hexacontahedral Configuration

- Proposed by Lobo & Serrano
- Allows 11 transducers to be placed
  - Ten for two quadrupole modes
  - One for monopole mode for scalar gravitational wave detection



Other Ideas for Spherical Antennas

- Hollow spheres
  - Lower frequency without unreasonably large mass
  - Preserves most of the cross section of an equivalent diameter sphere
  - Problems with construction
- Sphere within a sphere
  - Broader band response
  - Problems with construction and suspension

Noise in Resonant Mass Detectors

- Seismic
  - Can be eliminated at frequencies of resonant modes (~ 1 kHz) by seismic isolation stacks
- Thermal
  - Use high Q materials (sapphire, Al 5056, Nb, Cu-Al) and cool antennas to 4 K or below
- Amplifier noise
  - Superconducting quantum interference device (SQUIDs) properly impedance matched to antenna

Parameters for Sphere Noise: Transducer

- Paik style, inductive transducers
- Three modes
- Niobium bodies
- Q = 40 million
- Mass ratio 100:1
- Relative bandwidth 10%

Parameters for Sphere Noise: Thermal Noise

- Aluminum sphere
- Sphere Q = 40 million
- Aluminum intermediate mass
- Intermediate mass Q = 40 million
- Temperature 50 mK

$$S_{sph(thermal)} = 2k_B T \cdot Re[y_{22}(f)]$$

where y(f) is the admittance matrix of the sphere and y_{22}(f) depends on the Q’s

Parameters for Sphere Noise: SQUID Noise

- SQUID noise number $N_n = 1$
- Pick-up coil diameter $d_c = 9$ cm
- Noise resistance $R_n \alpha d_c^2$
- Velocity (forward action) noise

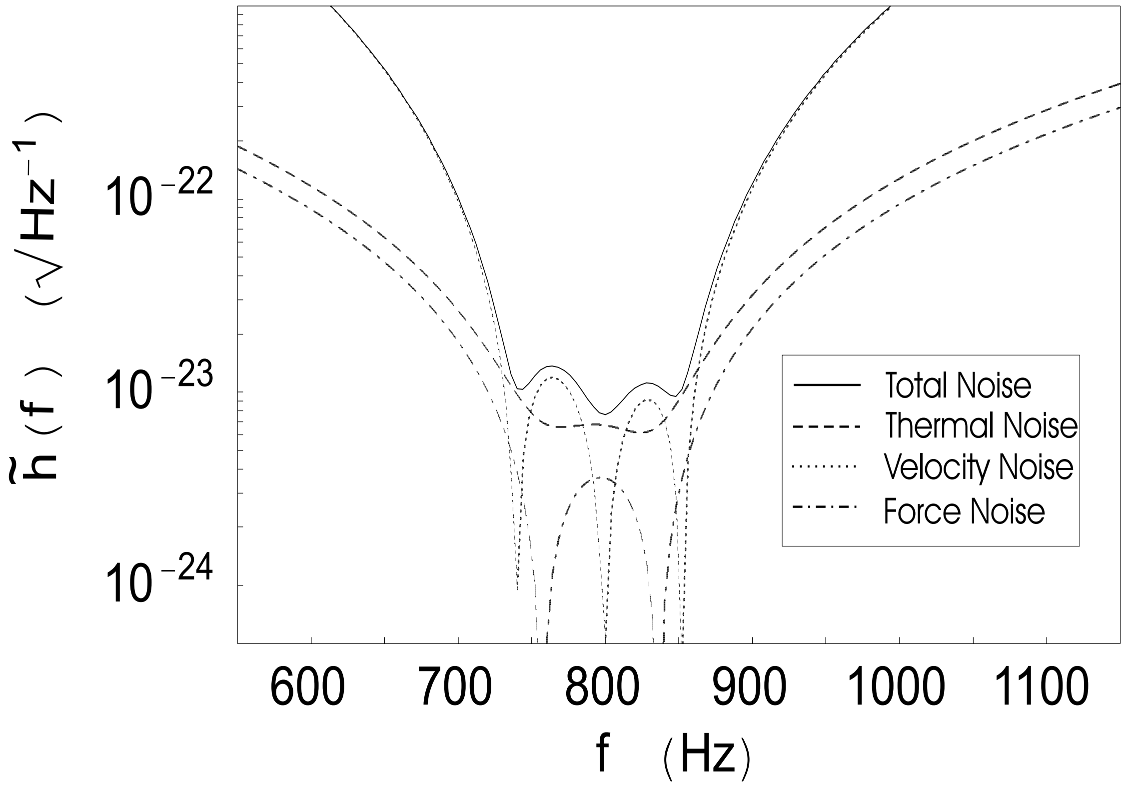
$$S_u(f) = \frac{N_n h f_0}{R_n}$$

- Force (back action) noise

$$S_{F(out)}(f) = N_n h f_0 R_n |y_{22}(f)|^2$$

Sphere strain spectral density

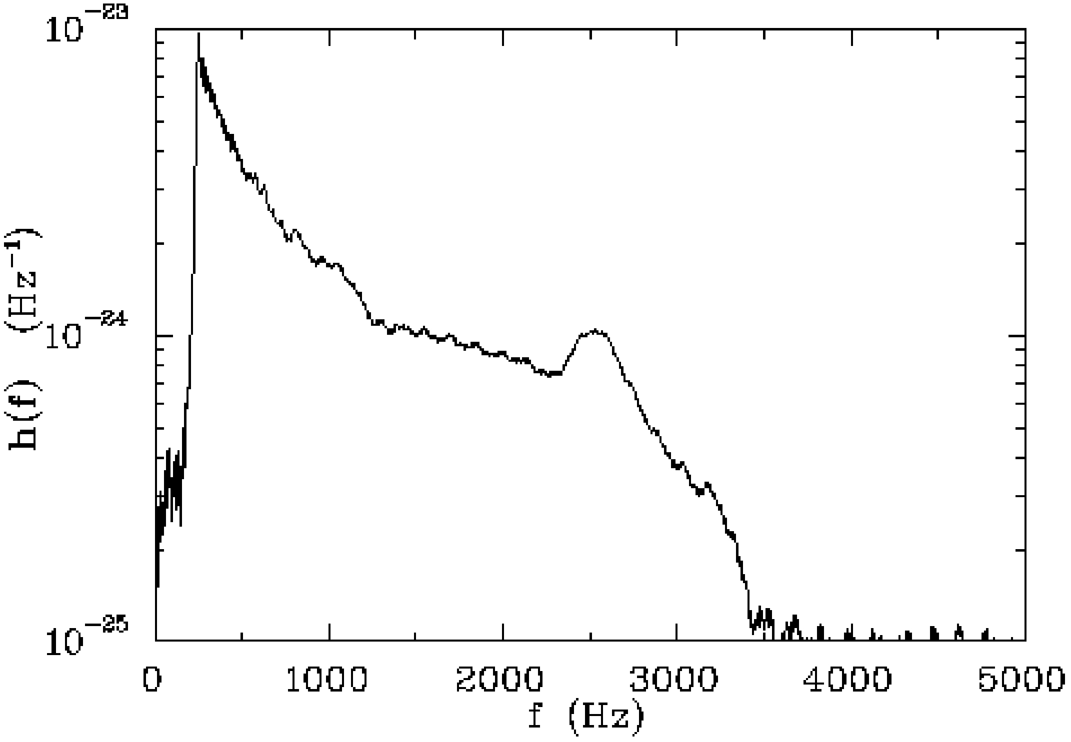
- 3.25 m sphere aluminum
- 50 tonne mass
- 795 Hz
- Six 3-mode transducers
- 40 million Q's
- Quantum limited SQUID
- 50 mK temperature



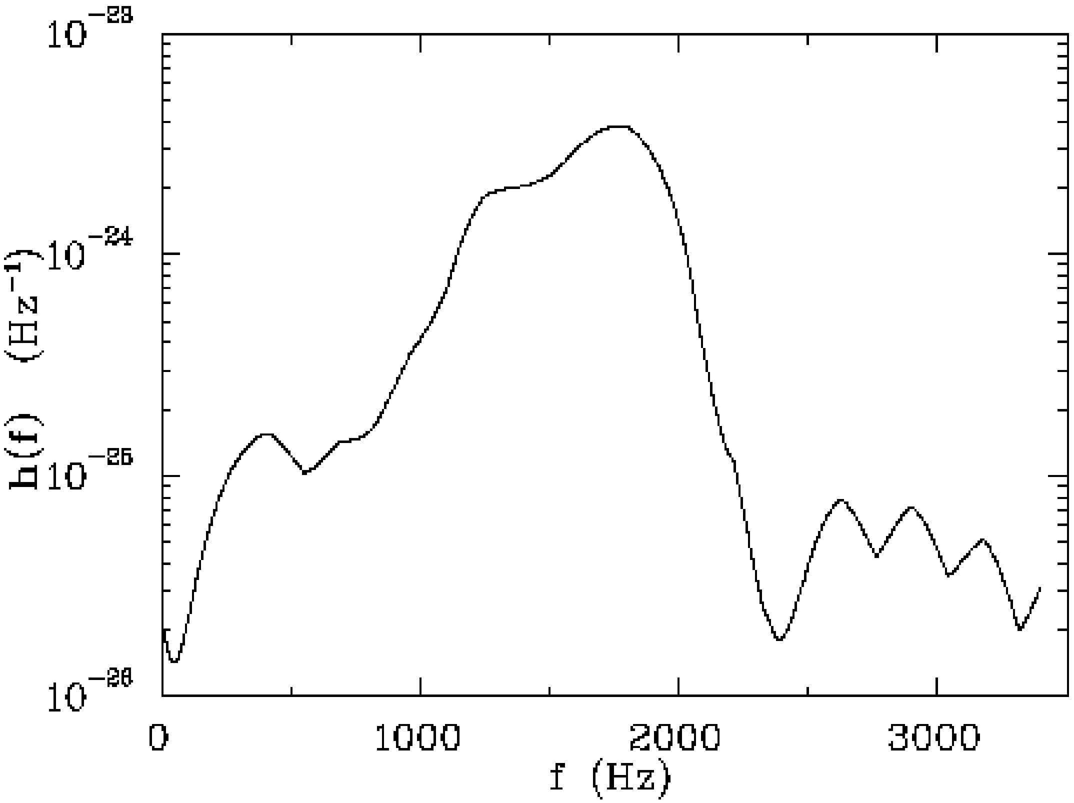
Sources for Comparison Between Spheres and Initial LIGO

- Binary neutron star inspiral and coalescence
  - Newtonian model
  - Quadrupole approximation
  - Smooth particle hydrodynamics
  - Separated into inspiral phase and coalescence phase
- Bar mode instability in a rapidly rotating star
  - Newtonian model with polytropic equation of state
  - Centrifugal hangup at 20 km
  - Quadrupole approximation
  - Gravitational wave backreaction ignored

Gravitational Waveform for Binary Neutron Star Inspiral and Coalescence



Gravitational Waveform for Bar Instability in Rapidly Rotating Star



Energy Signal to Noise Ratios for Initial LIGO and Spheres

Angle averaged BNS source at 15 Mpc

Frequency	Coalescence	Inspiral	Total
795 Hz	**0.0113**	**10.6**	**11.3**
940 Hz	**0.00985**	**5.79**	**6.28**
1100 Hz	**0.0146**	**3.43**	**3.88**
1292 Hz	**0.00948**	**1.40**	**1.64**
1520 Hz	**0.00853**	**0.907**	**1.09**
1782 Hz	**0.0104**	**0.558**	**0.719**
2096 Hz	**0.0197**	**0.285**	**0.449**
2461 Hz	**0.126**	**0.0886**	**0.407**
Xylophone	**0.210**	**22.8**	**25.6**
Initial LIGO	**0.00406**	**58.2**	**58.8**

Energy Signal to Noise Ratios for Initial LIGO and Spheres

Angle averaged RRS source at 1 Mpc

Frequency	Signal-to-noise Ratio
795 Hz	**0.0661**
940 Hz	**0.220**
1100 Hz	**1.22**
1292 Hz	**4.75**
1520 Hz	**6.95**
1782 Hz	**9.91**
2096 Hz	**0.935**
2461 Hz	**0.00168**
Xylophone	**24.1**
Initial LIGO	**0.197**

Parameters for Interferometer Noise

- Used whitepaper values as much as possible
- Laser power 125 W
- Gaussian width of beam 6 cm
- Seismic wall 10 Hz
- Sapphire mirrors
  - Thermoelastic noise included
  - Coating thermal noise NOT included
  - Q = 200 million
- Silica ribbon suspensions
- Did not use Buonanno & Chen model of optical noise
- Power at beamsplitter 9.3 kW

Conclusions

- With our parameters, interferometers with RSE are more sensitive than spheres
- Spheres have enough sensitivity to detect sources beyond our galaxy
- Advanced LIGO can see BNS out to near 200 Mpc
- Coalescence phase of BNS can be detected at a distance where an event is likely in a multiyear run
- High frequency, narrowband interferometers do better at detecting high frequency sources like rapidly rotating star
- Addition of back reaction to BNS model makes little difference to SNRs

A Niche for Spheres

- Simultaneous detection of a gravitational wave by two separate techniques adds confidence to discovery
- Spheres operating near interferometers can help detect stochastic background of gravitational waves
- Symmetry of spheres makes searches for scalar gravitational waves natural. This would allow for exploration of gravity beyond general relativity.