Cross-Correlating ALLEGRO and LIGO Data for Stochastic Background Searches

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Members of Stochastic Sources Upper Limits Group

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LIGO E7 Engineering Run

- 2001 Dec 28 – 2002 January 14:
  LIGO Livingston & Hanford Observatories;
  collaboration with GEO-600 & ALLEGRO

- Perform Upper Limits searches with engineering data

- Four groups: Inspiral, Periodic, Burst & Stochastic

- ALLEGRO data to be used in Burst & Stochastic searches

Outline of Talk

I Data Analysis Issues

- Cross-Correlation & Optimal Filtering
- Overlap Reduction Function (Geography)

II Technical Interface Issues

- Data Analysis Environment
- Sampling & Heterodying

III What to Expect

Data Analysis Techniques & Issues

Stochastic Background

Assume cosmological in origin, thus isotropic, unpolarized, gaussian, & stationary.

Describe i.t.o. GW contribution to $\Omega = \frac{\rho}{\rho_{\text{crit}}}$:

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_{\text{crit}}} \frac{d\rho_{\text{GW}}}{d \ln f} = \frac{f}{\rho_{\text{crit}}} \frac{d\rho_{\text{GW}}}{df}$$

Note $\rho_{\text{crit}} \propto H_0^2$, so $h_{100}^2 \Omega_{\text{GW}}(f)$ is independent of

$$h_{100} = \frac{H_0}{100\,\text{km/ s/ Mpc}}$$

How to Tell Stochastic Signal from Random Noise

- Need correlations among detectors
  - Detector 1: $h_1 = s_1 + n_1$, Detector 2: $h_2 = s_2 + n_2$
- Assume noise uncorrelated with signal & between detectors
- Cross-correlation:

$$\langle h_1 h_2 \rangle = \langle n_1 n_2 \rangle + \langle n_1 s_2 \rangle + \langle s_1 n_2 \rangle + \langle s_1 s_2 \rangle$$

Optimally Filtered Cross-Correlation Statistic

$$
\begin{aligned}
Y_Q &= \int dt_1\, dt_2\, h_1(t_1)\, Q(t_1 - t_2) h_2(t_2) \\
&= \int df\, \tilde{h}_1^*(f)\, \tilde{Q}(f)\, \tilde{h}_2(f)
\end{aligned}
$$

Combine detector outputs using an *Optimal Filter* to maximize signal-to-noise ratio:

- Signal $\equiv$ mean of cross-correlation statistic $Y \propto T$
- Noise $\equiv$ variance of cross-correlation statistic $Y \propto \sqrt{T}$

Optimal Filter

$$\widetilde{Q}(f) \propto \frac{f^{-3}\Omega_{\mathrm{GW}}(f)\gamma_{12}(f)}{P_1(f)P_2(f)}$$

- Enhanced by signal $f^{-3}\Omega_{\mathrm{GW}}(f)$
  $\longrightarrow$ depends on target signal model

- Suppressed by noise $P_1(f)$,$P_2(f)$

- Geometry via overlap reduction fcn (Flanagan, astro-ph/9305029)

$$\gamma_{12}(f) = d_{1ab}\ d_2^{cd}\ \frac{5}{4\pi}\int_{S^2} d\widehat{\Omega}\ e^{i2\pi f\widehat{\Omega}\cdot\Delta\vec{x}/c}\ P^{ab}_{cd}(\widehat{\Omega})$$

Sensitivity to Stochastic Background

Upper limit from cross-correlation of two instruments, assuming const $\Omega_{\mathrm{GW}}(f)$ (Allen & Romano, gr-qc/9710117)

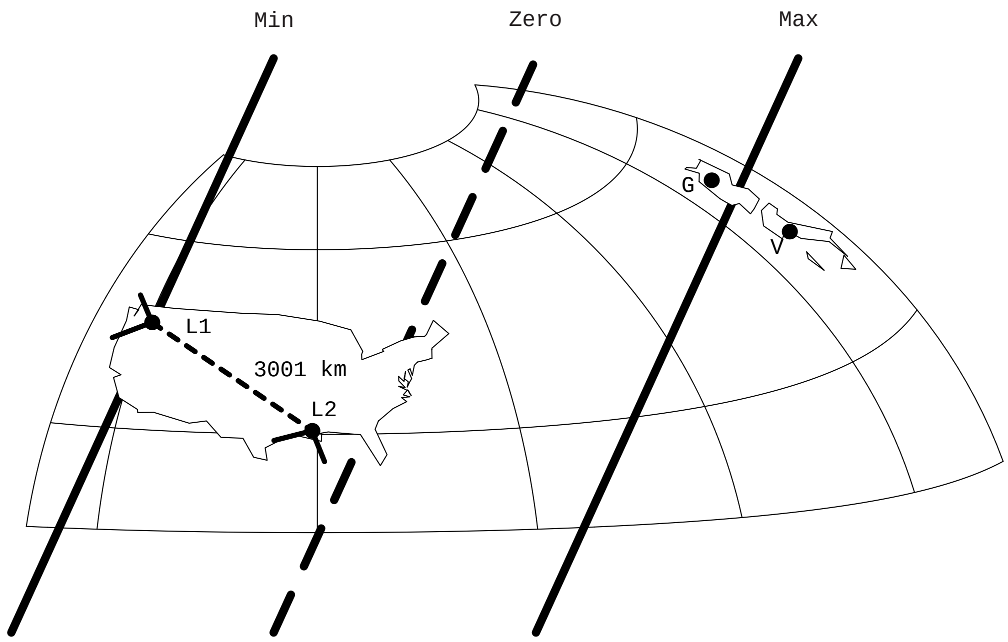
$$\Omega^{\mathrm{UL}} = \frac{10\pi^2}{3H_0^2}\left(T\int df\,\frac{\gamma_{12}^2(f)}{f^6 P_1(f)P_2(f)}\right)^{-1/2}\sqrt{2}\,\mathrm{erfc}^{-1}(2\times(1-\mathrm{CL}))$$

- Improves with (square root of) time
- Hurt by noise $P_1(f)$,$P_2(f)$
- Geometry via overlap reduction fcn (Flanagan, astro-ph/9305029)

$$\gamma_{12}(f) = d_{1ab}\ d_2^{cd}\ \frac{5}{4\pi}\int_{S^2} d\widehat{\Omega}\ e^{i2\pi f\widehat{\Omega}\cdot\Delta\vec{x}/c}\ P^{ab}_{cd}(\widehat{\Omega})$$

Overlap Reduction Function

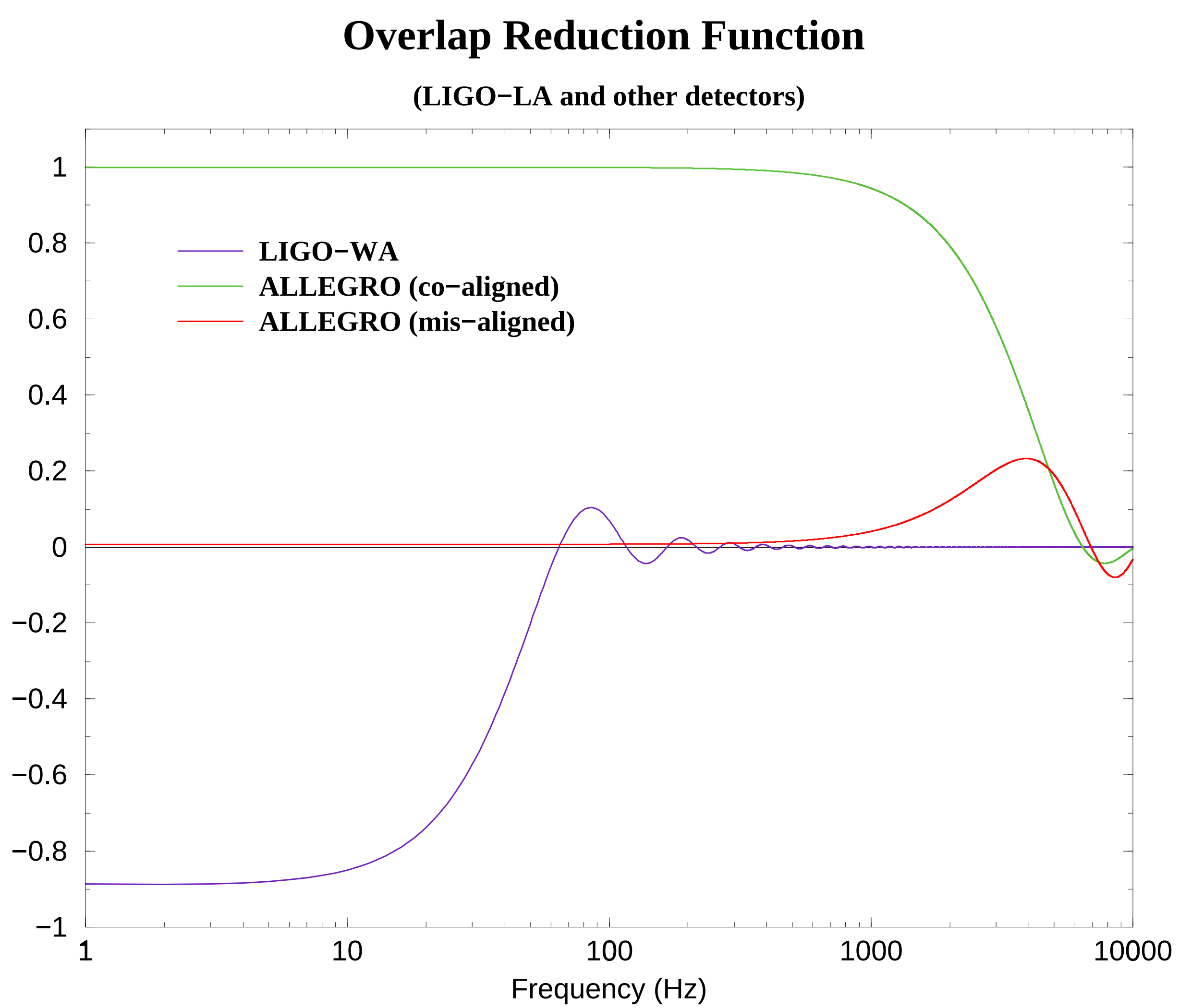
Depends on distance between & relative alignment of detectors

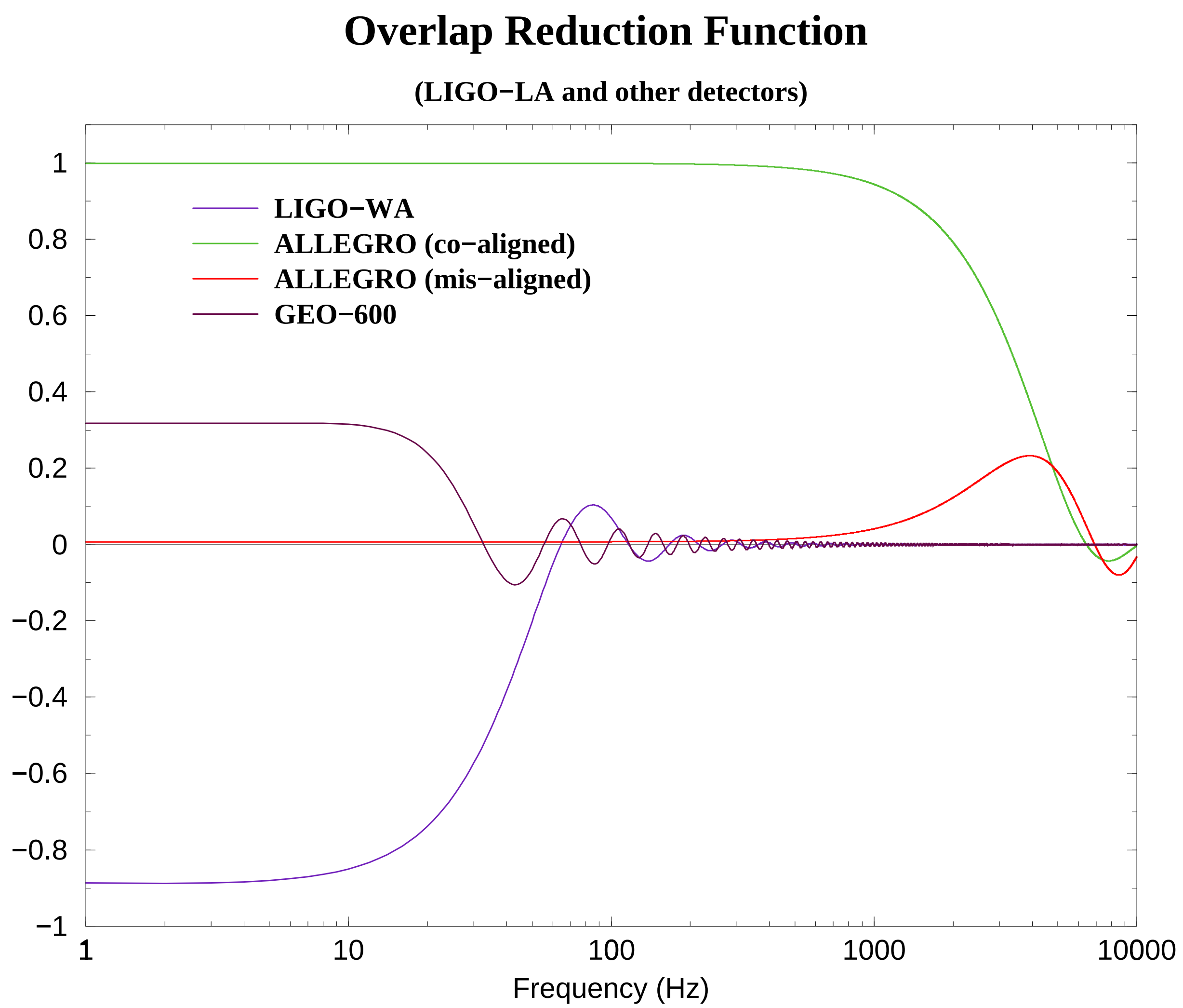


(figure from Allen & Romano, gr-qc/9710117)

ALLEGRO-LLO Overlap Reduction Function

- ALLEGRO and LIGO-Livingston only 40km apart
- For optimal alignment, $\gamma(900\,\mathrm{Hz}) \approx 95\%$
- Compare LLO-LHO Overlap Reduction Fcn; already oscillating by 100 Hz


Overlap Reduction Function
(LIGO−LA and other detectors)
LIGO−WA
ALLEGRO (co−aligned)
ALLEGRO (mis−aligned)
1
0.8
0.6
0.4
0.2
0
−0.2
−0.4
−0.6
−0.8
−1
1
10
100
1000
10000
Frequency (Hz)


Overlap Reduction Function
(LIGO−LA and other detectors)
LIGO−WA
ALLEGRO (co−aligned)
ALLEGRO (mis−aligned)
GEO−600
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10
100
1000
10000
Frequency (Hz)

Alignment of ALLEGRO

- ALLEGRO can be rotated

- Overlap reduction fcn orientation-dependent

- Finn & Lazzarini (gr-qc/0104040) propose comparing cross-correlations for co-aligned and mis-aligned (or anti-aligned) configurations to separate stochastic GW signal from correlated noise

Technical Interface Issues

- LIGO building its own infrastructure (LIGO Data Analysis System, common frame format with VIRGO, etc.)

- ALLEGRO operating for years within IGEC collaboration

- LIGO sampled at 16384 Hz, real signal

- ALLEGRO heterodyned at 907 Hz, sampled at 250 Hz; complex signal

- How to interface infrastructure & data formats?

Incorporating ALLEGRO data into LIGO Infrastructure

- ALLEGRO data to be written to frames

- Analysis in LDAS environment:
  - One set of LAL library routines
    for both IFO-IFO & IFO-bar correlations
  - Separate driver routines (LALWrapper shared objects)
    for IFO-IFO (Anderson, Romano & JTW)
    and IFO-bar (Heng & McHugh) searches

Conditioning LIGO data for correlation with ALLEGRO

- Data conditioning software will heterodyne LIGO data at 907 Hz & resample it to 250 Hz to allow straightforward correlation with ALLEGRO

- Subtleties can arise when cross-correlating complex (heterodyned) time streams; different interpretations depending on knowledge of state of reference oscillator (McHugh & JTW 2002)

Expectations from E7 Upper Limits Run

Previous Results

- Current best upper limit: correlation between EXPLORER & NAUTILUS bars (Astone et al, 1999): $\Omega_{\mathrm{GW}}(900\,\mathrm{Hz}) \leq 60$
- Upper limit from single bar (Astone et al, 1996): $\Omega_{\mathrm{GW}}(900\,\mathrm{Hz}) \leq 100$
- Correlation between Garching & Glasgow prototype IFOs (Compton et al, 1994): $\Omega_{\mathrm{GW}}(f) \lesssim 3 \times 10^5$

17 Days of ALLEGRO-LLO Correlations

Assume:

- No correlated noise
- ALLEGRO bandwidth & noise as in PRD **54**, 1264 (1996)
- (duty cycle) × 17 days of coïncident operation
- 90% confidence level upper limit

Then expect to set a limit around

$$\Omega_{GW}(900\,\mathrm{Hz}) \lesssim 0.2 \times (\text{duty cycle})^{-1/2} \times \left(\frac{\text{LLO ASD}(900\,\mathrm{Hz})}{10^{-22}\,\mathrm{Hz}^{-1/2}}\right)$$

For Comparison:

17 Days of LHO-LLO Correlations

Assume:

- Amplitude spectral density for E7 has same shape as design sensitivity, just scaled up
- $\Omega_{\text{GW}}(f)$ constant
- 90% confidence level

$$\Omega_{\text{GW}}(f) \lesssim 6 \times 10^{-6} \times (\text{duty cycle})^{-1/2} \times \left(\frac{(\text{LHO ASD})(\text{LLO ASD})}{(\text{LIGO-1 ASD})^2} \right)$$

Note: most of the correlations should come from lower frequencies (50–250Hz)