CONDITIONS FOR STEADY GRAVITATIONAL RADIATION FROM ACCRETING NEUTRON STARS

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GOVERNING RELATIONS

Consider a fluid displacement

$$\vec{\xi} = \vec{f}(r,\theta)e^{i(m\phi+\sigma t)} \sim \alpha R\ , \qquad m \neq 0\ , \qquad \alpha \ll 1\ ;$$

which induces adiabatic perturbations of a Newtonian equilibrium star (angular momentum J_*). This produces a total angular momentum

$$J = J_*(M,\Omega) + (1-K_j)J_c\ , \qquad J_c = -K_c\alpha^2 J_*\ . \tag{1}$$

The canonical angular momentum obeys the relation

$$dJ_c/dt = 2J_c[(F_g(M,\Omega) - F_v(M,\Omega,T)]\ . \tag{2}$$

For $l = m = 2$ r-modes, the gravitational radiation growth rate is

$$F_g = \left(\frac{1}{\tau_{gr}}\right)\left(\frac{\Omega}{\Omega_c}\right)^6\ , \quad \tau_{gr} \cong 3.26\ \mathrm{s}\ , \quad \Omega_c \equiv \sqrt{\pi \mathrm{G}\langle\rho\rangle}\ .$$

The viscous damping rate is

$$F_v \cong F_{sh}(T) + F_{bl}(\mathcal{S}_n, \mathcal{S}_s, \Omega, T, B) + F_{hb}(T_h, \Omega, T)\ .$$

We have modified the viscous and magnetic boundary layer damping rate (F_{bl}) of Kinney & Mendell (2002). The hyperon bulk viscosity damping rate (F_{hb}) due to $n + n \rightleftharpoons n + \Lambda$, etc. employs results of Lindblom & Owen (2002) and Haensel, Levenfish & Yakovlev (2002). (See Figure 1.)

$$dJ/dt = 2J_cF_g + \dot{J}_a(t)\ , \tag{3}$$

where $\dot{J}_a = j_a\dot{M}(t)$ is the rate of accretion of angular momentum.

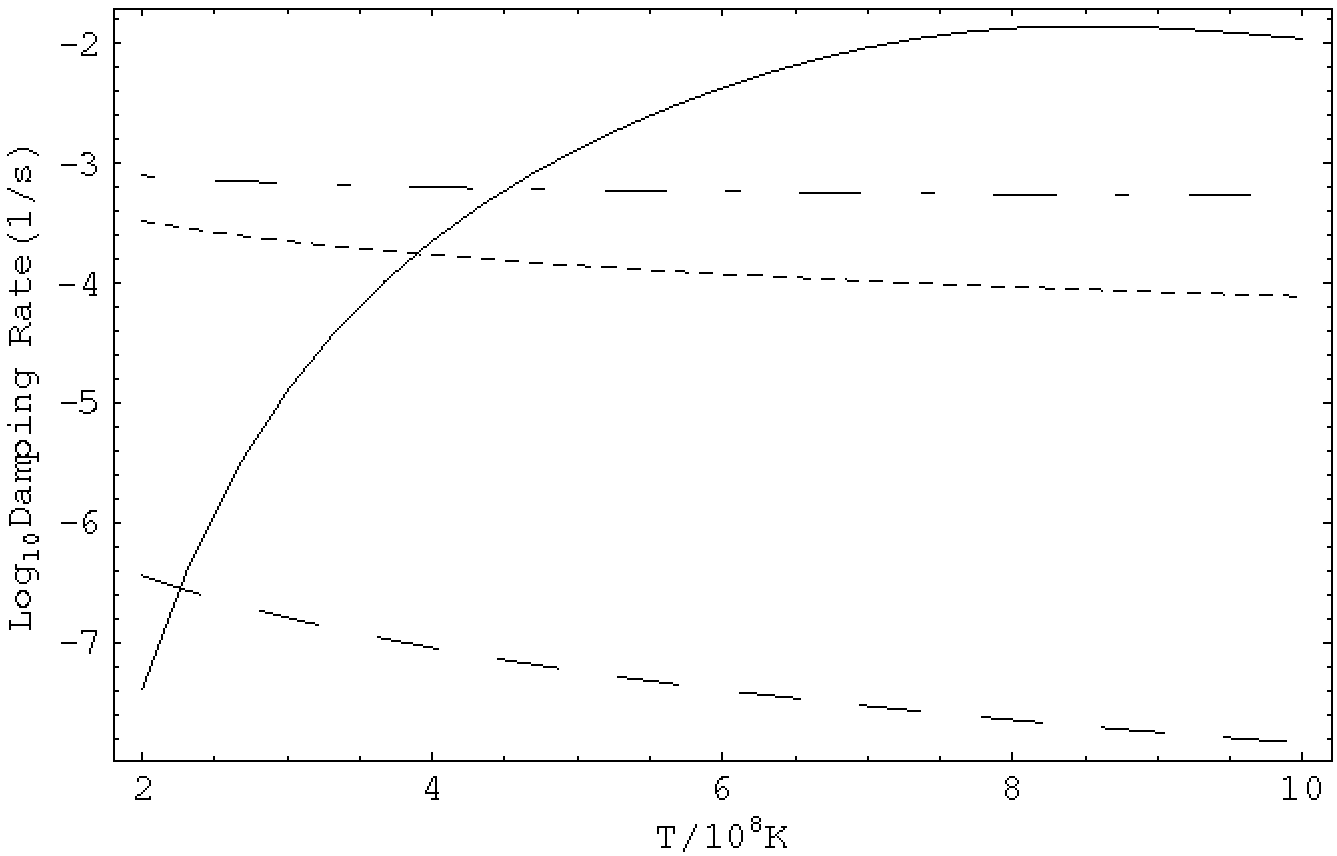

Fig. 1. The dependence on temperature of the three contributions to the damping rate F_v: core shear viscosity (F_{sh}, long dashed), boundary layer viscosity for $B \lesssim 10^9$ Gauss (short dashed) and $B = 10^{11}$ Gauss (short and long dashed), and hyperon bulk viscosity (solid). The model chosen has a hyperon superfluid transition temperature $T_h = 2 \times 10^9$ K, core–crust slippage factors $\mathcal{S}_n = \mathcal{S}_s = 0.2$, and $\Omega = 0.30\Omega_c$.

Combining equations (1), (2), and (3) then gives

$$\frac{1}{\alpha}\frac{d\alpha}{dt} = F_g - F_v + [K_j F_g + (1-K_j)F_v]K_c\alpha^2 - \left(\frac{j_a}{2J_+}\right)\dot{M}(t)\ , \quad (A)$$

$$\left(\frac{I_+}{J_+}\right)\frac{d\Omega}{dt} = -2[K_j F_g + (1-K_j)F_v]K_c\alpha^2 + \left[\frac{(j_a - j_+)}{J_+}\right]\dot{M}(t)\ ; \qquad (B)$$

where $I_+(M,\Omega) = \partial J_+/\partial\Omega$ and $j_+(M,\Omega) = \partial J_+/\partial M$.

Thermal energy conservation for the star gives

$$\int \frac{\partial T}{\partial t} c_v dV \equiv C(T)\frac{dT}{dt} \cong 2\tilde{E}_c F_v(\Omega, T) + K_{nuc}\langle\dot{M}\rangle c^2 - L_\nu(T)\ , \quad (4)$$

where the rotating frame canonical energy $\tilde{E}_c = -(\sigma/m+\Omega)J_c = K_e\Omega J_+\alpha^2$. We have assumed that the thermal conductivity timescales are short enough to allow the use of a single spatially-averaged temperature T.

Comparison of observations of thermal emission from isolated neutron stars with computed cooling histories has led Kaminker, Yakovlev & Gnedin (2002) to propose the following maximum values of the (density dependent) superfluid transition temperatures:

(a) $T_n \lesssim 10^8$ K for the (triplet) core neutrons,

(b) $T_p \gtrsim 5 \times 10^9$ K for the (singlet) core protons,

(c) $T_n \gtrsim 5 \times 10^9$ K for the (singlet) inner crust neutrons.

The neutrino luminosity is then given by (see Figure 2)

$$L_\nu = \mathcal{C}_{du}T^6 R_{du}(T/T_p) + \mathcal{C}_{mu}T^8 R_{mu}(T/T_p) + \mathcal{C}_{ei}T^6 + \mathcal{C}_{nn}T^8 + \mathcal{C}_{cp}T^7\ .$$

servation:

$$L_{\gamma,acc} \approx (3GM/4R)\dot{M}(t)\ .$$

We have taken $\dot{M} = \dot{M}_{Edd}/3 \approx 10^{-8} M_\odot\ \mathrm{yr}^{-1}$.

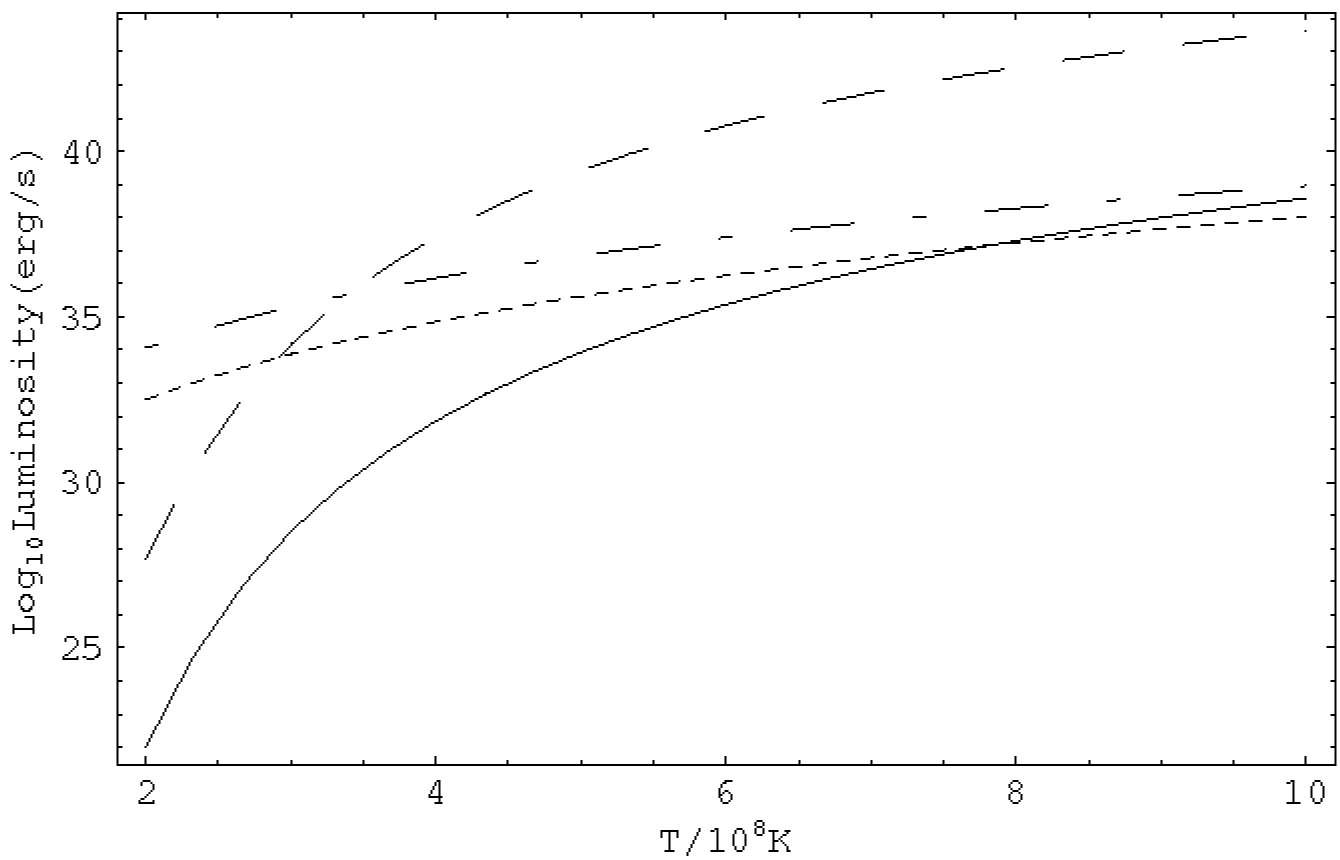

Fig. 2. The dependence on temperature of the four contributions to the neutrino luminosity L_ν: direct Urca (long dashed), modified Urca (solid), neutron-neutron and electron-ion bremsstrahlung (short dashed), and Cooper pairing of inner crust neutrons (short and long dashed). The proton superconducting transition temperature is taken to be $T_p = 5 \times 10^9$ K.

INITIAL AND EQUILIBRIUM STATES

We are interested in the evolution of neutron stars after they have been spun up to the point where the gravitational radiation growth rate has become equal to the viscous damping rate:

$$F_g(\Omega_0, M_0) = F_v(\Omega_0, M_0, T_0) \; .$$

This equality defines our initial state, where the perturbation can begin to grow. (See Figure 3.) The initial temperature T_0 is determined by the vanishing of equation (4), with the nuclear heating in the inner crust balanced by the neutrino emission.

In contrast to the initial state, the equilibrium state of our dynamical variables is defined by the vanishing of the evolution equation (3), in addition to equations (2) and (4). The equilibrium amplitude is then given by

$$\alpha_e = \left[\frac{F_a}{2K_c F_g} \right]^{1/2} \sim \left(10^{-6} - 10^{-5} \right) \; .$$

These values of α_e are much less than the saturation amplitude of the r-mode instability [Arras et al. (2002)].

The linearized analysis of Wagoner, Hennawi & Liu (2001) shows that stability of the equilibrium requires that

$$\left(\frac{1}{\ } \frac{\partial L_v}{\ } \right) > \left(\frac{1}{\ } \frac{\partial F_v}{\ } \right) > 0$$

assuming that $|\partial/\partial T| \sim 1/T$.

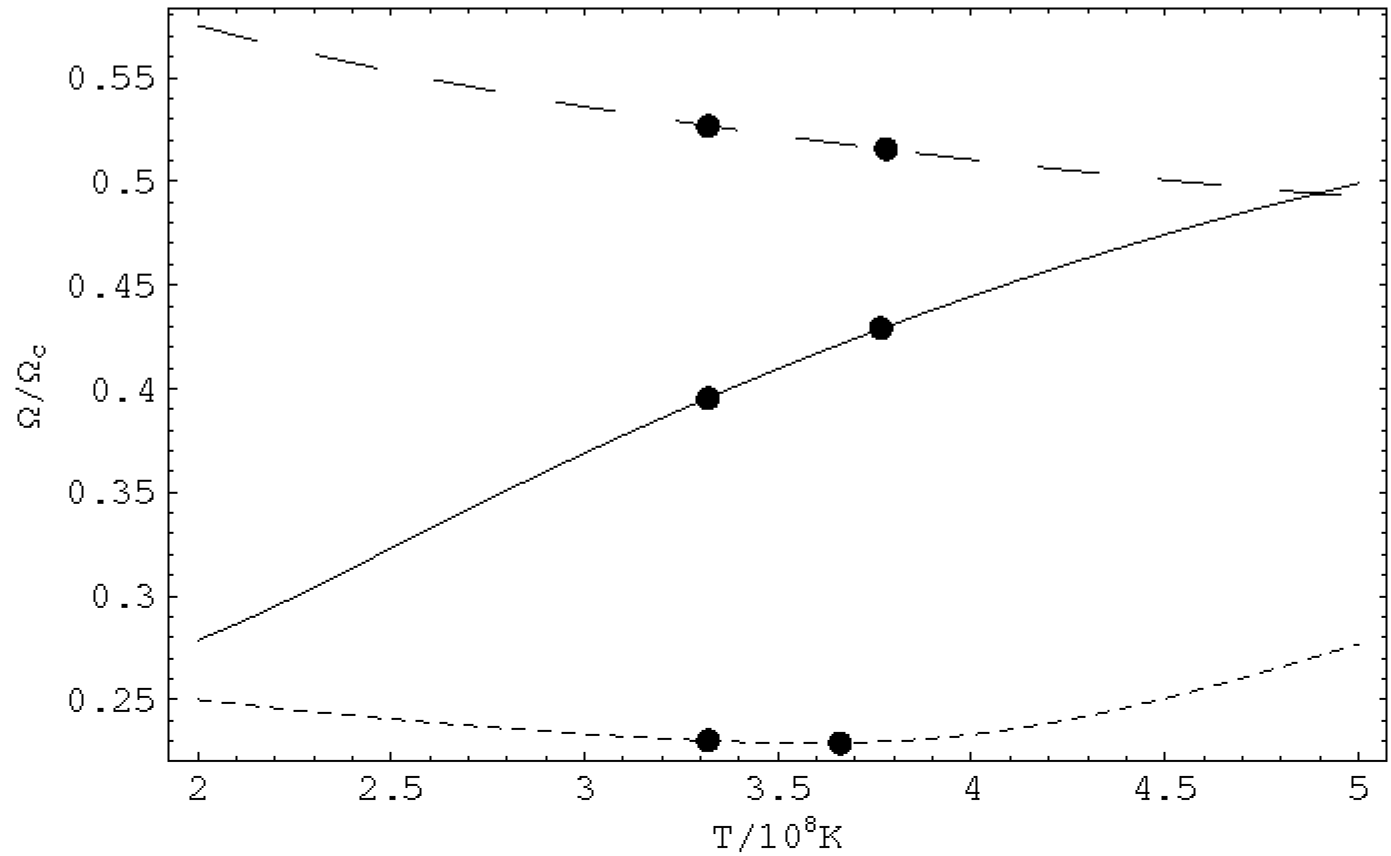

Fig. 3. The relation bewtween Ω and T on the critical curve defined by $F_g = F_v$ is shown for the choices (a) $T_h = 3 \times 10^9$ K, $\mathcal{S}_{ns} = 1.0$ (long dashed), (b) $T_h = 3 \times 10^9$ K, $\mathcal{S}_{ns} = 0.1$ (short dashed), and (c) $T_h = 1 \times 10^9$ K, $\mathcal{S}_{ns} < 0.3$ (solid). [$\mathcal{S}_{ns}^2 \equiv (2\mathcal{S}_n^2 + \mathcal{S}_s^2)/3$.] The magnetic boundary-layer viscosity is assumed negligible. Also shown are the initial and equilibrium states (at the higher temperature on each curve).

The maximum rotation rate (due to shedding) of our chosen neutron star model is $f_{max} \cong (2/3)(\Omega_c/2\pi) = 856$ Hz. Coherent oscillations in Type I X-ray bursts have been observed at frequencies $F \leq 590$ Hz, which would then correspond to $\Omega \lesssim 0.46\Omega_c$ if they represented the rotation rate. There is evidence that some of these are the first harmonic, in which case the highest spin frequency is 350 Hz. (For isolated neutron stars, $f \leq 642$ Hz.)

EVOLUTION

For a typical value $\Omega \sim 0.3\Omega_c$, the time scale $\tau_g \equiv 1/F_g \sim 10^4$ sec. Two other key time scales are that due to cooling and accretion,

$$\frac{1}{\tau_c} \equiv F_c \equiv \frac{L_\nu(T_0)}{C(T_0)T_0} \sim \frac{1}{10^3\ \mathrm{yr}} , \quad \frac{1}{\tau_a} \equiv F_a \equiv \left(\frac{j_a}{J_0}\right) \langle \dot{M} \rangle \sim \frac{1}{10^7\ \mathrm{yr}} .$$

It can be shown that the evolution from the initial state is also controlled by the sign of $(\partial F_v/\partial T)_0$, which is equal to the sign of the slope of the critical curve.

• If the slope is negative [case (a)], there will be a thermal runaway with a growth rate that is of the same magnitude as found by Levin (1999).

• If the slope is close to zero [case (b)], there will initially be overstable oscillations of the type found by Wagoner, Hennawi, and Liu (2001).

• If the slope is positive [case (c)], the oscillations of the growing amplitude are damped out on a timescale τ_c, after which it slowly increases to its equilibrium value [$\alpha_e = 1.9 \times 10^{-6}$ for the parameters of case (c)], along with Ω and T (see Figures 4, 5, and 6). The time required to reach equilibrium is

$$\Delta t \approx (\Delta\Omega/\Omega)\tau_a .$$

Throughout, F_g remains very close to F_v, so the evolution is along the critical curve.

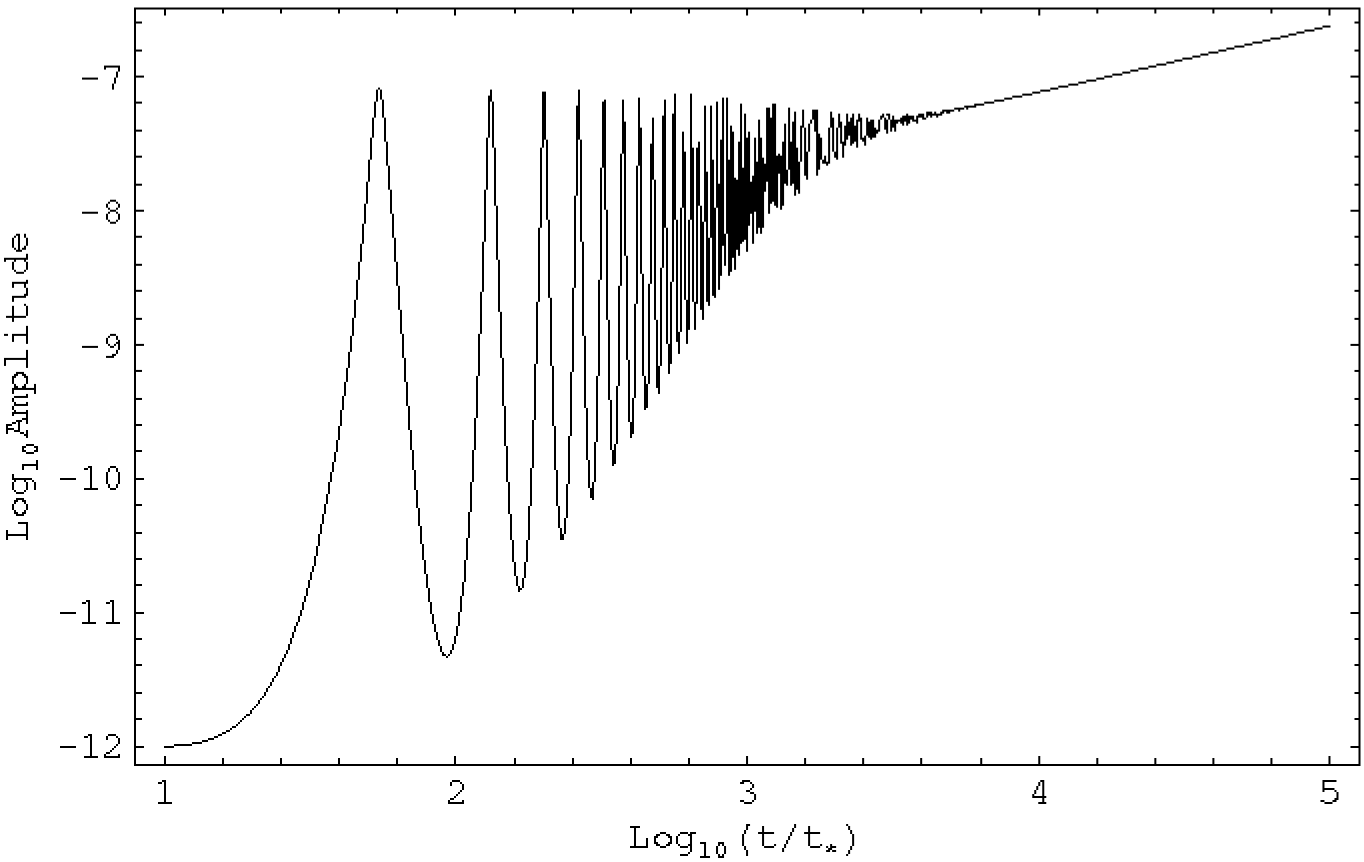

Fig. 4. The early evolution of the r-mode amplitude α, for case (c). The initial amplitude was chosen to be $\alpha_0 = 10^{-12}$, and $t_* = 1$ year.

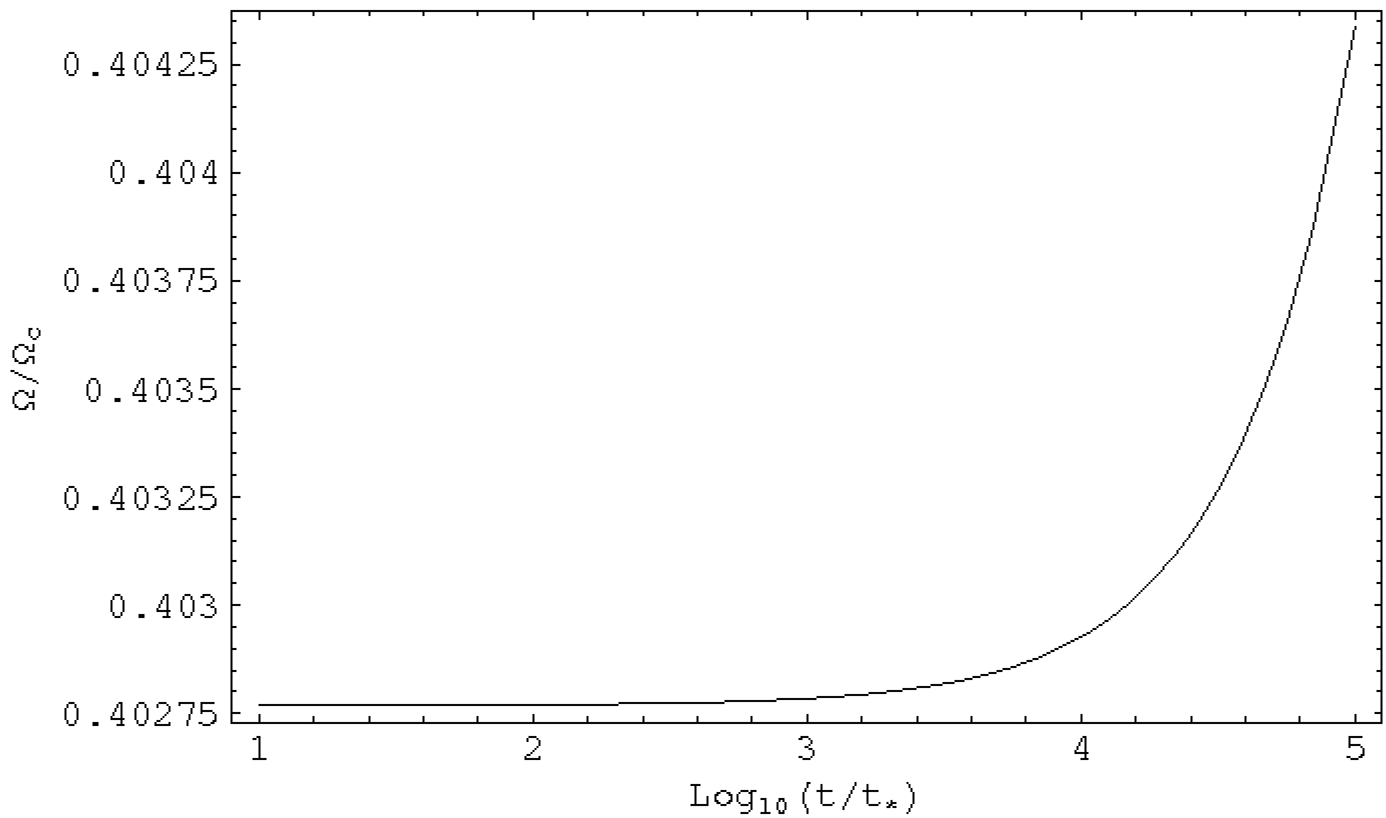

Fig. 5. The early evolution of the angular velocity, for case (c)

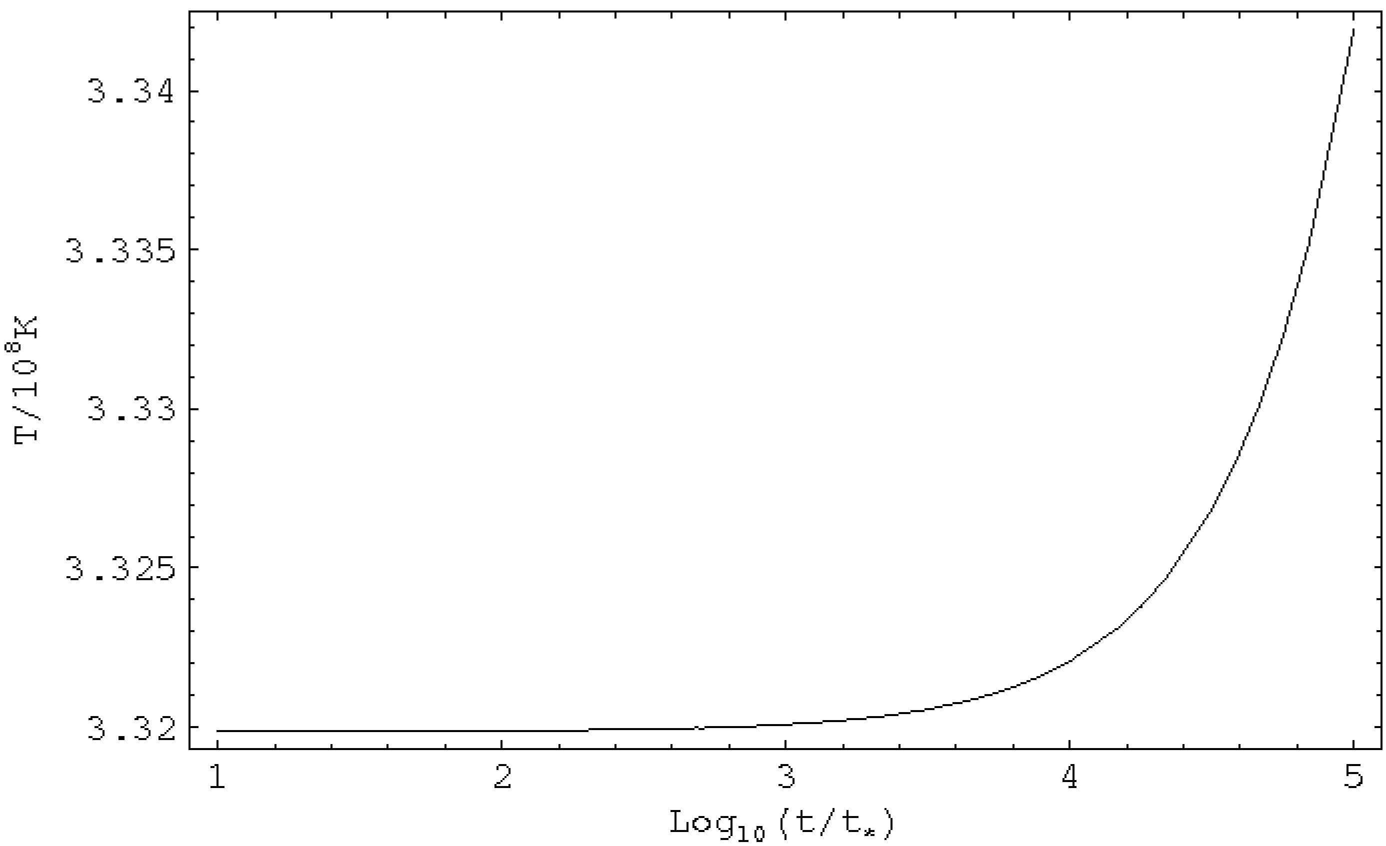

Fig. 6. The early evolution of the temperature, for case (c)

CONCLUSIONS

Evolution to a stable equilibrium state can occur if

(a) a significant fraction of the neutron star is above the density threshold [$(5-8)\times 10^{14}$ g cm^{-3}] for hyperons,

(b) their superfluid transition temperature $T_h \lesssim 2\times 10^9$ K,

(c) the core neutrons near the crust are not a superfluid whose vortices are strongly pinned to the crust,

(d) the magnetic field $B \lesssim 10^{10}$ G in that core–crust boundary layer.

If Sco X-1 has been spun up by accretion to such a stable equilibrium state (in which gravitational-wave flux is proportional to X-ray flux), it should be detectable by the second generation LIGO detectors. (However, its spin period P remains unknown; with $f_{gw} = 4/3P$. Also, varying $\dot{M}(t) \Rightarrow$ phase change by one cycle in $\gtrsim 3$ days.)

When signal recycling ('narrow-banding') is employed, a few additional LMXB's may also be detectable [Cutler & Thorne (2002)].