Inspiral Parameter Estimation via Markov Chain Monte Carlo (MCMC) Methods

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LIGO-G020104-00-Z

Inspiral Detected by One Interferometer

- From data, estimate m_1, m_2 and amplitude of signal
- Generate a probability distribution function for these parameters => statistics

More Parameters - MCMC

- MCMC methods are a demonstrated way to deal with large parameter numbers.
- Expand one-interferometer problem to include terms like spins of the masses.
- Multiple interferometer problem: Source sky position and polarization of wave are additional parameters to estimate and to generate PDFs for.

Initial Study

- Used “off the shelf” MCMC software
- See Christensen and Meyer, PRD **64**, 022001 (2001)

Present Work

- Develop a MCMC routine that operates within LAL
- Uses LAL routine “findchirp” with some modifications
- Using Metropolis-Hastings Algorithm

Bayes' Theorem

z = the data $\vec{\theta} = (\theta_1, \theta_2, \ldots, \theta_n)$ The n parameters

$$p(\vec{\theta} \mid z) \propto p(\vec{\theta}) p(z \mid \vec{\theta})$$

$p(\vec{\theta} \mid z)$ Posterior PDF

$p(\vec{\theta})$ a priori PDF for $\vec{\theta}$

$p(z \mid \vec{\theta})$ Likelihood

Bayes – Normally Hard to Calculate

$$p(\theta_i \mid z) = \int \ldots \int p(\vec{\theta} \mid z) d\theta_1 \ldots d\theta_{i-1} d\theta_{i+1} \ldots d\theta_n$$

The PDF for parameter θ_i

$$\hat{\theta}_i = \int \ldots \int \theta_i p(\vec{\theta} \mid z) d\theta_1 \ldots d\theta_{i-1} d\theta_{i+1} \ldots d\theta_n$$

Estimate for parameter θ_i

MCMC Does Integral

- Parameter space sampled in quasi-random fashion.
- Steps through parameter space are weighted by the likelihood and a priori distributions
- **z = s + n** Data is the sum of signal + noise

$$p\left(z \mid \vec{\theta}\right) \propto \exp\left[2\left\langle z, s\left(\vec{\theta}\right)\right\rangle - \left\langle s\left(\vec{\theta}\right), s\left(\vec{\theta}\right)\right\rangle\right]$$

$$\langle a, b \rangle = \int_{-\infty}^{\infty} df \, \tilde{a}(f) \tilde{b}^{*}(f) / S_n(f)$$

Markov Chain of Parameter Values

Start with some initial parameter values: $\left(\theta_1^{(1)}, \theta_2^{(1)}, \ldots, \theta_n^{(1)}\right)$

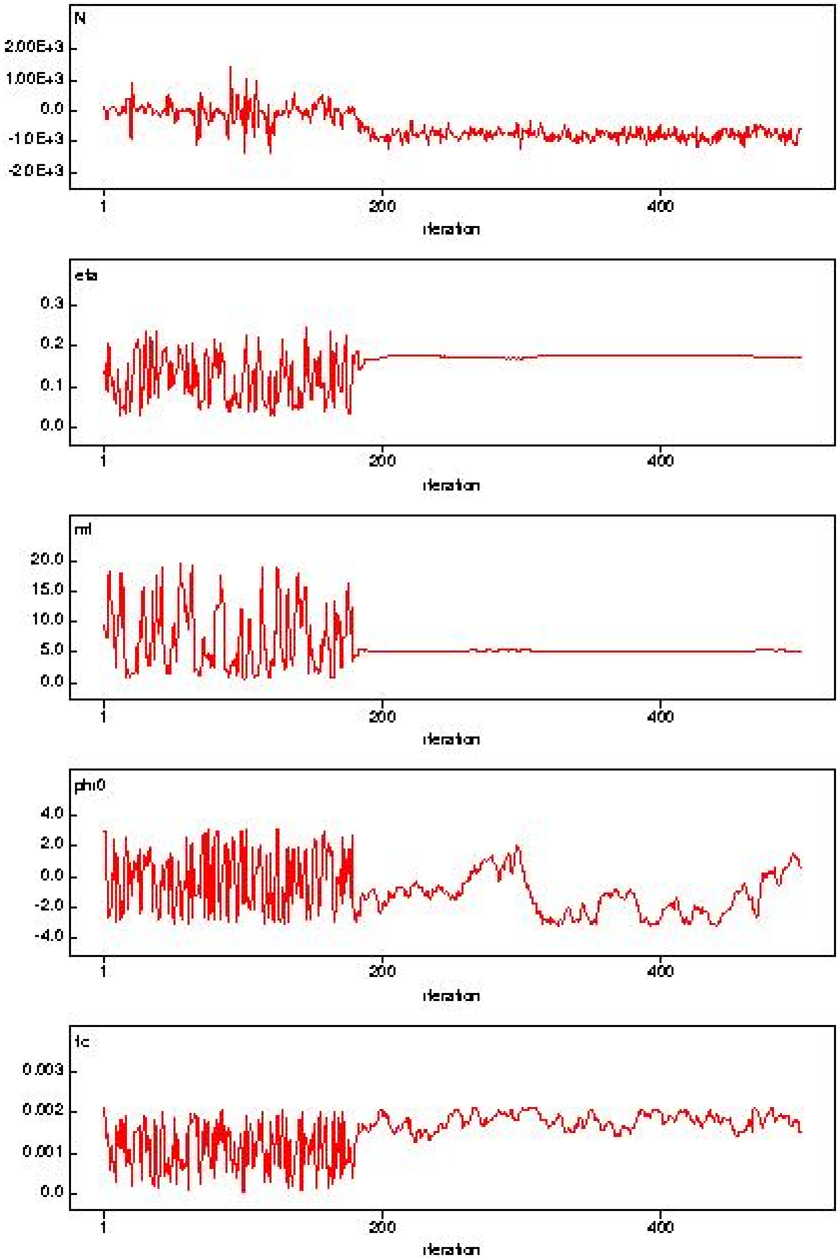
In some “random” way, select new candidate parameters:

$$\left(\theta_1^{(2)}, \theta_2^{(2)}, \ldots, \theta_n^{(2)}\right)$$

Calculate: $$\alpha = \frac{p\left(\vec{\theta}^{(2)}\right) p\left(z \mid \vec{\theta}^{(2)}\right)}{p\left(\vec{\theta}^{(1)}\right) p\left(z \mid \vec{\theta}^{(1)}\right)}$$

Accept candidate as new chain member if $\alpha > 1$
If $\alpha < 1$ then accept candidate with probability $= \alpha$
If candidate is rejected, last value also becomes new chain value.
Repeat 250,000 times or so.

Example from "off the shelf" software



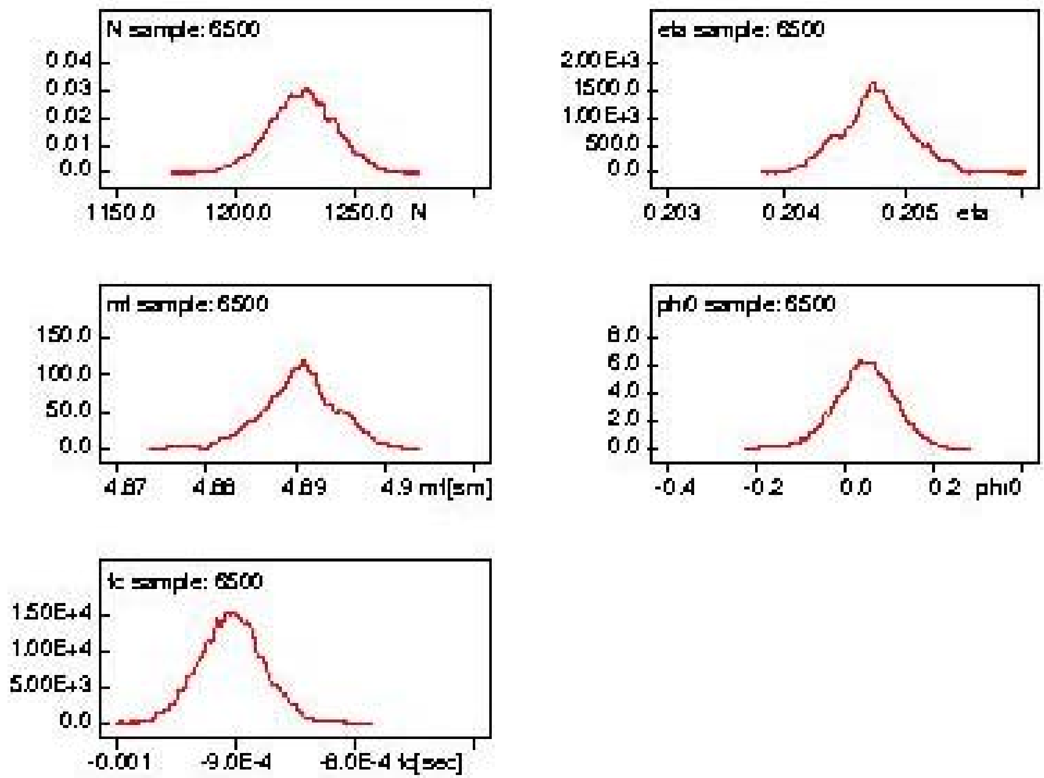
$$m_1 = 1.4 M_{solar}$$

$$m_2 = 3.5 M_{solar}$$

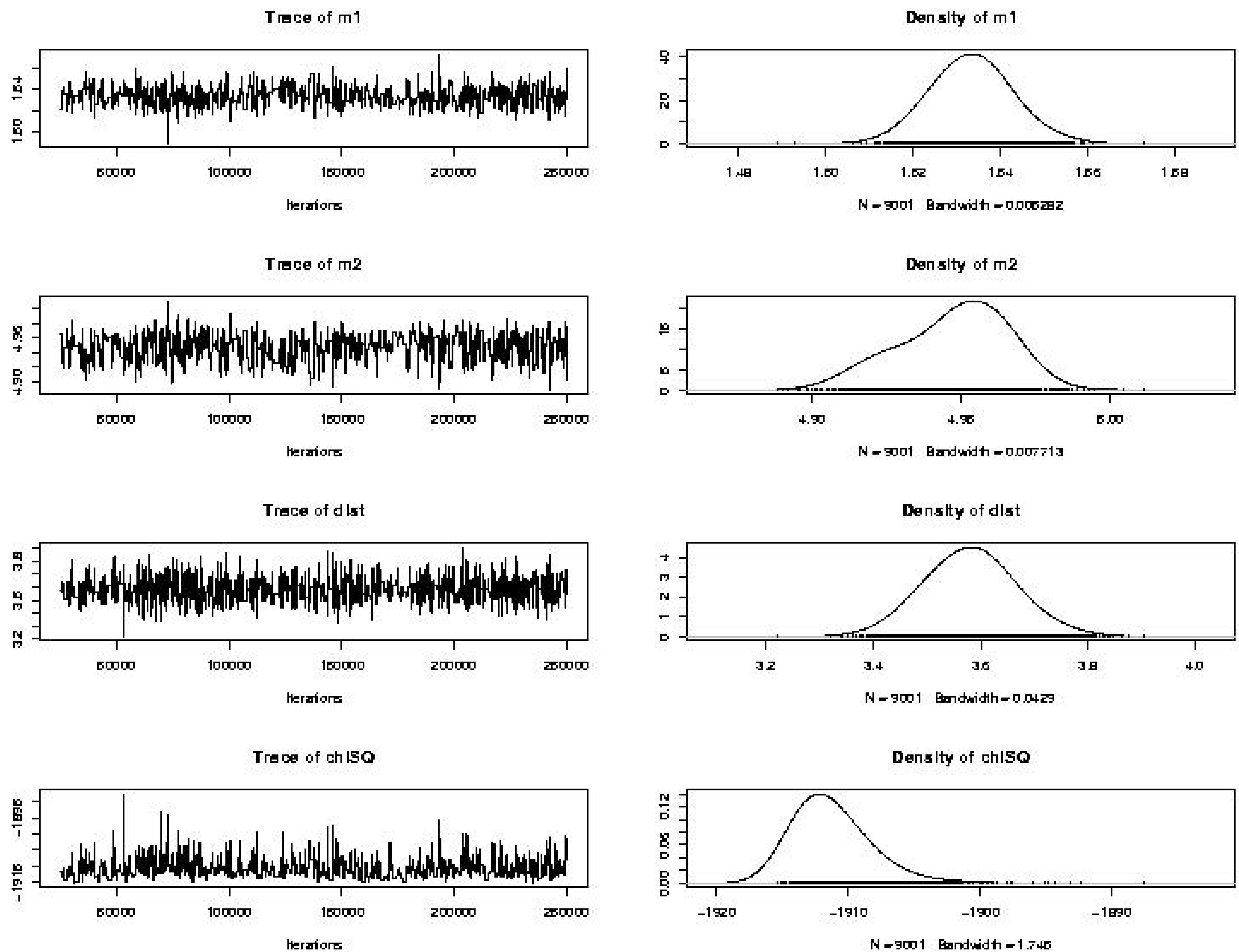
$$m_t = m_1 + m_2 = 4.9 M_{solar}$$

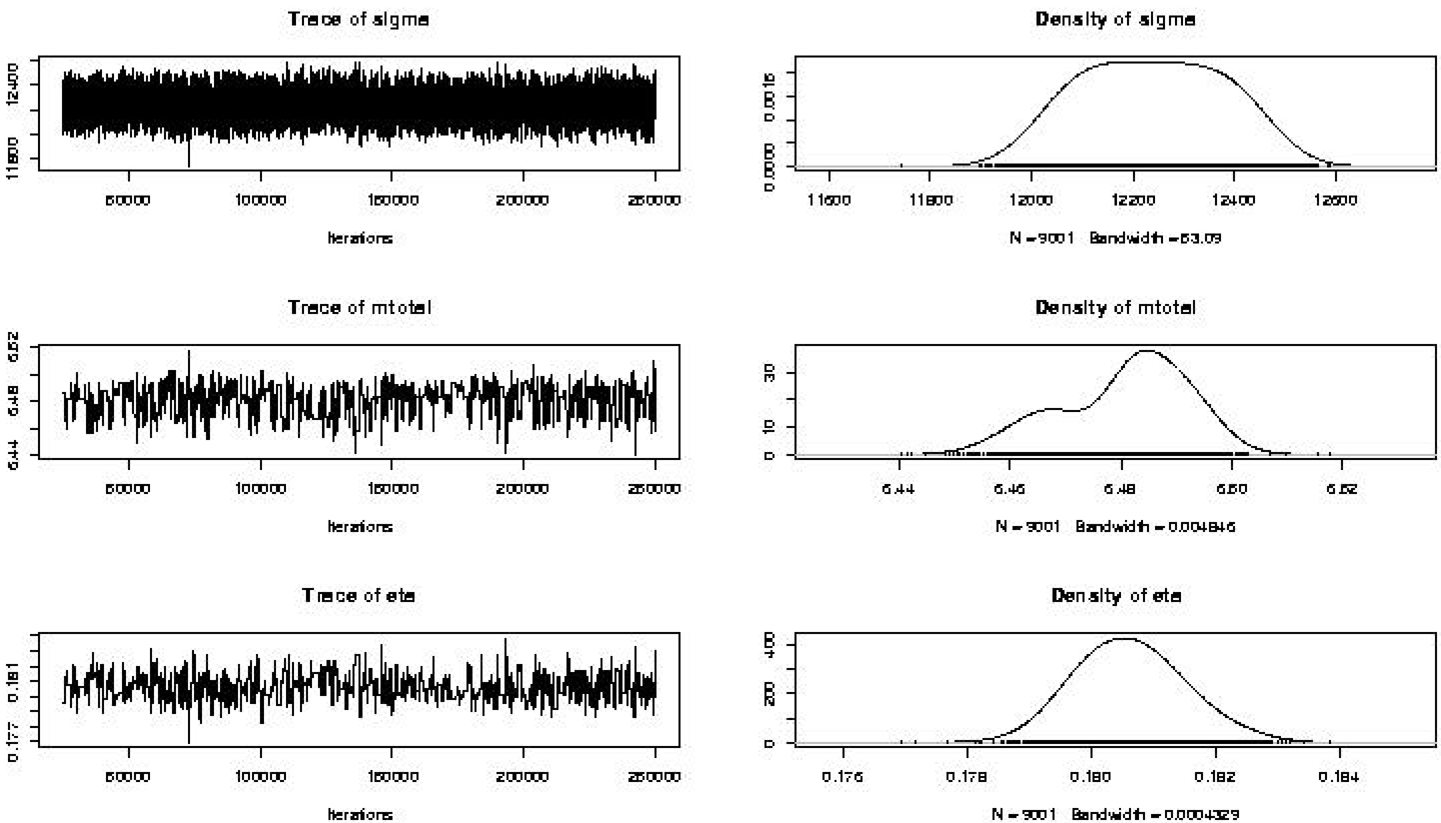
$$\eta = m_1 m_2 / (m_1 + m_2)^2 = 0.204$$

Markov Chain Represents the PDF



New Metropolis-Hastings Routine




Trace of sigma
12400
11800
50000
100000
150000
200000
250000
Iterations
Density of sigma
0.0015
0.0000
11600
11800
12000
12200
12400
12600
N = 9001 Bandwidth = 53.09
Trace of mtotal
6.52
6.48
6.44
50000
100000
150000
200000
250000
Iterations
Density of mtotal
30
10
0
6.44
6.46
6.48
6.50
6.52
N = 9001 Bandwidth = 0.004846
Trace of eta
0.181
0.177
50000
100000
150000
200000
250000
Iterations
Density of eta
400
200
0
0.176
0.178
0.180
0.182
0.184
N = 9001 Bandwidth = 0.0004329

Metropolis-Hastings Algorithm

- Have applied this method to estimating 10 cosmological parameters from CMB data. See Knox, Christensen, Skordis, ApJ Lett **563**, L95 (2001)
- Metropolis-Hastings algorithm described in Christensen et al., Class. Quant. Gravity **18**, 2677 (2001)