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LSC Meeting
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Study of the behaviour of an interferometer with non-spherical mirrors

Purpose

Analysis of the main problems related with the implementation of non-spherical mirrors inside the Fabry-Perot cavities of a gravitational wave interferometer.

Tools

A FFT-code simulation program and several Fortran codes I wrote for processing the electromagnetic field grids.

Conclusions

Comprehension of various phenomena

- stability for misalignment pertubations;
- noise related with diffraction;
- noise related with tilt.

Preliminaries before the simulations

1. Design of the laser beam as the Gaussian field that has the best coupling with the flat top beam resonating inside the Fabry-Perot with reshaped mirrors.

$$|\int \Psi^*_{Norm}(\frac{R}{R_0})u_{Norm}(R,\rho)2\pi R\mathrm{d}R|^2 = 0.943$$

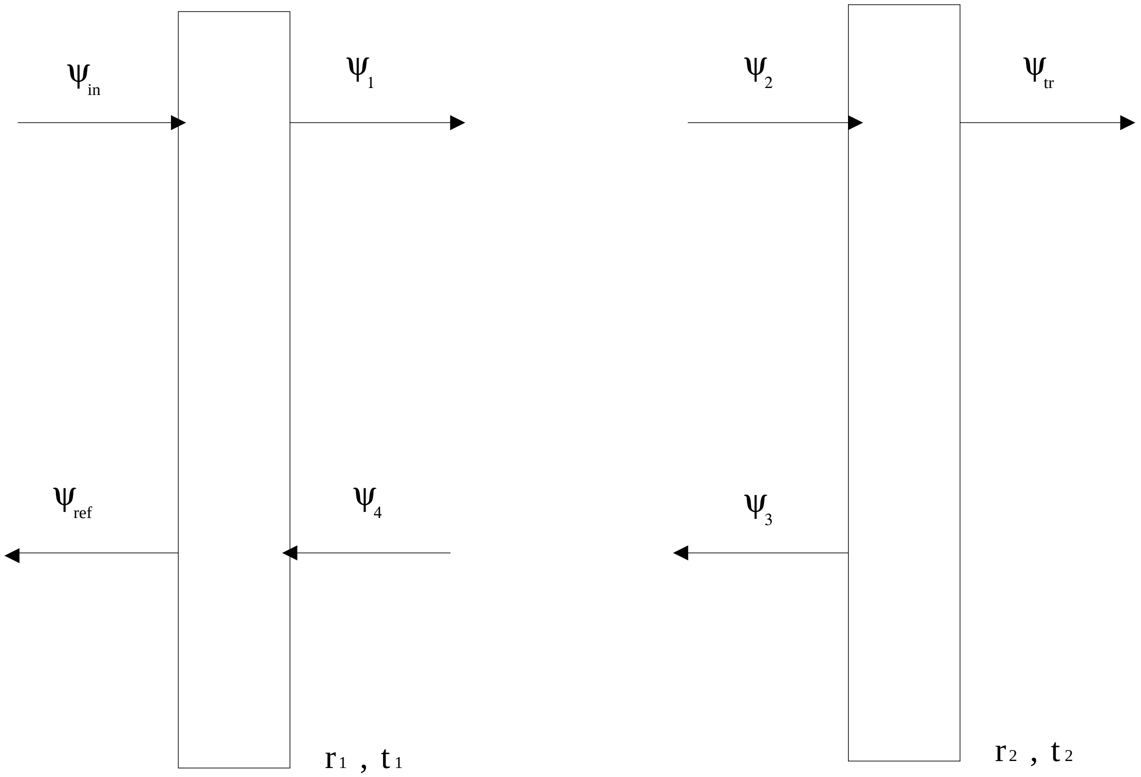
2. Limit on diffraction losses

$$L_{diffr} = 21ppm \qquad R_m = 16cm$$

3. Evaluation of nominal gains

$$Gain_{Analytical} = 372.986 \qquad Gain_{Numerical} = 372.886$$

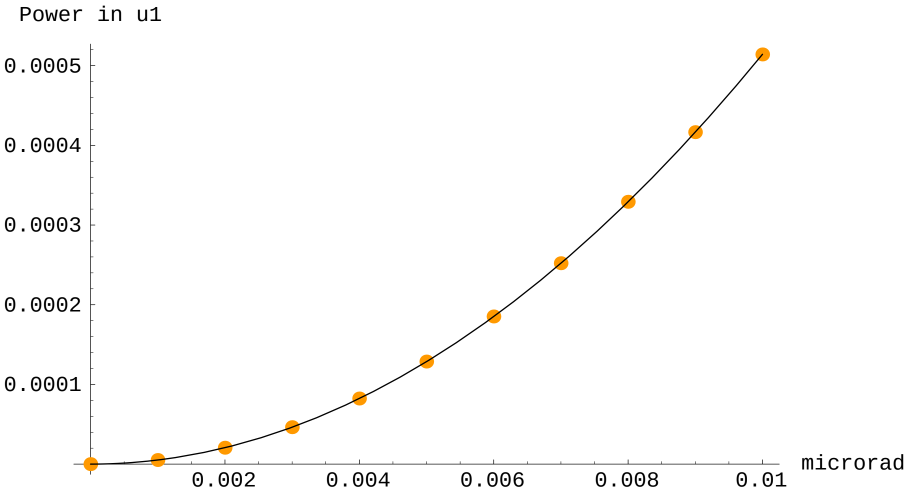
Estimation of the excitation of the higher order modes by misalignment

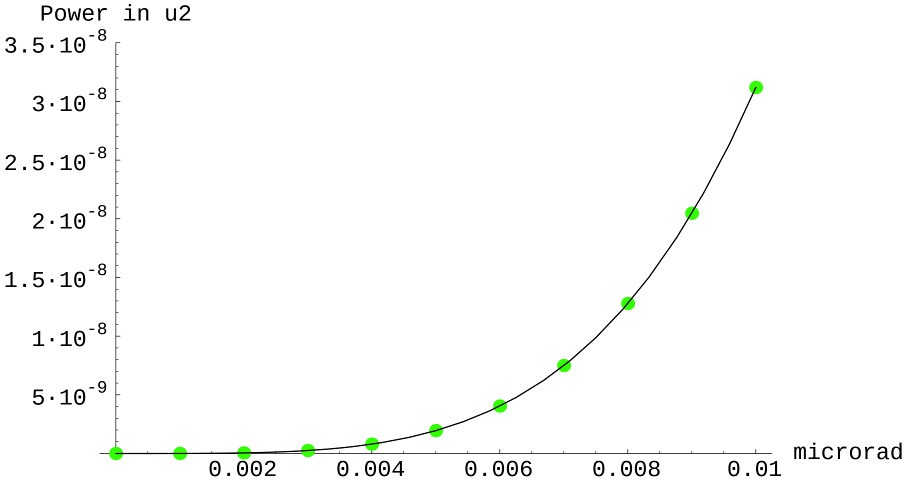


$$\alpha_1 \sim \frac{kR_0 \sin\theta}{\sin\theta_G/2}$$

$$u_0^{'}(\vec{R}) = \alpha_0 u_0(\vec{R}) + \alpha_1 u_1(\vec{R}) + \alpha_2 u_2(\vec{R}) + \dots \qquad (1)$$

Results of the evaluation of the portion of power due to higher order modes when the cavity is misaligned





$$|\alpha_0|^2 = 1 - 5.14211(\frac{\theta}{\mu\mathrm{rad}})^2 = 1 - 5.14\,10^{-4}(\frac{\theta}{10^{-8}\mathrm{rad}})^2$$

$$|\alpha_1|^2 = 5.14262(\frac{\theta}{\mu\mathrm{rad}})^2 = 5.14\,10^{-4}(\frac{\theta}{10^{-8}\mathrm{rad}})^2$$

$$|\alpha_2|^2 = 3.1196(\frac{\theta}{\mu\mathrm{rad}})^4 = 3.12\,10^{-8}(\frac{\theta}{10^{-8}\mathrm{rad}})^4$$

Details on the decomposition of the field grids inside the Fabry-Perot

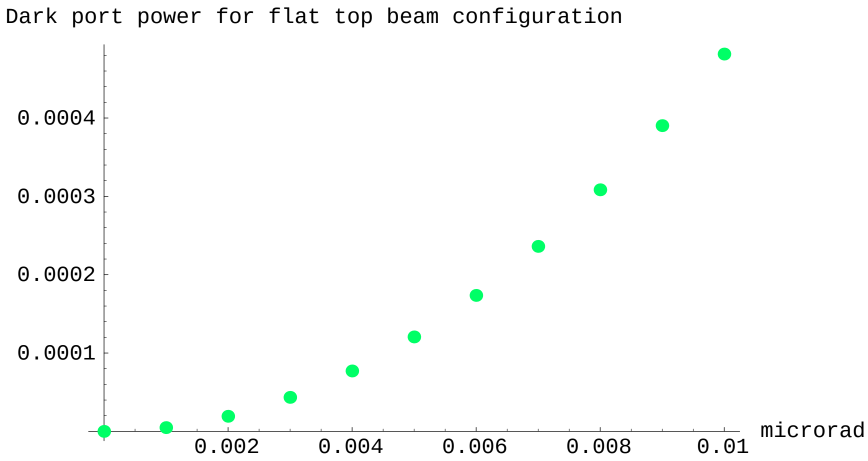
$$u_0^{'}(\vec{R}) = \alpha_0 u_0(\vec{R}) + \alpha_1 u_1(\vec{R}) + \alpha_2 u_2(\vec{R}) + \ldots$$

Step 1 Normalization of the fields.

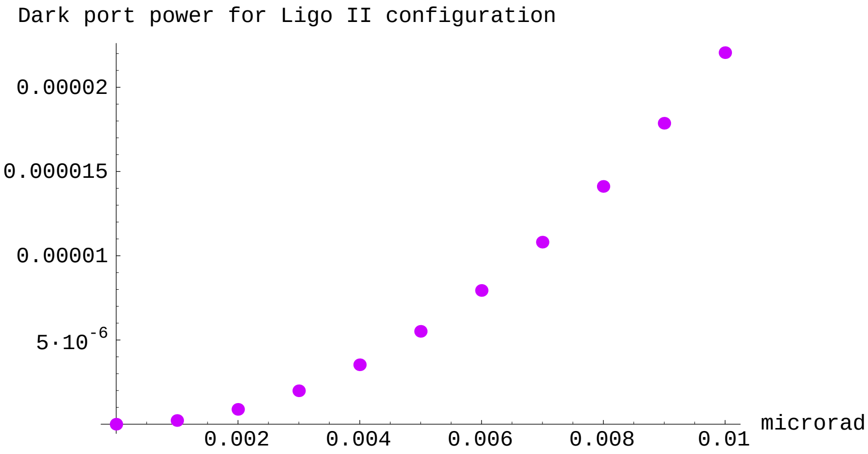
Step 2 Projection of $u_0^{'}(\vec{R})$ on the field $u_0(\vec{R})$ to get α_0.

Step 3 The remaining orthogonal part is decomposed in an odd and an even contribution.

Influence of misalignment on the total power at the antisymmetric port



Flat top beam

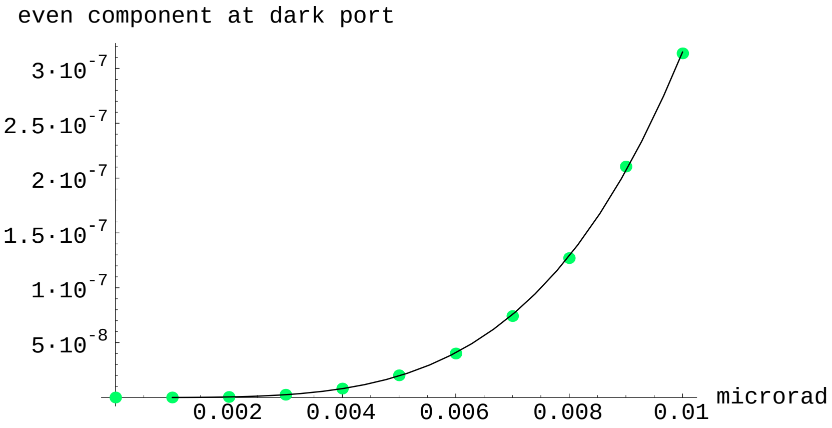


Gaussian beam

Filtering the dipolar component

$$P_{dipolar} = 4.81808(\frac{\theta}{\mu\text{rad}})^2$$

$$P_{non-dipolar} = 31.4982(\frac{\theta}{\mu\text{rad}})^4$$

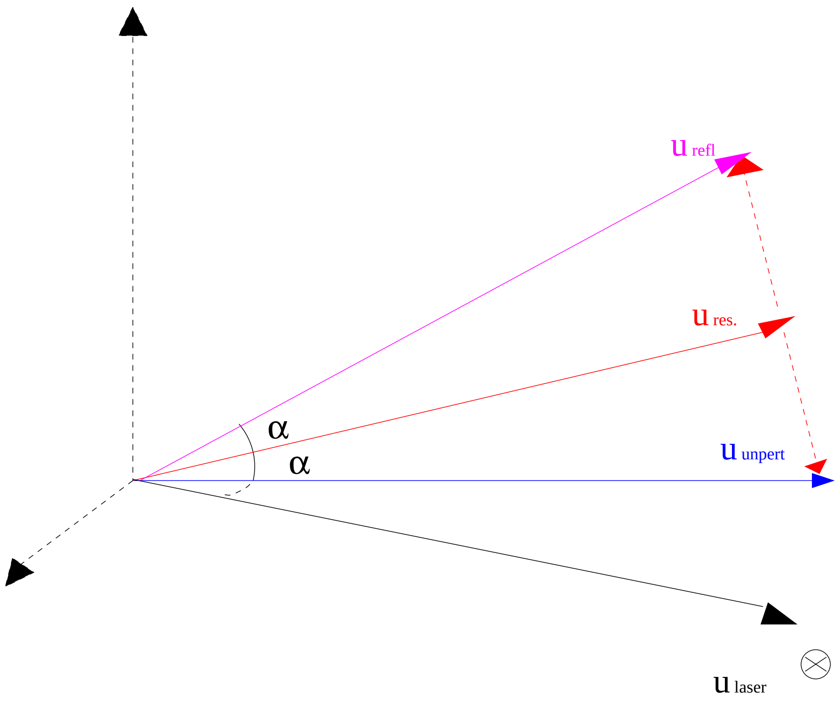


$$P_{dipolar} = \mathcal{C}|\alpha_1|^2(\frac{\theta}{\mu\text{rad}})^2$$

$$|\int \Psi^*_{Norm}(\frac{R}{R_0})u_{Norm}(R,\rho)2\pi R\text{d}R|^2 = 0.943$$

$$\mathcal{C} = 0.937$$

Interpretation of the results by means of a geometrical representation



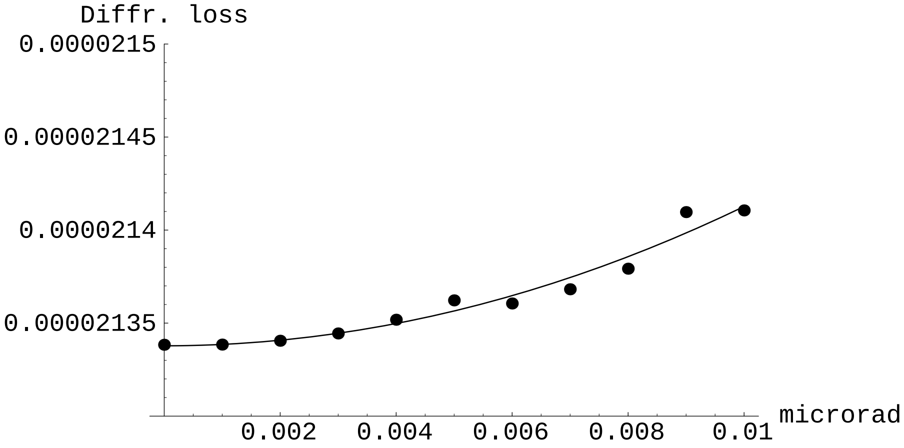
$$P_{non-dipolar} = 31.4982(\frac{\theta}{\mu\text{rad}})^4$$

$$\mathcal{C}(|\alpha_1|^4 + |\alpha_2|^2)(\frac{\theta}{\mu\text{rad}})^4 = 27.76(\frac{\theta}{\mu\text{rad}})^4$$

Interpretation of diffraction losses

$$u_0^{'}(\vec{R}) = \sqrt{1-|\alpha_1|^2}u_0(\vec{R}) + \alpha_1 u_1(\vec{R}) + \alpha_2 u_2(\vec{R})$$

$$L_{total} = L_0 + |\alpha_1|^2(L_1 - L_0) + \int_{R>R_M} 2\mathrm{Re}[\alpha_2 u_0^*(\vec{R})u_2(\vec{R})]\mathrm{d}^2\vec{R}$$



$$L_{diffr} = [21.34 + 748.6(\frac{\theta}{\mu\mathrm{rad}})^2]\,ppm$$

Summary of addressed phenomena

- Numerical studies of the characteristic behaviour of an optical resonator with special non-spherical mirrors that make a flat mode resonate inside the cavity.
- Comparison between perturbative estimations and values obtained by simulations.
- Comprehension of diffraction losses.
- Analysis of misalignment effects.

Reduction of thermoelastic noise in gravitational wave antennas by using particular flattened cavity modes

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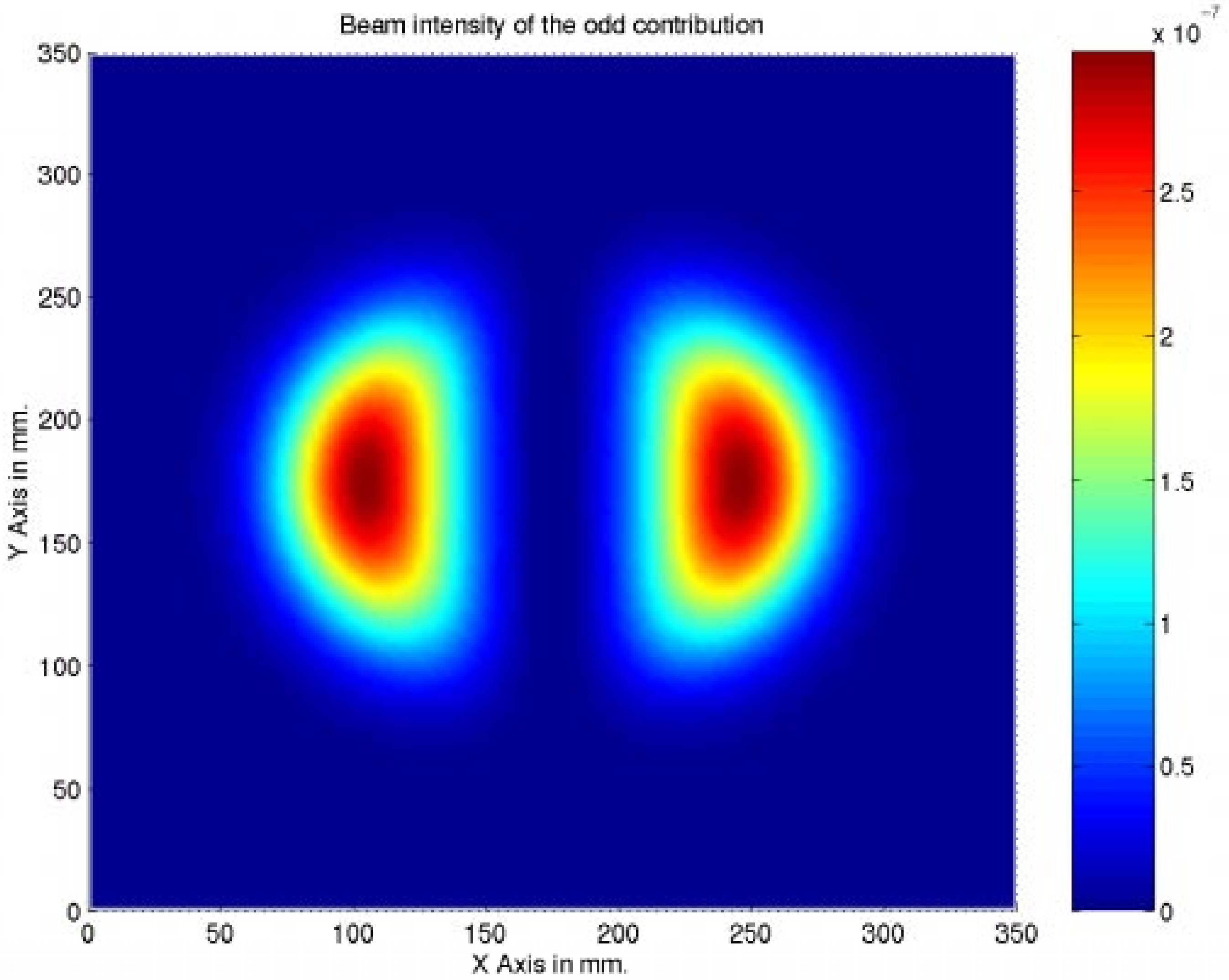
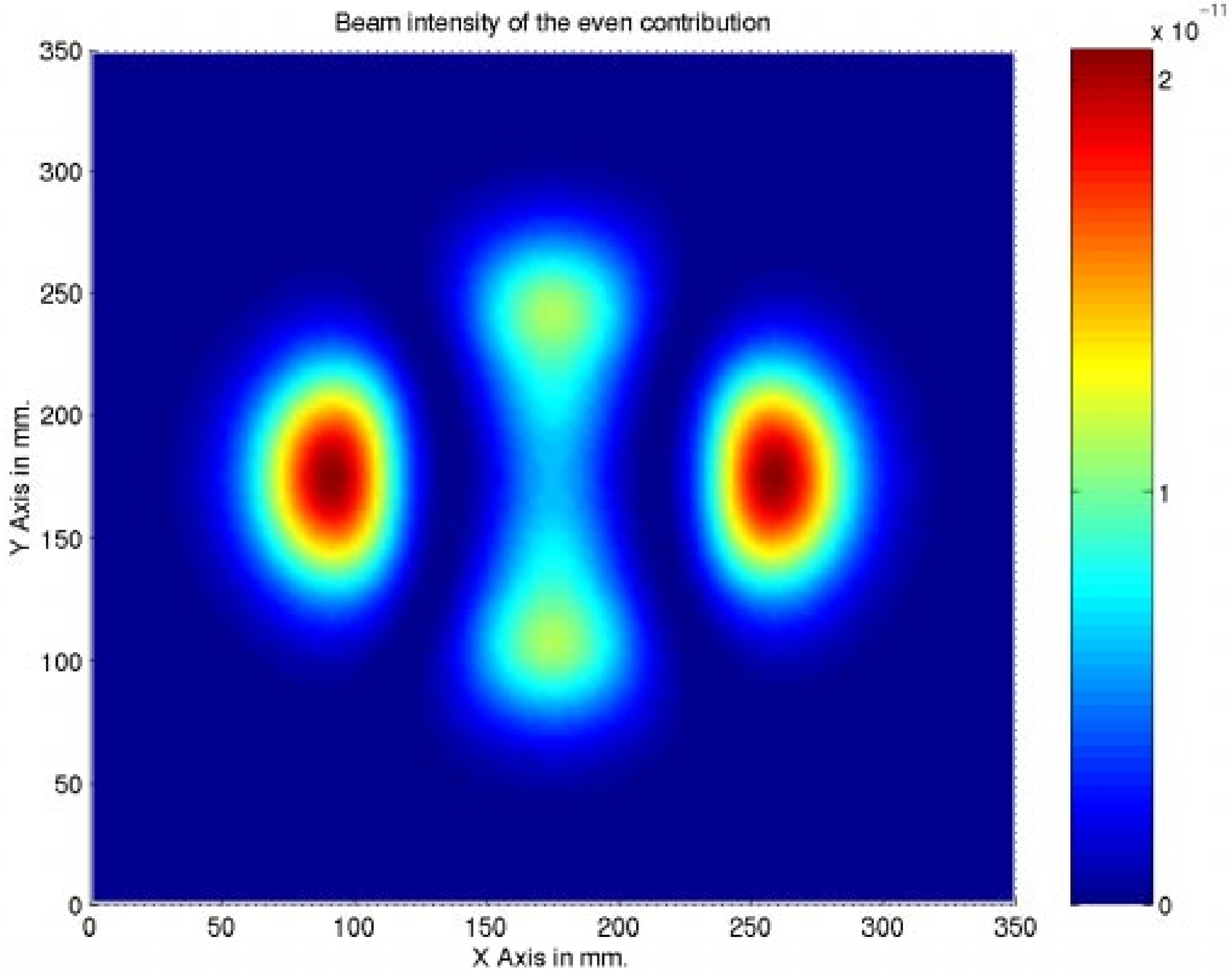
We explicitly contruct a non-gaussian paraxial cavity mode that has a flat top beam intensity by properly designing the end mirrors of a resonator. The aim is to reduce the thermoelastic noise due to the interaction between the field and the mirror by optimizing the shape of both. We present analytical and numerical results for the spectral density of thermoelastic noise in the special case of a resonator as long as the Fabry-Perot arms of the gravitational wave detector Ligo. We also discuss the alignment stability of such cavity and we mention the most important investigations that are in progress in order to properly design the phase profile of each mirror of the gravitational wave interferometer Ligo. Several numerical simulations have been done in order to understand the impact of a small misalignment in the Fabry-Perot cavity on the power built up inside the arm and the signal picked at the dark port of the beamsplitter. A model for those effects is proposed that takes into account mismatch problems and losses.


Beam intensity of the odd contribution
Y Axis in mm.
X Axis in mm.
0
50
100
150
200
250
300
350
0
50
100
150
200
250
300
350
x 10^-7
0
0.5
1
1.5
2
2.5


Beam intensity of the even contribution
Y Axis in mm.
X Axis in mm.
0
50
100
150
200
250
300
350
x 10^{-11}
2
1
0