S. R. Valluri

Gravitational Wave Signal from a Pulsar and the Zak-Gelfand Transform

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1) Introduction

2) The Gravitational Wave Signal from a Pulsar

3) The Fourier Transform of the GW Signal

4) The Zak-Gelfand Transform (ZGT)

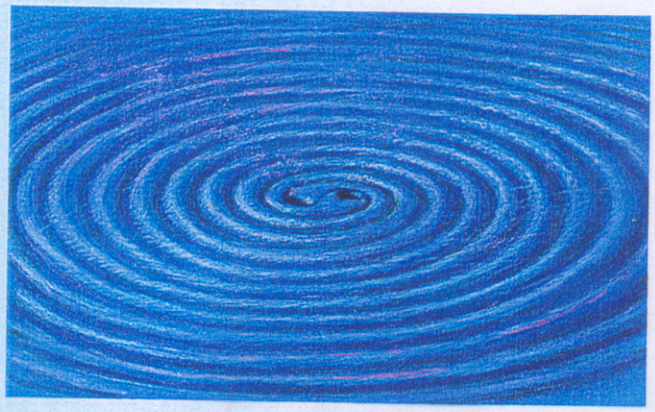
Analytic Calculations of the FT and the ZGT and Pulsar Spindown

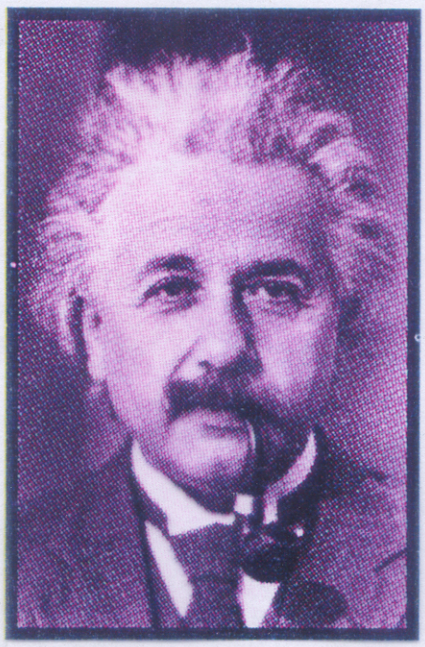
5) Numerical Plots

6) Conclusions

1.

CATCHING THE GRAVITATIONAL WAVE





Albert Einstein,
the father of general
relativity theory

1: The Detection of Gravitational Waves.

A CHALLENGE FOR THE SOCIETY OF TRUE SEARCHERS !!

Albert Einstein - 1916 - predicted existence of GW

Estimated the amount of GW - Long period binary Star. Radiation had "negligible practical effect"

Felix Pirani (Graduate of UWO - CANADA)

1962 : Gravitation - An Introduction To Current Research

"The weakness of the gravitational interaction makes it exceedingly unlikely that gravitational radiation will ever be the subject of direct (direct) observation"

Joe Weber, Joe Taylor, Hulse, Many others proved otherwise !

Nature outruns the most daring imagination of humans far behind. !!

Motivation for the Fourier Transform (FT)

a) Study the extent to which the peak of FT is:

1) Smudged

2) how much the FT Spreads in frequency
FT - ideal filter for Constant f. Space.

b) Useful for Data Analysis, Doppler Shift Study and Stepping Around the Sky Method (Schutz 1991) which rely on the FT.

c) An interesting Study pease of Special Functions (the generalised Hypergeometric Function) Transforms

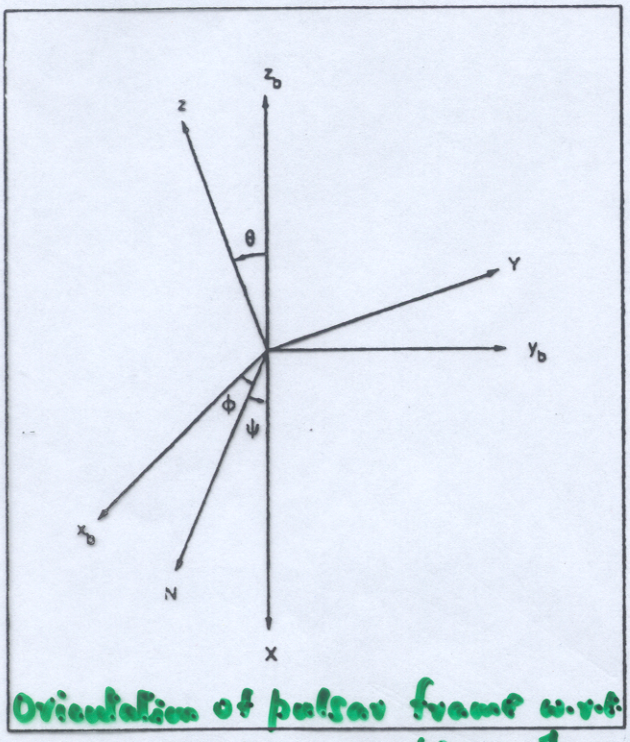
Numerical Schemes } Padé Schemes
Asymptotic Expansions }

Sequence Transformations

First Step
Only Rotational Motion of Earth Considered
Orbital Motion around Sun was neglected - now Considered

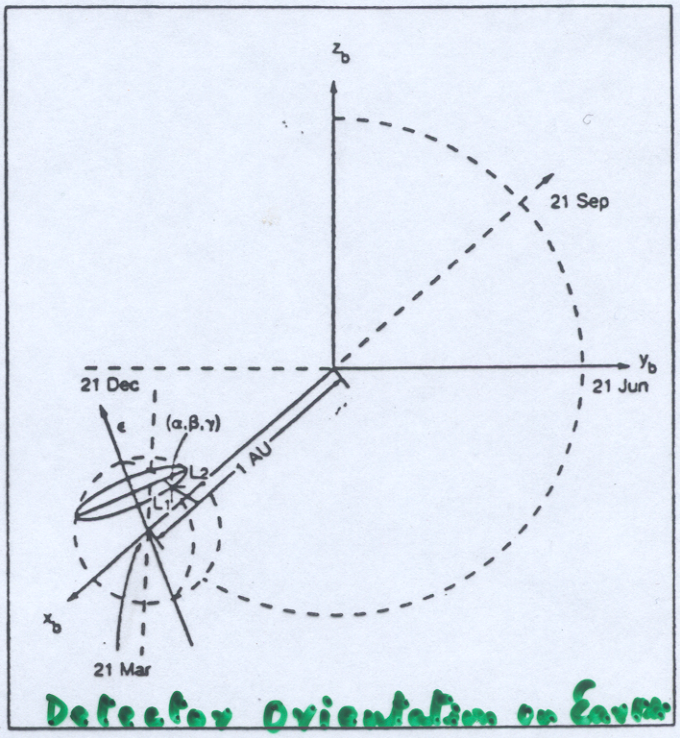
d) Computational Cost, Strategy

2) GW Signal from a Pulsar:



Orientation of pulsar frame w.r.t. Wave frame

Figure I



Detector Orientation on Earth

Figure II

α = Colatitude, γ = Angle bisector of detector with local meridian
β = azimuth

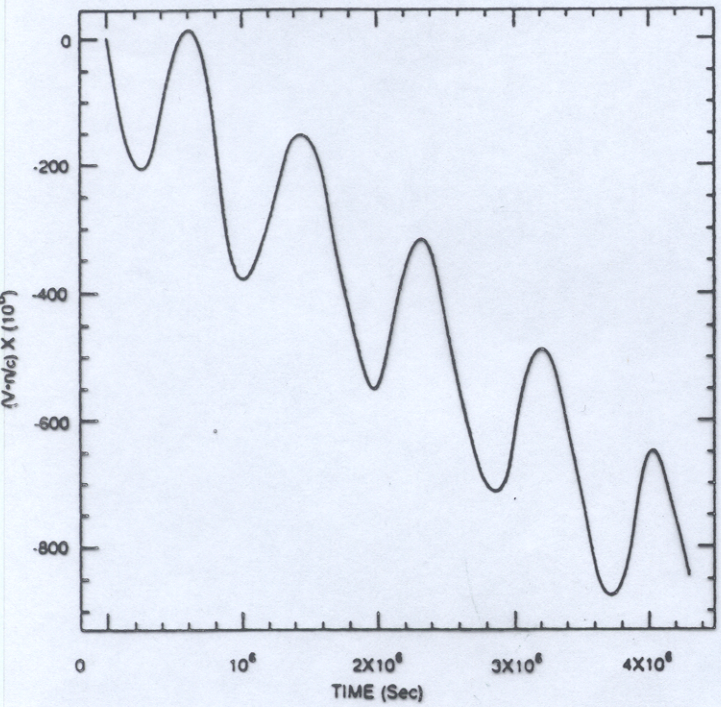


Figure III

Doppler velocity variation $\vec{V}\cdot\vec{n}/c$
Diurnal + Orbital (Period = 1 yr)

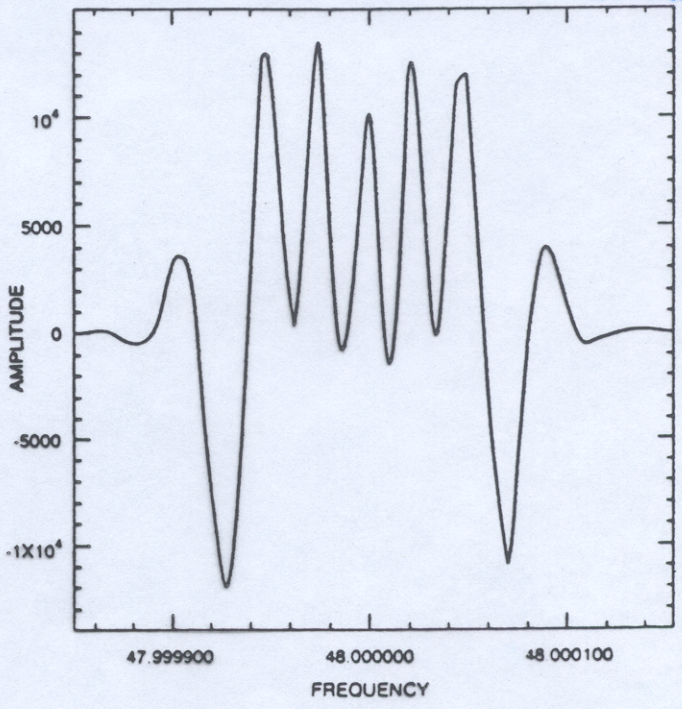


Figure IV

FT of pulsar signal (includes Orbital correction)

Detector's Motions

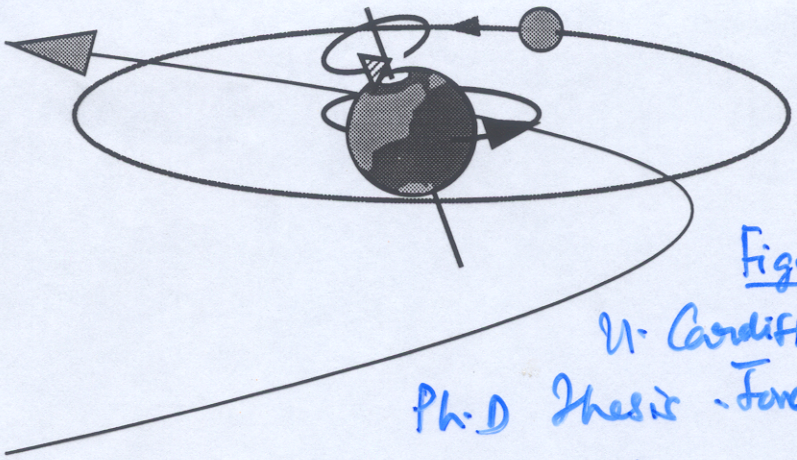


Figure: U. Cardiff. 1997 Ph.D Thesis - Jones

Doppler Shifts Vary with time. - Large Effect!

Figure 2.1: A Detector's overall motions: *A detector on the Earth's surface will be constantly moving and accelerating. The Earth's rotation, the Earth-Moon orbiting system, the Earth orbiting the Sun, the Sun's motion through the galaxy will all introduce accelerations on the detector, some more important than others.*

pulsars. One would not expect to see a sinusoidal signal in a time series data set, the noise would normally be too high. A correlation between detectors (similar to the method employed in a stochastic search) at any frequency may be possible, if the detectors are stationary. However as the two detectors would have to be far apart, they would 'see' different Doppler shifts at different times thus the signal would not be able to be correlated. Also the wide-band noise in the correlation would overwhelm the signal. Searches must separate the narrow-band signals from the wide-band noise which does not happen in a correlation analysis.

Doppler modulation

A detector on the Earth's surface would be constantly moving. The Earth's rotation, its motion about the Earth-Moon barycentre and its motion about the Sun would all contribute to accelerations of the detector (Figure 2.1). These accelerations would produce Doppler shifts on a received signal that vary with time. Thus over time the sinusoid will have a spread of frequencies which depends on the relative positions of the detector to the source.

Long Integration Times Needed.
Better S/N ratio.

3: The Fourier Transform of the GW Signal:

Other Motions like

1) the Earth-Moon Orbiting
2) the Sun's motion through the galaxy
* 3) Jupiter's perturbations of the Earth

(will be attempted next)

will all introduce accelerations on the detector.

The Doppler Shift considered next;

Both the rotational and orbital motions included

The FM output of the Signal $h(t)$ with amplitude unity. is:

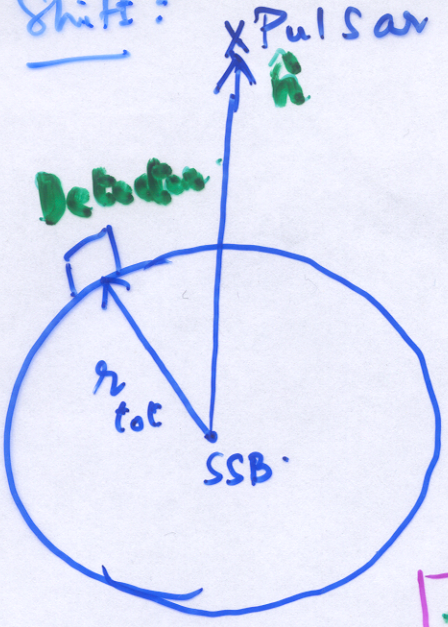
$$h(t) = \mathrm{Re}\ \exp(i\phi(t)) = \cos\phi(t) \quad (1)$$

$\phi(t)$ = Phase of the Doppler shifted received signal for a single direction (θ, ϕ) Sky Search

$$= 2\pi \int_0^t f_{\text{received}}(t')\, dt' \quad (2)$$

$$\tilde{h}(f) = \int_0^T h(t)\, e^{-i2\pi f t}\, dt \quad (3)$$

Doppler Shift:



Two instants t_b and t_d referring to the same phase of the GW signal:

$$t_b = t_d + \left(\frac{\vec{r}_{tot} \cdot \hat{n}}{c} \right)_{t_d}$$

Or via

$$\vec{r}_{tot}(t_d) = \vec{r}_{tot}(t_b + t_d - t_b) \approx \vec{r}_{tot}(t_b) + (t_d - t_b)\, \dot{r}_{tot}\Big|_{t_b}$$

$$t_d \approx t_b - \left(\frac{\vec{r}_{tot} \cdot \hat{n}}{c} \right)_{t_b}$$

t_b = Time in SSB frame (Solar System Barycenter)

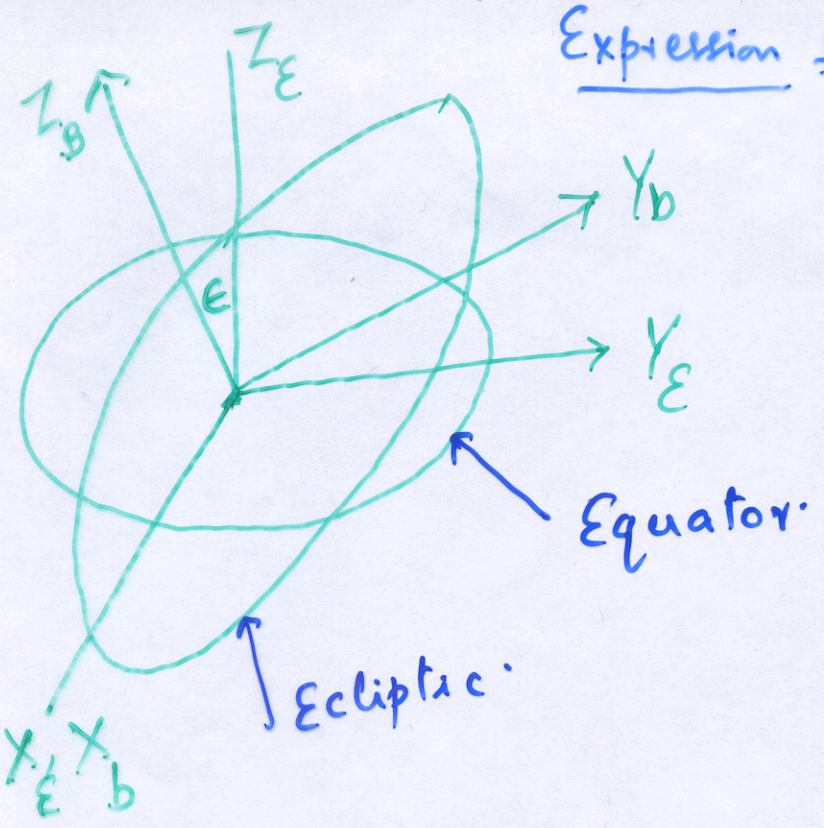
t_d = Time in Detector frame

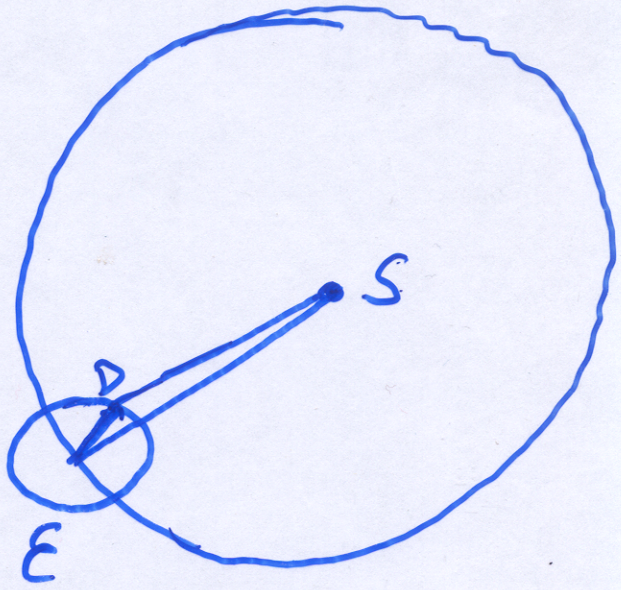
$$f_0 = \frac{1}{\Delta t_b} \qquad \Delta t_b = \Delta t_d \left(1 + \frac{\vec{n} \cdot \dot{\vec{r}}}{c}\Big|_{t_d} \right)$$

$$f = \frac{1}{\Delta t_d} \quad \text{i.e.} \quad f = f_0 \left[1 + \frac{\hat{n} \cdot \dot{\vec{r}}}{c}\Big|_{t_d} \right]$$

Doppler Shift → depends on the Source direction

Expression for $\bar{r}_{tot}$:





$\bar{SD} = \bar{SE} + \bar{ED}$, i.e. $\bar{r}_{tot} = \bar{A} + \bar{R}$

SSB and Earth axes connected by rotation ϵ about common X axis

$$\begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}_{SSB} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\epsilon & -\sin\epsilon \\ 0 & \sin\epsilon & \cos\epsilon \end{pmatrix} \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix}_{\varepsilon}$$

$$\vec{R} = \left(R \sin\alpha \cos(\beta_0 + \omega_{rot} t_b),\ R \sin\alpha \sin(\beta_0 + \omega_{rot} t_b),\ R\cos\alpha\right)$$

$$\begin{pmatrix} A_x \\ A_y \\ A_z \end{pmatrix}_{SSB} = \begin{pmatrix} A \cos \omega_{orb} t_b \\ A \sin \omega_{orb} t_b \\ 0 \end{pmatrix}$$

$$\bar{r}_{tot}(t) - \bar{r}_{tot}(0) = \begin{bmatrix} A(\cos(\omega_{orb} t) - 1) \\ A(\sin(\omega_{orb} t)) \\ 0 \end{bmatrix} = \bar{R}_{orb} + \bar{R}_{rot}$$

$$+ \begin{bmatrix} R\sin\delta\left(\cos(\omega_{rot} t) - 1\right) \\ R\sin\delta \quad \sin(\omega_{rot} t)\cos\epsilon \\ R\sin\delta \quad \sin(\omega_{rot} t)\sin\epsilon \end{bmatrix} \quad (4)$$

R = Earth radius

A = Earth-Sun distance

$\hat{n}$ = Unit Vector in direction of Source

$= (\sin\theta\cos\phi,\ \sin\theta\sin\phi,\ \cos\theta)$

We need to Consider the expression:

$$e^{i\phi_{Phase}(t)} = e^{i 2\pi f_0 \left\{ t + \hat{n}\cdot\left(\bar{R}_{orb} + \bar{R}_{rot}\right)/c \right\}}$$

$$= \exp\left\{ i 2\pi f_0 t + i 2\pi f_0 \left(\hat{n}\cdot\frac{\bar{R}_{rot}}{c}\right)\right\} \exp\left\{ i \frac{2\pi f_0}{c} \hat{n}\cdot\bar{R}_{orb} \right\}$$

The first of these exponentials has been already done.

and treats purely rotational motion case.

The plane wave expansion for $\exp\left\{ i 2\pi f_0 \frac{\hat{n}\cdot\bar{R}_{rot}}{c} \right\}$ follows next:

Useful in many applications.

$$e^{i\phi(t)} = e^{i2\pi f_0\left\{t+\hat{n}\bullet\left(\frac{\bar{R}_{orb}+\bar{R}_{rot}}{c}\right)\right\}} = \exp\{i2\pi f_0 t\}\bullet\exp\left\{i2\pi f_0\left(\hat{n}\bullet\frac{\bar{R}_{rot}}{c}\right)\right\}\bullet\exp\left\{i2\pi f_0\left(\hat{n}\bullet\frac{\bar{R}_{orb}}{c}\right)\right\} \quad (5)$$

The second and third of these exponential expressions incorporate the purely rotational and orbital effect terms.

The plane wave expansion for the rotational part of eq (5) is of the form[4]:

$$\exp\left\{i2\pi f_0\frac{\hat{n}}{c}\bullet\bar{R}_{rot}\right\} = 4\pi\sum_{\ell m} i^{\ell}Y_{\ell m}(\theta,\phi)Y^{*}_{\ell m}\left(\alpha,\frac{\omega_r t}{2}\right)j_{\ell}\left(k\,Sin\left(\frac{\omega_r t}{2}\right)\right),\ -\ell\le m\le\ell,\ 0\le\ell\le\infty \quad (6)$$

Here,

$k=\frac{4\pi f_0 R_e Sin\alpha}{c}$, α is the colatitude of the detector, j_ℓ is the spherical Bessel function and $Y_{\ell m}$ are the Spherical Harmonics. Plane wave expansions have significant applications in a variety of problems in quantum and electromagnetic theory, etc.

Similarly, the orbital part upon expansion gives:[8]

$$\exp\left\{i\frac{2\pi f_0}{c}\bar{R}_{orb}(t)\bullet\hat{n}\right\} = \exp\left[i\frac{2\pi f_0 A}{c}\sin\theta\bullet\{\cos(\omega_{orb}t)\cos\phi+\sin(\omega_{orb}t)\sin\phi-\cos\phi\}\right]$$

$$= e^{-i\frac{2\pi f_0 A}{c}\sin\theta\cos\phi}\bullet e^{i\frac{2\pi f_0 A}{c}\cos(\omega_{orb}t-\phi)\sin\theta} \quad (7)$$

$$= \underbrace{e^{-i\frac{2\pi f_0 A}{c}\sin\theta\cos\phi}}_{\substack{Independent\ of\ t,\\ depends\ on\ f_0, A, \theta\ and\ \phi}}\bullet\sum_{n=-\infty}^{\infty} i^n\ \underbrace{e^{in\omega_{orb}t}}_{\substack{will\ be\ combined\\ with\ e^{i2\pi f_0 t}e^{-i\frac{m}{2}\omega_r t}}}\ e^{-in\phi}\ \underbrace{J_n\left(\frac{2\pi f_0 A}{c}\bullet\sin\theta\right)}_{Independent\ of\ t}$$

The Fourier transform $\tilde{h}(f)$ of the signal can therefore be written in terms of an indexed summation over $S_{n\ell m}(f_0,\alpha,t)$ which incorporates the time integral:

The Fourier transform $\tilde{h}(f)$ of the signal can therefore be written in terms of an indexed summation over $S_{n\ell m}(f_0,\alpha,t)$ which incorporates the time integral:

$$\sum_{n.\ell.m}\int_0^T S_{n\ell m}(\omega_0,\omega_r,\omega_{orb},\alpha,t,A,k,\theta,\phi)dt = \left[\begin{array}{l} \sum_{n\ell m} 4\pi i^{\ell} Y_{\ell m}(\theta,\phi) N_{\ell m} P_{\ell}^{m}(\cos\alpha)\sqrt{\frac{\pi}{2}} e^{-i\frac{2\pi A f_0}{c}\sin\theta\cos\phi} \\ \bullet \int_0^T e^{-i2\pi(f-f_0)t}\frac{J_{\ell+\frac{1}{2}}\left(k\sin\left(\frac{\omega_r t}{2}\right)\right)}{\sqrt{k\sin\left(\frac{\omega_r t}{2}\right)}} e^{-im\frac{\omega_r}{2}t - in\omega_{orb}t}dt \\ \bullet i^n e^{-in\phi} J_n\left(\frac{2\pi f_0 A\sin\theta}{c}\right) \end{array}\right] \quad (8a)$$

the $Y_{\ell m}^{*}\left(\alpha,\frac{\omega_r t}{2}\right)$ have been expressed in terms of the Legendre polynomials $P_{\ell}^{m}(\cos\alpha)$, the

$$N_{\ell m} = \sqrt{\frac{(2\ell+1)(\ell-|m|)!}{4\pi(\ell+|m|)!}} \quad (8b)$$

is the normalization constant for the P_{ℓ}^{m}, and J is the cylindrical Bessel function.
For conciseness, we define:

$$B_{orb} = 2\left(\frac{\omega-\omega_0}{\omega_r}+\frac{m}{2}+\frac{n}{2}\frac{\omega_{orb}}{\omega_{rot}}\right) \quad (9)$$

$$\omega_r = R\,\omega_{orb},\ (R\approx 365)$$

4: Zak-Gelfand Transform:

Next, we consider the integral:

$$I = \int_0^{2\pi R} e^{-iB_{orb}\frac{\omega_r t}{2}} j_{\ell}\left(k\sin\left(\frac{\omega_r t}{2}\right)\right)dt \quad (10a)$$

Defining

$$R\omega_{orb}t = s + 2\pi(j-1), \quad (10b)$$

Using the relation

$$j_\ell\left(k\sin\left(\frac{s}{2}+\pi(j-1)\right)\right)=e^{i\ell(j-1)\pi}j_\ell\left(k\sin\frac{s}{2}\right), \qquad (11a)$$

the integrand with the summation becomes:

$$\sum_{j=1}^{R} e^{-\frac{i}{2}B_{orb}2\pi(j-1)}j_\ell\left(k\sin\left(\pi\left(\frac{s}{2\pi}+j-1\right)\right)\right)=Z[f(s,B_{orb})] \qquad (11b)$$

where Z[ ] is the *Zak-Gelfand Transform*[5] of:

$$f(s,B_{orb})=e^{-i\frac{B_{orb}}{2}\cdot 2\pi(j-1)}j_\ell\left(k\sin\pi\left(\frac{s}{2\pi}+j-1\right)\right) \qquad (12)$$

This integral transform is useful, for example in the treatment of Bloch wave functions for an electron in periodic potentials arising in condensed matter physics[6] and other mathematical applications.

Setting

$$\frac{s}{2}=\xi=\frac{\omega_r t}{2}, \qquad (13a)$$

we have

$$I=\frac{2}{\omega_r}\sum_{j=1}^{R}\int_0^\pi e^{i\pi(j-1)\{\ell-B_{orb}\}}e^{-iB_{orb}\xi}j_\ell(k\sin\xi)d\xi\,. \qquad (13b)$$

The series related to the *Zak* transform can be summed:

$$\sum_{j=1}^{R} e^{i\pi(j-1)(\ell-B_{orb})}=\frac{1-e^{i\pi(\ell-B_{orb})R}}{1-e^{i\pi(\ell-B_{orb})}},\ (R=365), \qquad (14)$$

Here $(\ell-B_{orb})$ is real and $e^{i\pi R(\ell-B_{orb})}$ is a complex number.

Equation (14) is an exact result analogous to that obtained in diffraction maxima in a crystal.

The integral

$$I_1=\int_0^\pi e^{-iB_{orb}\xi}j_\ell(k\sin\xi)d\xi \qquad (15)$$

can be evaluated exactly as well as numerically. The analytic expression is given below:

Analogy with the Crystal lattice.

Periodic Pattern of the Rays in each year.

Electron in a periodic lattice potential and a Constant field (eg, the magnetic field)

Eigenvalues of finite translations - Complete set of States in QM.

Zak - Gelfand Transform.

Zak → Finite Translations in Solids

Gelfand - Periodic Potentials for Schrödinger equ.

Weil-Brezin map - known to Gauss

Radar - Ambiguity Functions (Time Delay Range determination

Uncertainty Principle

for Detectors at (τ_0, ω_0) and $(\tau_0+\tau, \omega_0+\omega)$ to be distinguished from each other.

The series related to the Zak transform can be summed (as Seemabai) the integral.

$$I = \int_0^{\pi} e^{-iB_{orb}\xi} \, j_\ell(k \sin\xi) \, d\xi$$

(j_ℓ = Spherical Bessel function)

Can be evaluated exactly as well as numerically.

$$I = 2 e^{iB_{orb}\pi/2} \int_0^{\pi/2} \cos(B_{orb}\xi) \, j_\ell(k\cos\xi) \, d\xi$$

$$= 2 e^{iB_{orb}\pi/2} \cdot \sqrt{\frac{\pi}{2}} \int_0^1 \cos\left(B_{orb}\cos^{-1}x\right) \frac{J_{\ell+\frac{1}{2}}(kx)}{\sqrt{kx}} \frac{dx}{\sqrt{1-x^2}}$$

$$= 2 e^{iB_{orb}\pi/2} \cdot \pi \sqrt{\frac{\pi}{2}} k^{\ell+\frac{1}{2}} \cdot \Gamma\left[\begin{matrix} \ell+1 \\ \ell+\frac{3}{2}, \frac{\ell+B_{orb}+2}{2}, \frac{\ell-B_{orb}+2}{2} \end{matrix}\right]$$

$$\cdot {}_1F_3\left(\ell+1; \ell+\frac{3}{2}, \frac{\ell+B_{orb}+2}{2}, \frac{\ell-B_{orb}+2}{2}; \frac{-k^2}{16}\right) \quad (16)$$

The $\Gamma[\ \ ]$ denote one gamma function in numerator, three in denominator

The ${}_1F_3$ denotes the Generalized Hypergeometric Function with one parameter (here $l+1$) and three in denominator.

Variable $= -\frac{k^2}{16}$.

Hence: $\sum S_{nlm}(\omega_0, \omega_{sv}, \omega_{orb}, d, t, A, k, \theta, \phi)$

$$= \left[\sum_{n=-\infty}^{\infty} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} 4\pi\, i^l\, Y_{lm}(\theta,\phi)\, N_{lml}\, P_l^m(\cos\delta)\right]_{\omega_{sv}=1}^{\omega_{sv}=2}$$

$$\cdot \sqrt{\frac{\pi}{2}}\, e^{-i\frac{2\pi f_0}{c} A \sin\theta \cos\phi} \cdot i^n\, e^{-in\phi}\, J_n\left(\frac{2\pi f_0}{c} A \sin\theta\right)$$

$$\cdot \left\{\frac{1 - e^{i\pi(l - B_{orb})R}}{1 - e^{i\pi(l - B_{orb})}}\right\} \cdot 2\, e^{iB_{orb}\pi/2} \cdot \pi \cdot k^{l+\frac{1}{2}}$$

$$\cdot \frac{\Gamma(l+1)}{\Gamma\left(l+\frac{3}{2}\right)\Gamma\left(\frac{l+B_{orb}+2}{2}\right)\Gamma\left(\frac{l-B_{orb}+2}{2}\right)} \cdot {}_1F_3\left(l+1; l+\frac{3}{2}, \frac{l+B_{orb}+2}{2}, \frac{l-B_{orb}+2}{2}; -\frac{k^2}{16}\right)$$

(17)

$|S_{nlm}|^2$ Can be Calculated

Searches for periodic or quasiperiodic sources:

- limited primarily by computational resources available.

Detect sources of GW with slowly changing frequency.

Vector λ denotes the search parameters:

$$\lambda = (\lambda^0, \lambda^1, \lambda^2, \lambda^3, \lambda^4, \dots) = (f_0, \theta, \phi, f_1, f_2, \dots)$$

$$|f_j| \leq \left(\frac{1}{\tau_{min}}\right)^j, \quad \tau_{min} = f/\dot{f} = \text{characteristic time scale over which } f \text{ changes by a factor of unity.}$$

Observations of Radio Pulsars: $\tau_{min} = 40$ yr

(young fast pulsars, $f_{max} = 1$ kHz)

Old Slow pulsars with

$f_{max} = 200$ Hz, $\tau_{min} = 1000$ yr.

Metric Formalism - determine the # of parameter space points sampled in a search that accumulates S/N - Summing over power spectra.

Search / Signal parameters - never precisely matched

Metric on parameter space of waveforms:

Pulsar Spindown

Intrinsic frequency model for pulsar rotation:

$$f(t) = \frac{f_0}{2}\left(1 + \sum_k f_k t^k\right) \qquad t_{obs} << \tau \qquad (18)$$

τ = Spindown age

$$\tau = \frac{f}{\dot{f}} \quad \text{(in years)}$$

$$|f_k| \leq \tau_{min}^{-k}$$

$\tau_{min} \sim 40$ years for pulsars in our galaxy.

$\tau_{min} \geq 10^7$ years for millisec pulsars

Gravitational wave frequency measured at detector

$$f_{GW}(t) = f_0\left(1 + \sum f_k t_{Sd}^k\right)\left(1 + \frac{\bar{v}\cdot\hat{n}}{c}\right) \qquad (19)$$

$\bar{v}$ = detector velocity.

Phase ϕ now involves quadratic, cubic ... terms in t.

The integrals now involve additional trigonometric functions besides $e^{iB\Sigma} J_\ell(K\sin\Xi)$

$$t_{Sd} = t + \frac{\bar{x}}{c}\cdot\hat{n}, \quad f_k = (\tau_{min})^{-k}$$

$$\phi = \frac{2\pi f_0}{2}\left[\left\{t + \frac{\bar{x}}{c}\cdot\hat{n}\right\} + \sum_{k=0}^{} \frac{f_k}{k+1}\left\{t + \frac{\bar{x}}{c}\cdot\hat{n}\right\}^{k+1}\right] \qquad (20)$$

Consider the Second term $\sum_{k=0}^{\infty} \frac{f_k}{k+1} \left\{ t + \frac{\bar{x}}{c} \cdot \hat{n} \right\}^{k+1}$

$$= \sum_{k=0}^{\infty} \frac{T_{min}}{(T_{min})^{k+1}} \frac{1}{k+1} \left\{ t + \frac{\bar{x}}{c} \cdot \hat{n} \right\}^{k+1} \quad (21)$$

$$= -T_{min} \ln \left(1 - \frac{t + \frac{\bar{x}}{c} \cdot \hat{n}}{T_{min}} \right) \quad \left(\text{the } \log(1-x) \text{ Series} \right)$$

$$\therefore \phi(t) = e^{i 2\pi \frac{f_0}{2} \left\{ \left(t + \frac{\bar{x}}{c} \cdot \hat{n} \right) - T_{min} \ln \left(1 - \frac{t + \frac{\bar{x}}{c} \cdot \hat{n}}{T_{min}} \right) \right\}}$$

$$= e^{i 2\pi \frac{f_0}{2} \left(t + \frac{\bar{x}}{c} \cdot \hat{n} \right)} \cdot \exp\left\{ \ln \left(1 - \left(\frac{t + \frac{\bar{x}}{c} \cdot \hat{n}}{T_{min}} \right) \right) \right\}^{-T_{min} i 2\pi \frac{f_0}{2}}$$

$$= e^{i 2\pi \frac{f_0}{2} \left(t + \frac{\bar{x}}{c} \cdot \hat{n} \right)} \cdot \left[1 - \left(\frac{t + \frac{\bar{x}}{c} \cdot \hat{n}}{T_{min}} \right) \right]^{-i 2\pi \frac{f_0 T_{min}}{2}} \quad (22)$$

Binomial Expansion: ↓

$$= e^{i 2\pi \frac{f_0}{2} \left(t + \frac{\bar{x}}{c} \cdot \hat{n} \right)} \cdot \left[1 + i 2\pi \frac{f_0}{2} T_{min} \left(\frac{t + \frac{\bar{x}}{c} \cdot \hat{n}}{T_{min}} \right) + \frac{(i \pi f_0 T_{min})(i \pi f_0 T_{min} + 1)}{2} \cdot \left(\frac{t + \frac{\bar{x}}{c} \cdot \hat{n}}{T_{min}} \right)^2 + \cdots \right].$$

Define the Spin down Moment Integrals:

$$\int_0^T e^{i2\pi \frac{f_0}{2}\left(t+\frac{\bar{x}}{c}\cdot\hat{n}\right)} e^{-i2\pi f t} \cdot \left(t+\frac{\bar{x}}{c}\cdot\hat{n}\right)^k dt \qquad (23)$$

for $k = 1, 2, 3, \cdots$

$$I_{generic} = \int_0^T e^{i2\pi \frac{f_0}{2}\left(t+\frac{\bar{x}}{c}\cdot\hat{n}\right)} e^{-i2\pi f t}\, dt \qquad (24)$$

$$\frac{d^k I_{generic}}{df_0^k} = (i\pi)^k \int_0^T e^{i2\pi \frac{f_0}{2}\left(t+\frac{\bar{x}}{c}\cdot\hat{n}\right)} e^{-i2\pi f t} \cdot \left(t+\frac{\bar{x}}{c}\cdot\hat{n}\right)^k dt \qquad (25)$$

for $k = 1, 2, 3, \cdots$

The kth derivative gives the corresponding Spin down moment integral.

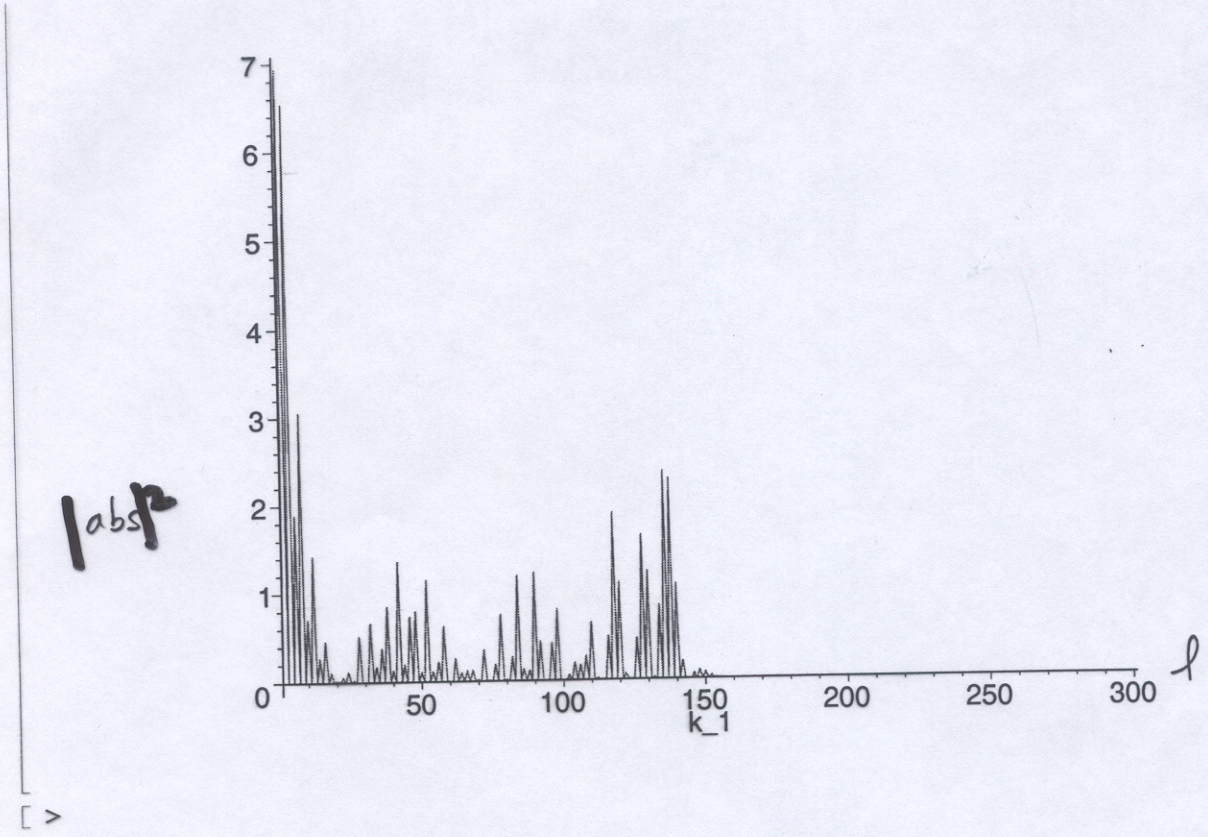
$I_{generic}$ has been analytically evaluated.

Its derivatives with respect to f_0 can be done by Maple, MatLab etc

5: Numerical Calculations, Plots

Orb + Rot. Motion

$B_{orb} = 150, \quad k = 150$



[>

Intensity ~ 0 for $l > k (\approx 150)$

Index Theorem of Bessel Functions

In the integrand: $e^{iB\xi} j_\ell(k\cos\xi)$

a) if $\ell >> 1$, B has to be small to control oscillations

b) If $k = \ell \sim 10$, $B \leq 10$

c) $k = \ell \sim 200$, B small to control oscillations

Index Theorem works itself around well!
(for Bessel functions)

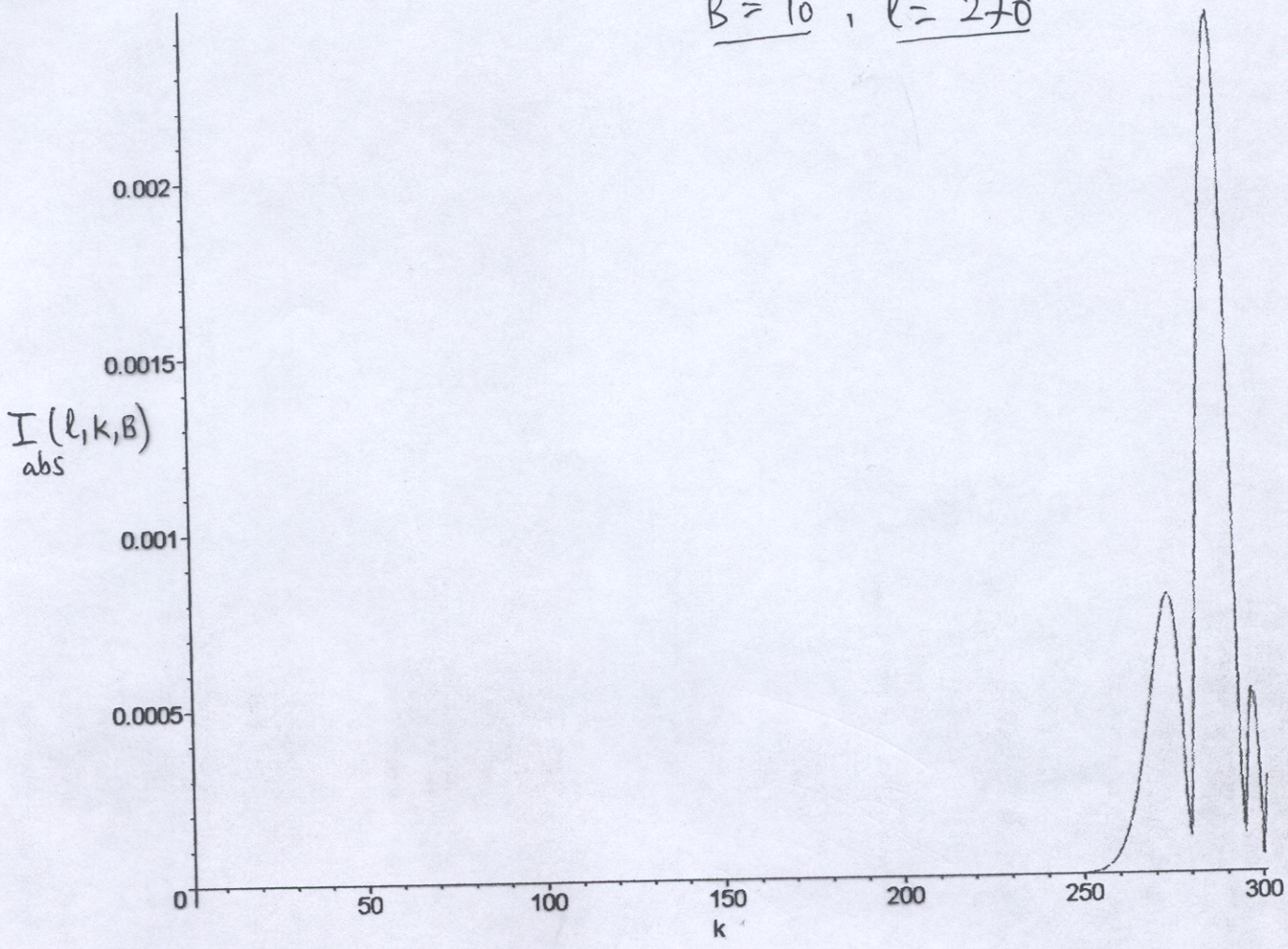
Parallel Implementation Needed to deal with N parameters

SHARCNET (Ontario Universities Computer Facilities for Parallel Implementation)

Perturbations of Inputs and Anisotropy of Detector important. (SHOULD BE CONSIDERED)

ABS Value of $I(l,k,B)$ vs k.

$B = 10$, $l = 270$



The Index Theorem of Bessel functions $J_\nu(x)$

when $\nu > x$, amplitude falls rapidly $\to 0$

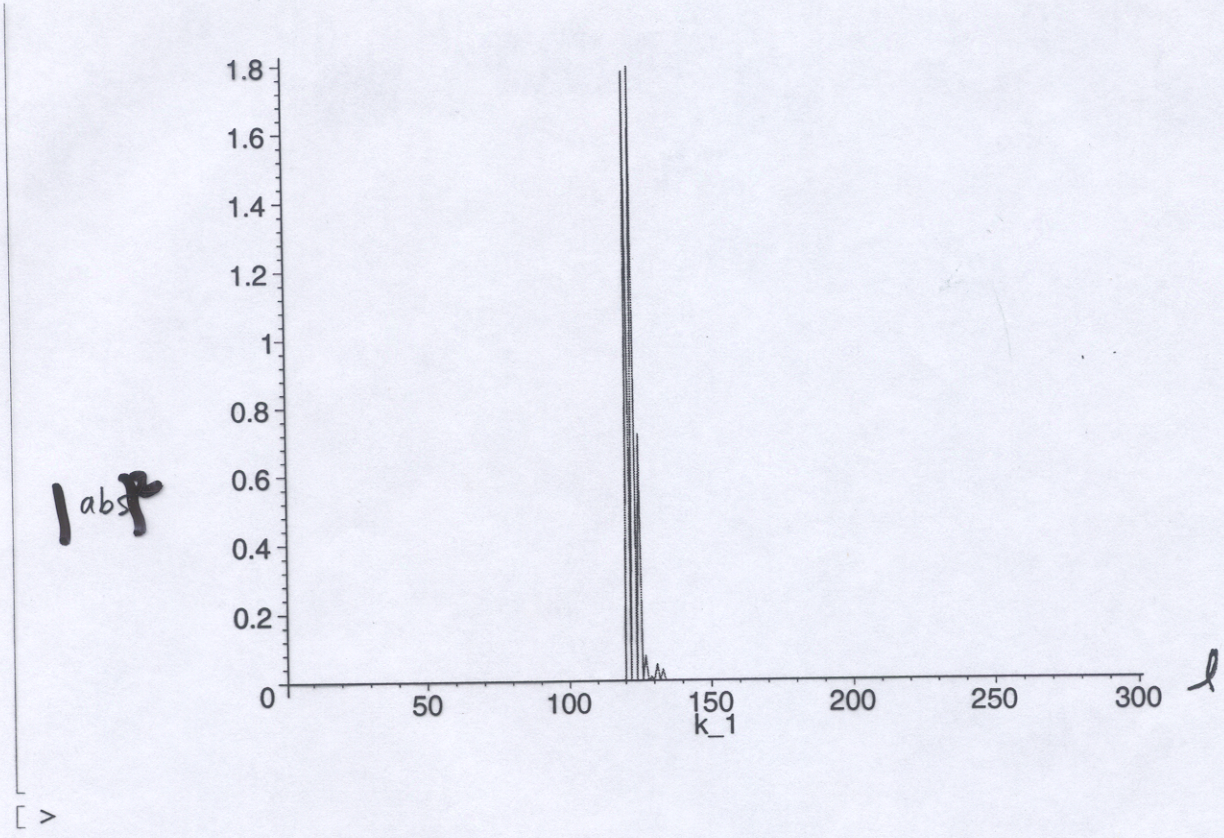
Might be of use in Hierarchical Searches

long data Stream - 2 or more stages.

$\nu = l + \frac{1}{2}$

$B_{orb} = 275$, $k = 150$

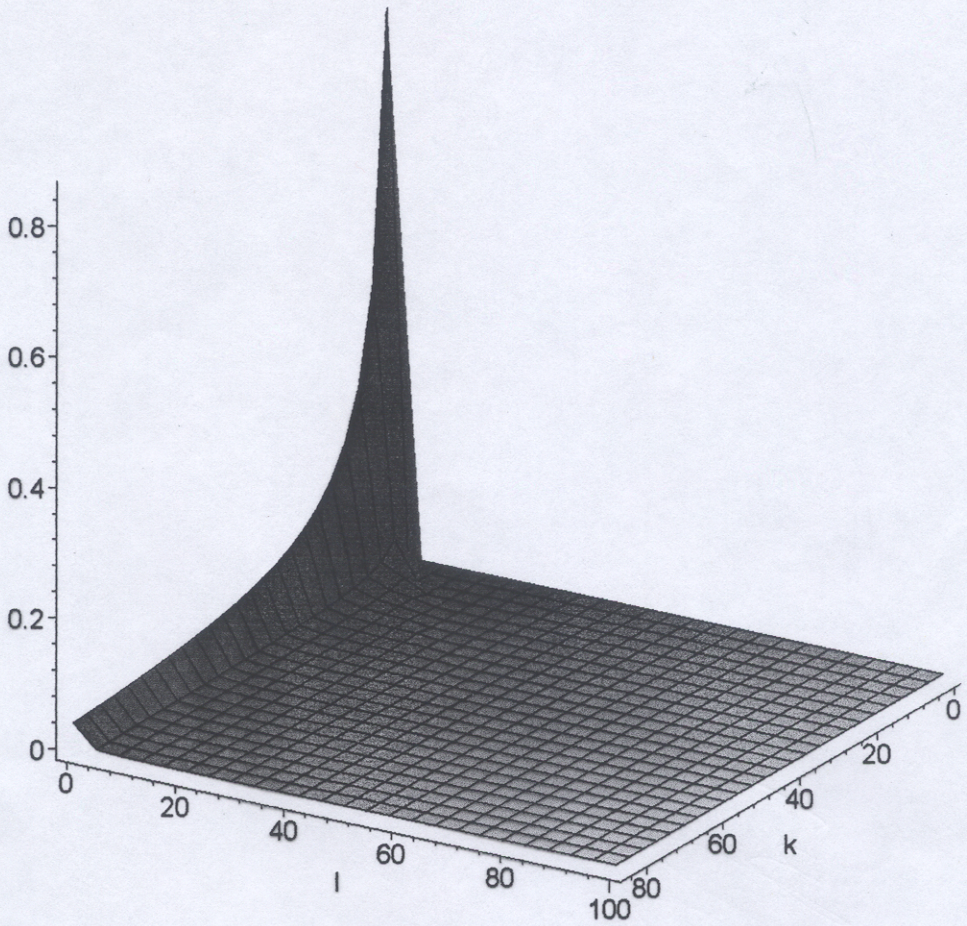
Orb + Ref. Motion



Peak close to $l \sim 120$ ($|l - B| \approx 150$ (= k)

```
> #Plot of Slm_squared versus l and k
```

$|S_{l,m}(k)|^2$



Vol 6 "Astronomical Papers for the use of the American Ephemeris and Nautical Almanac" 1898

Perturbations due to Jupiter on the Earth's Longitude over 1 arc Second:

$$7''.208 \cos(179\,\text{deg}\ 31'.9 + g' - g)$$
$$+ 2''.600 \cos(263\,\text{deg}.13' + g')$$
$$+ 2''.731 \cos(109\,\text{deg}.29'.6 - 2g' - g)$$

g' = mean anomaly of Jupiter

g = mean anomaly of Earth.

JPL and Naval observatory updated estimates but no drastic improvements in quantity important in principle - Relativistic measurement of time Considering the acceleration to which the clock is subject.

(Lunar Cutler - Analysis - Should be Studied in this AEI-Potsdam) Context, Barycenter Package.

6: Conclusions

1) Looking for Specific Sources of Continuous GW - has been tried

2) As yet no 'Search' of a large area of the sky attempted. Would increase the chances of discovering unpredicted GW Sources or invisible pulsars

3) GEO 600 - medium size detector - a fifth of LIGO and VIRGO

Can however look for Specific Sources of Continuous GW, i.e. known pulsars, before larger ones go on-line

4) Our present work - precursor to the all sky - all frequency Search of pulsars - Useful for Stepping around Sky (Schutz 1991)

Analytic Methods - reduce N_{patch}. Spindown parameters

Parallel Implementation on SHARCNET.

5) Unimagined Sources can exist. Exciting GW astronomy.

" There are more things in Heaven and Earth, Horatio, than are dreamt of in your philosophy! "

Ref: Vallisneri et al, Class. Quantum Grav. 19 (2002) 1327-1334

40 years have elapsed after the earlier pessimistic assessment of the detectability of GW. It is fitting to have a reminder of the role that CalTech played in realizing the correctness of General Relativity even in the 1930s. Prof Tolman was one of the earliest to invite Einstein to visit CalTech. Shown next are the houses of Tolman and Einstein in CalTech

These pictures were taken by me when Prof Kip Thorne invited me in 1996 to CalTech.

My thanks to him.

Prof R. C. TOLMAN HOUSE, CAL TECH

House where Einstein lived in 1930's, Cal Tech

707. S. OAKLAND.

PASADENA, CA.

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