Reducing Thermoelastic Noise by Reshaping the Light Beams and Test Masses

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MIT/LSC Meeting on Flat Topped Beams for Advanced LIGO

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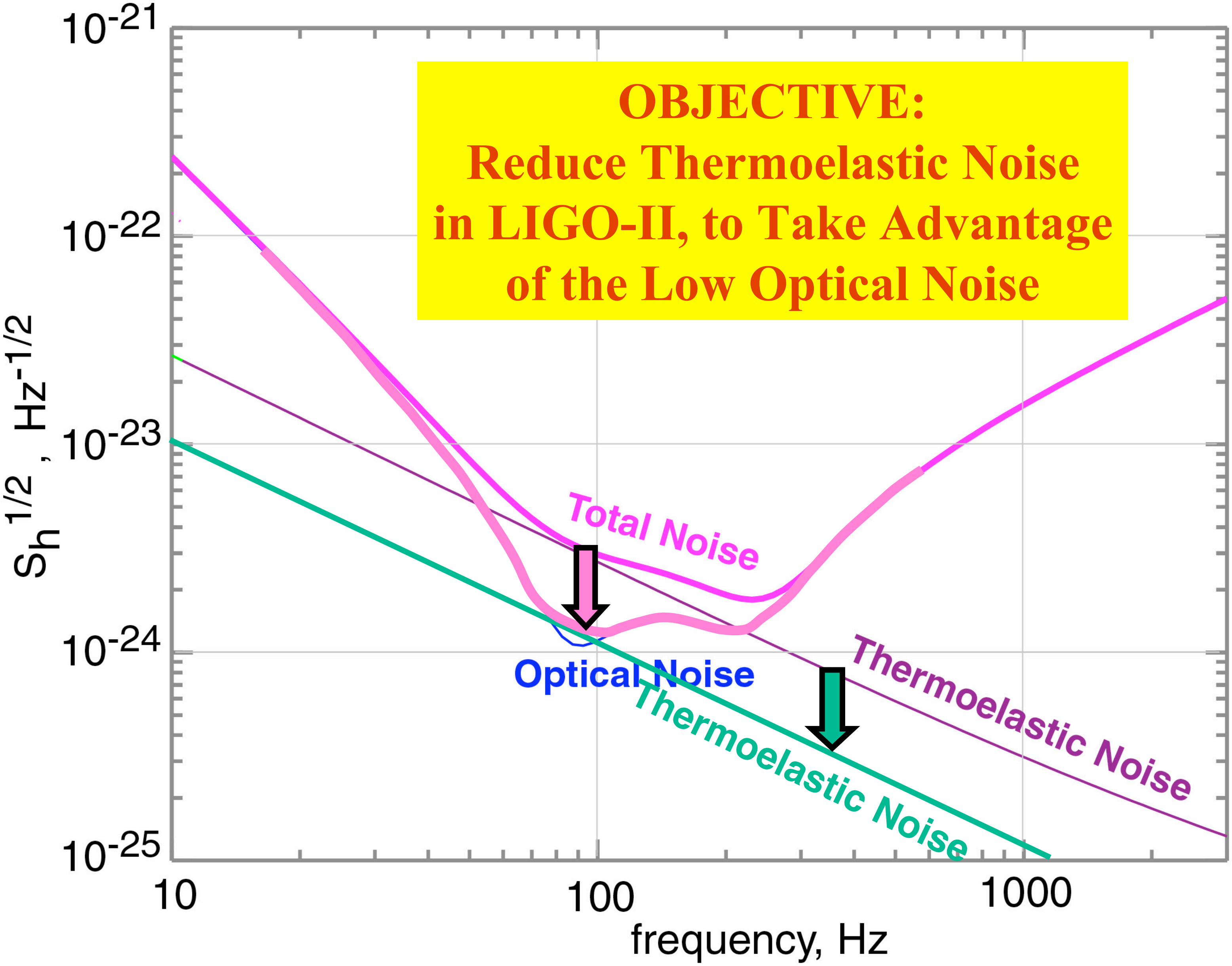
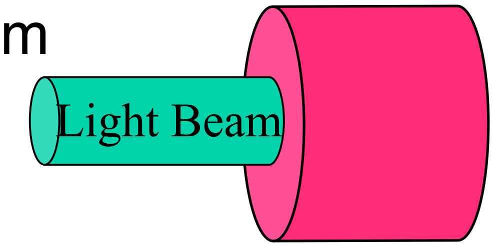
CONTEXT AND OVERVIEW

Sapphire Mirrors



KEY POINTS ABOUT THERMOELASTIC NOISE

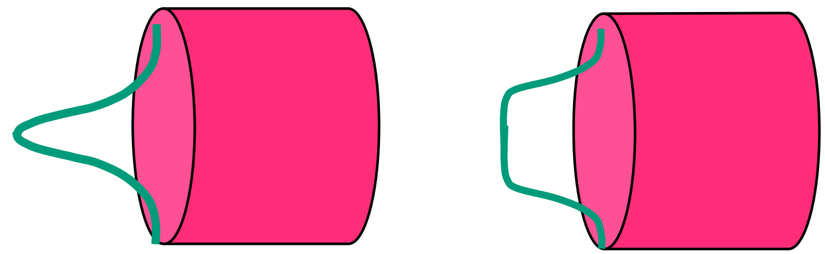
- **Physical Nature**
  - On timescale ~0.01 secs, random heat flow => hot and cold bumps of mean size ~0.5 mm
  - Hot bumps expand; cold contract
  - Light averages over bumps
  - Imperfect averaging => ***Thermoelastic noise***



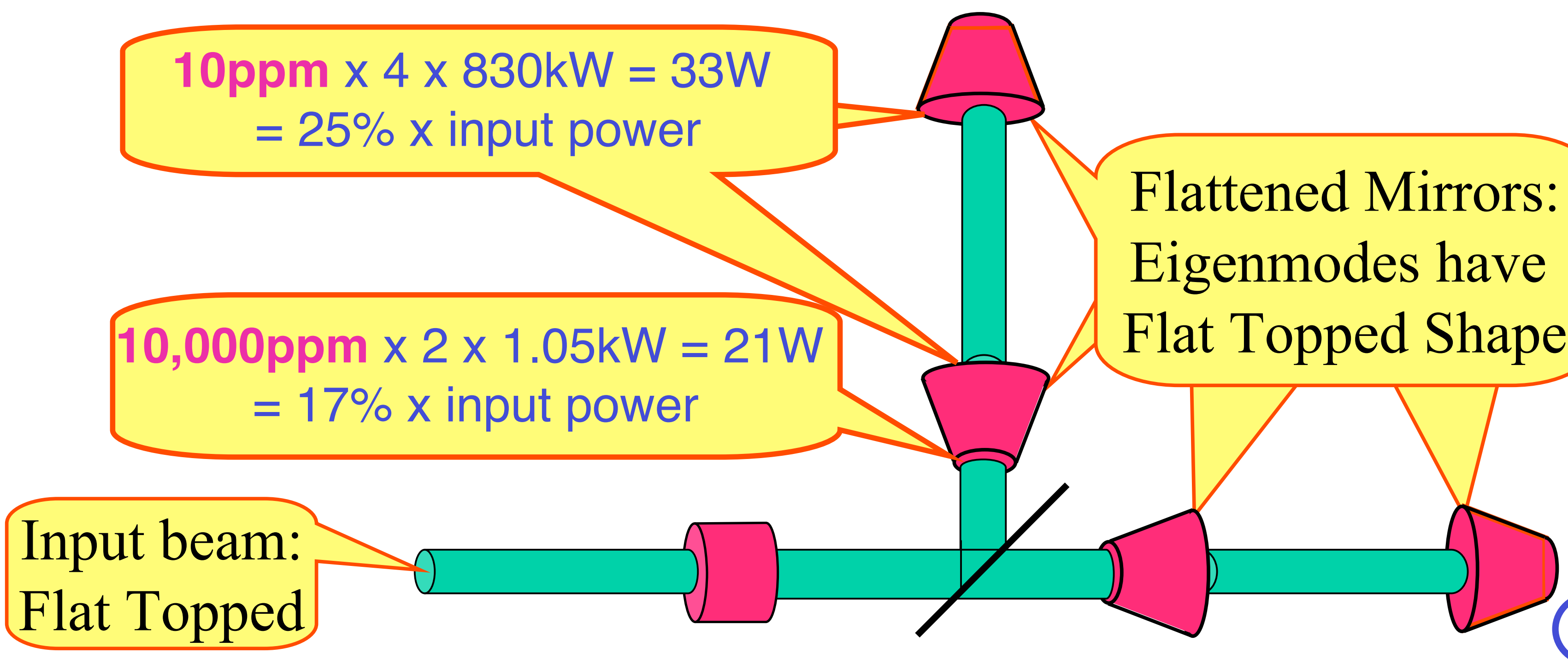
- **Computed via fluctuation-dissipation theorem**
  - Dissipation mechanism: heat flow down a temperature gradient => Computation highly reliable (by contrast with conventional thermal noise!)
  - This reliability gives us confidence in our proposal for reducing thermoelastic noise

Strategies to Reduce Thermoelastic Noise

- Gaussian beam averages over bumps much less effectively than a flat-topped beam.

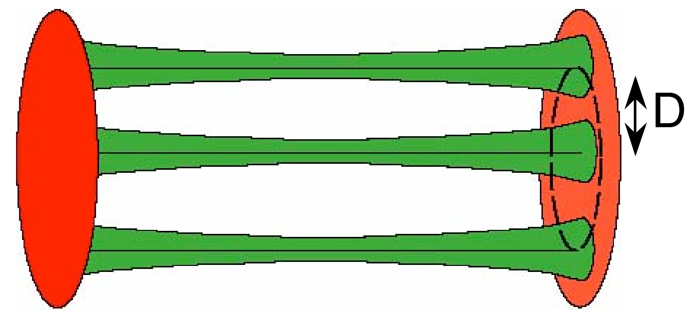


- The larger the beam, the better the averaging.
  - Size constrained by diffraction losses

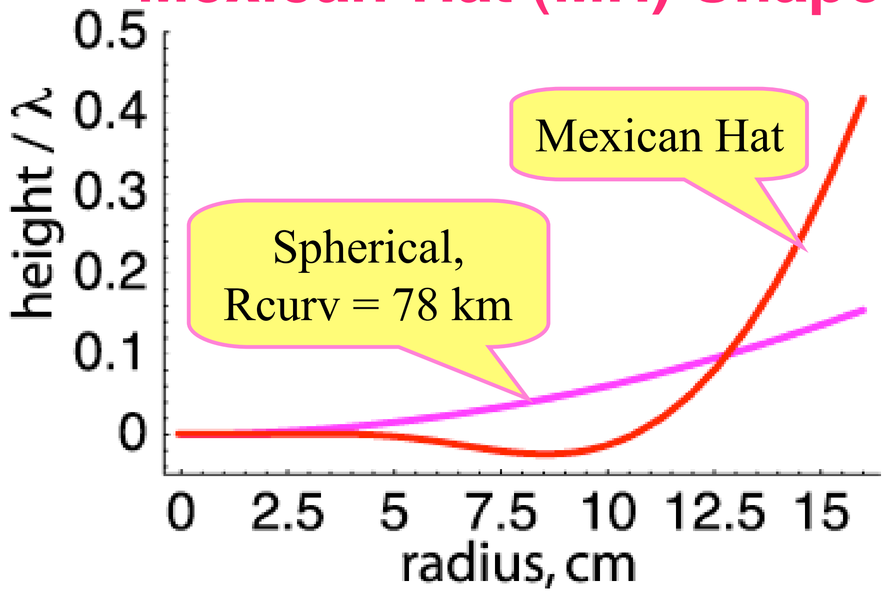


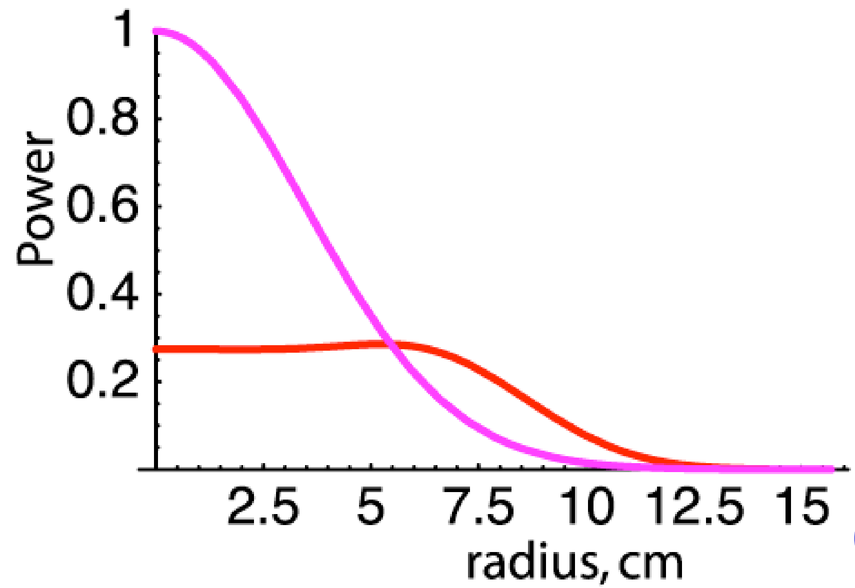
OUR FLATTENED MIRRORS & BEAMS

- **Compute desired beam shape:**
  - Superposition of minimal-spreading Gaussians -- axes uniformly distributed inside a circle of radius D
  - Choose D so diffraction losses are 10 ppm

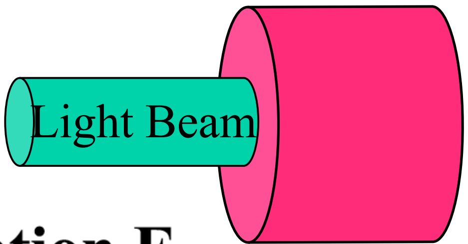


- **Compute shape of mirror to match phase fronts: Mexican-Hat (MH) Shape**





Computing noise: Fluctuation dissipation theorem



- **Thought experiment:**
  Static **pressure on mirror face**
  Shape is beam intensity profile, normalization F_o
  $\Rightarrow$

$$S_h = 4\left(\frac{k_b T \alpha E}{(1-2\nu)C_V \rho}\right)^2 \frac{1}{\omega^2} I \qquad I = \frac{1}{F_o^2}\int d^3 r \left|\nabla \theta\right|^2$$

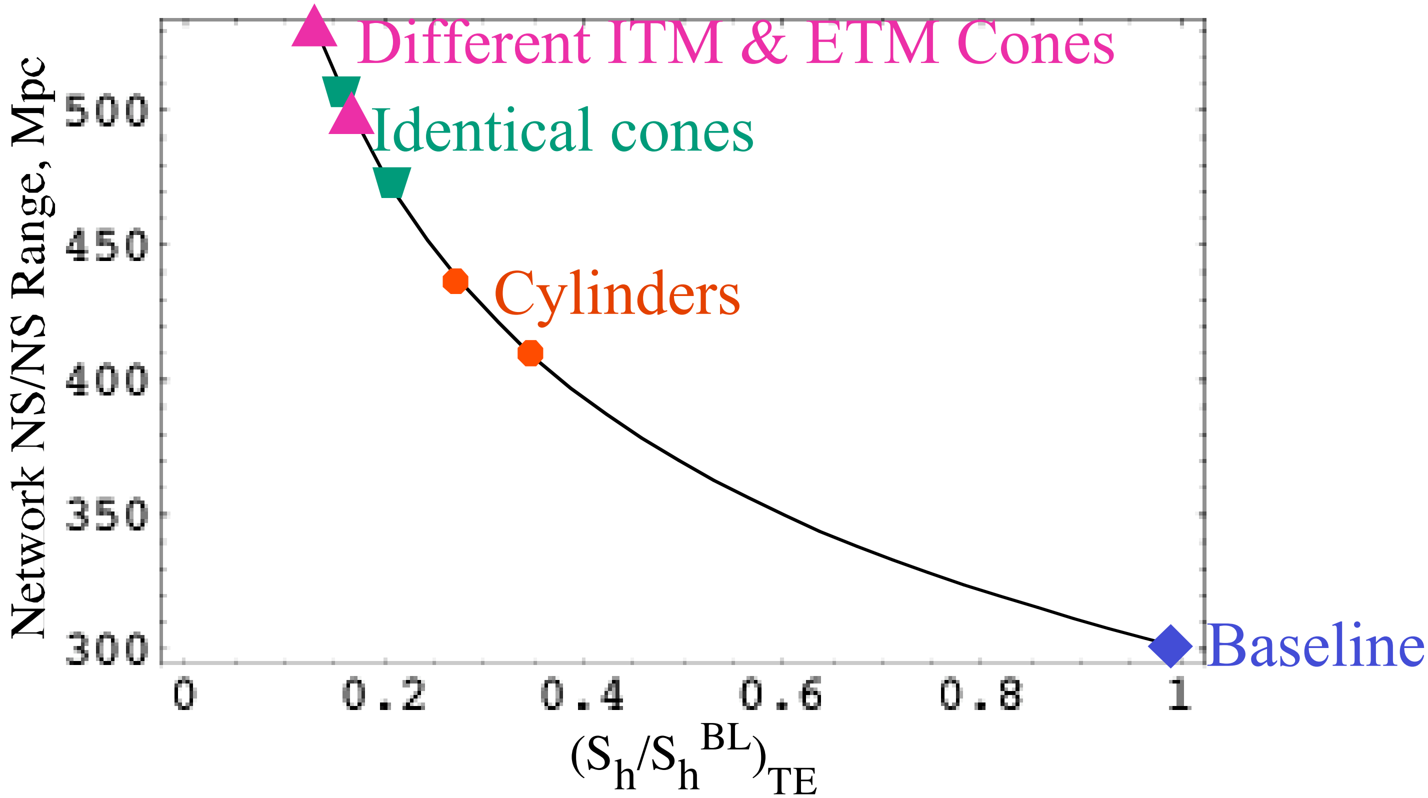
- ***I*** **contains information about beam, mirror shape and size**
- **Find *I* via standard elasticity code (finite-element)**

Independently computed by O'Shaughnessy, Strigin, Vyatchanin

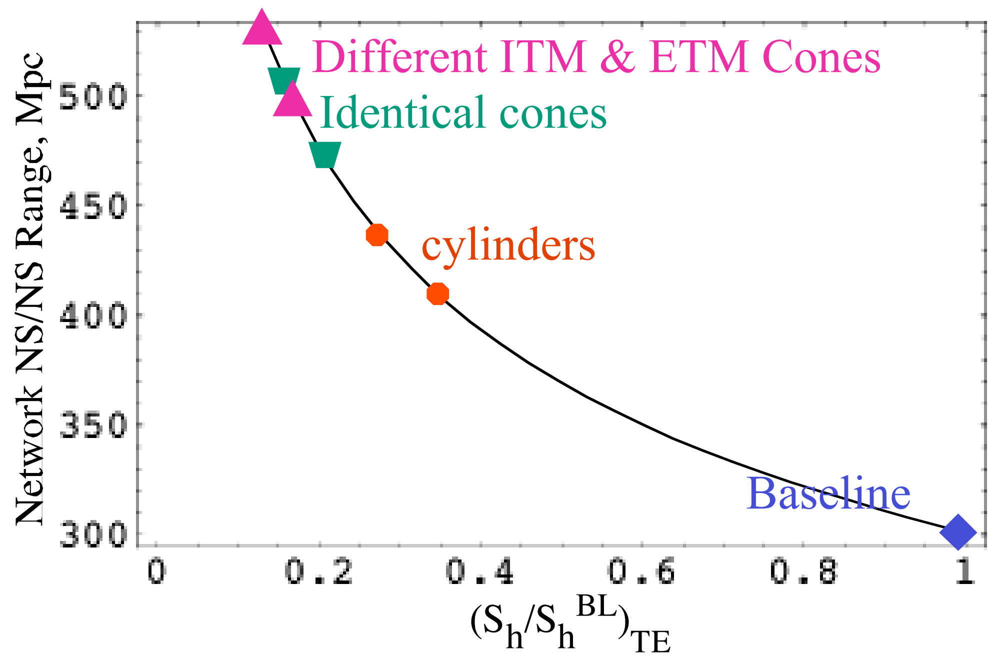
$$S_h/S_h^{BL} = I/I_{BL}$$

LIGO Network's NS/NS Range

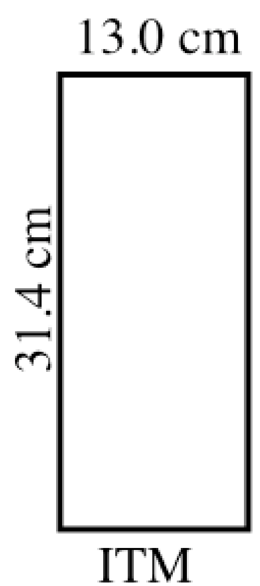
- Computed by Buonanno & Chen (private communication)
- Includes only thermoelastic noise and optical (unified quantum) noise --- assumes all others can be made negligible
- Optical parameters (SR mirror, homodyne phase) optimized for NS/NS range at each level of thermoelastic noise



Summary of Thermoelastic Predictions



Cylindrical Test Masses



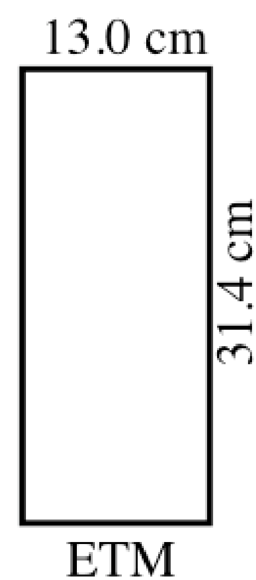
Baseline Test-Mass Shape

Coated to edge - 8mm:

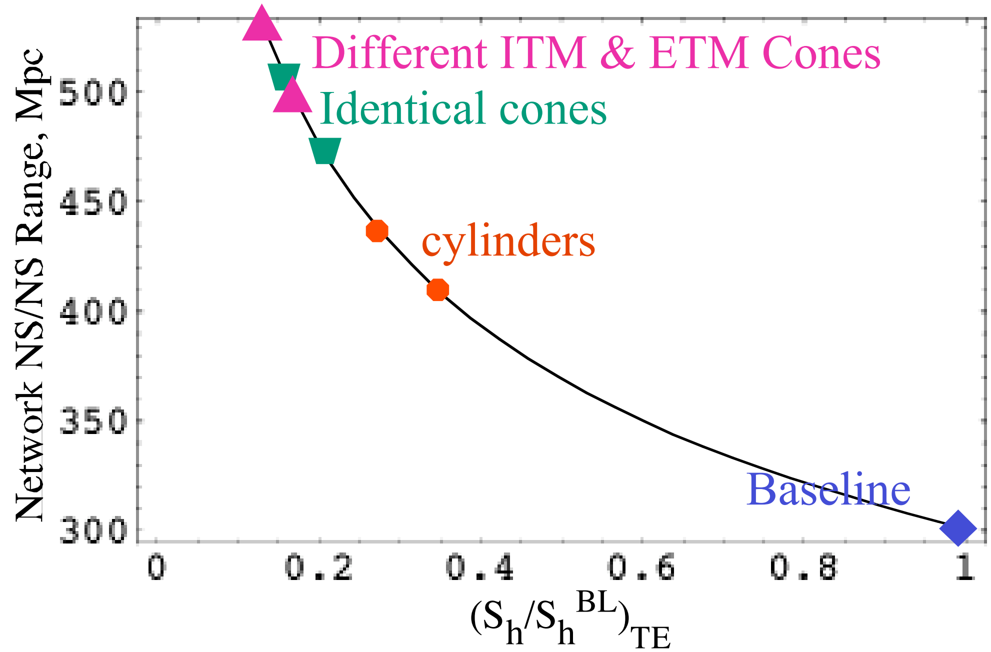
- Sh/ShBL = 0.364
- Range = 403 Mpc
- Rate/RateBL = 2.4

Coated to edge:

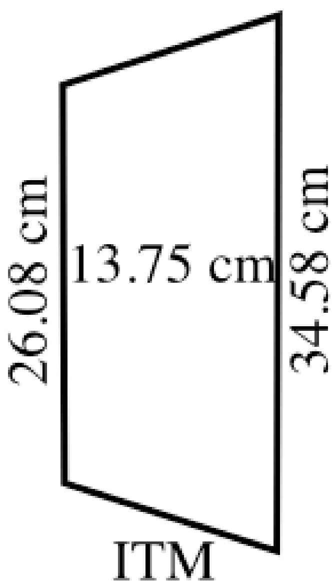
- Sh/ShBL = 0.290
- Range = 431 Mpc
- Rate/RateBL = 3.0



Summary of Thermoelastic Predictions



Identical Conical Test Masses



ETM & ITM Identical Cones

Coated to edge - 8mm:

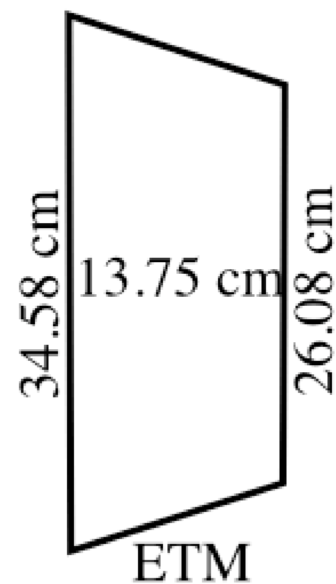
Sh/ShBL = 0.207

Range = 471 Mpc

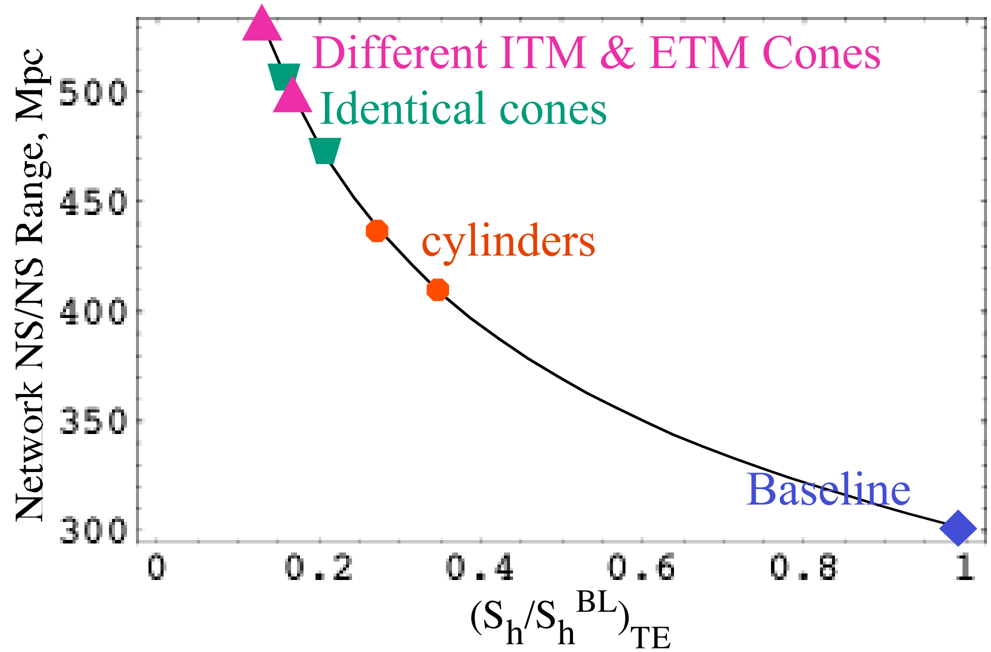
Coated to edge:

Sh/ShBL = 0.162

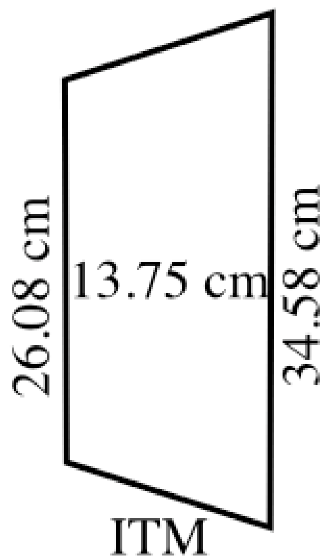
Range = 503 Mpc



Summary of Thermoelastic Predictions



Different Cones for ITM & ETM



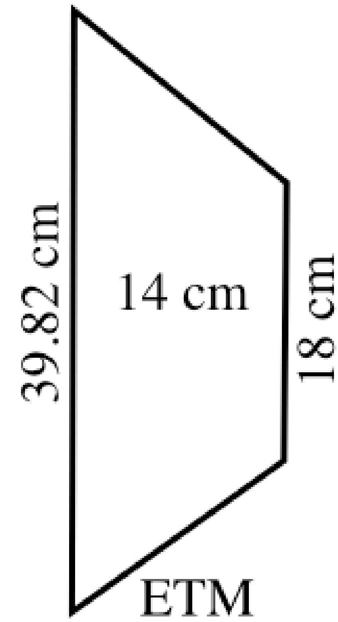
ETM & ITM Different Cones

Coated to edge - 8mm:

- Sh/ShBL = 0.170
- Range = 497 Mpc
- Rate/RateBL = 4.5

Coated to edge:

- Sh/ShBL = 0.135
- Range = 527 Mpc
- Rate/RateBL = 5.4

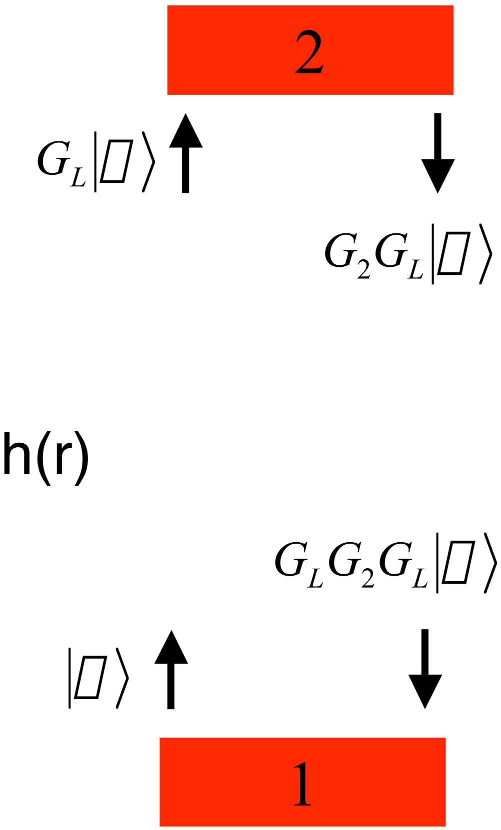


Practical Issues: Parasitic Modes & Tilt

- **Compare two configurations:**
  - ***Baseline*** Gaussian-Mode Interferometer
    - Mirror radius R= 15.7 cm
    - Gaussian beam radius ro = 4.23 cm
    - Diffraction losses (per bounce in arms) L0 = 1.9 ppm
  - ***Fiducial MH*** (Mexican-Hat) Interferometer
    - Mirror radius R = 16 cm
    - MH beam radius D = 10.4 cm
    - Diffraction losses (per bounce in arms) L0 = 18 ppm
  - [Conservative comparison]

- **Two sets of analysis tools:**
  - Arm-cavity integral ***eigenequation*** [-> orthornormal modes]
    - \+ 1st & 2nd order ***perturbation theory*** [-> mode mixing]
    - **O'Shaugnessy**
  - FFT simulation code [adapted from LIGO E-2-E model]
    - **D'Ambrosio**

Propagation and Eigenequation



Same tools as used with spherical mirrors:

- **Propagation Operators (=unitary!)**
  - ***Free propagator*** : If cavity length L,

$$G_L(r,r') = -i\frac{k}{2\pi L}\exp i\left[\frac{(r-r')^2}{2L/k} + kL\right]$$

  - ***Reflection off mirror***: If mirror heights are h(r)

$$G_{1,2}(r,r') = \delta(r-r')\exp\left[-2ikh_{1,2}(r)\right]$$

- **Eigenequation**
  - Beam returns after one round trip with similar shape if $\gamma|\psi\rangle = G_1 G_L G_2 G_L|\psi\rangle$
  - Eigensolutions $\{\gamma_k, |\psi_k\rangle\}$ complete and orthonormal, with $|\gamma_k| = 1$

Eigenproblem Method

- **Numerical Method**
  - Discretize the integral operators $\{G_L, G_{1,2}\}$ on finite circle $[0, R_{max}]$
  - Multiply matricies to form discrete round-trip operator $G_{net} = G_1 G_L G_2 G_L$
  - Diagonalize round-trip operator G_{net}
- **Output**
  - Discrete representation of $\{\gamma_k, |\psi_k\rangle\}$, appropriate to finite mirror of radius R_{max}. If mirror large, ~ infinite-mirror wavefunction
  - Losses:
    - Exact roundtrip losses: $L_{net} = 1 - |\gamma|^2$
    - Clipping losses:
      - Mirror 1: $L_1 = 1 - \int_{r<R_1} |\psi|^2 \, dA$
      - Mirror 2: if $\phi = G_L \psi$, then $L_2 = 1 - \int_{r<R_2} |\phi|^2 \, dA$

Eigenproblem Perturbation Theory

- **Perturbation theory**

  Conventional perturbation theory expansion: if $G_2' = G_2 + \square G_2$

$$\left|\square\right\rangle = \left|\square_o\right\rangle + \square_{k \neq 0} \left|k\right\rangle \frac{\left\langle k \middle| G_1 G_L \square G_2 G_L \middle| \square_o \right\rangle}{\square_o \,\square\, \square_k} + O(\square G_2{}^2)$$

etc

Both state and phase vary smoothly with perturbation parameter

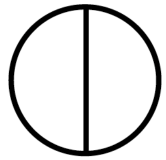
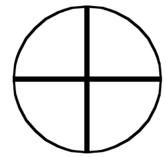
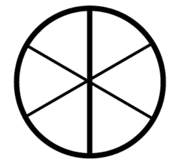
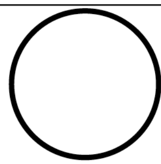
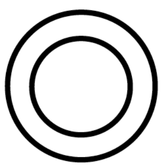
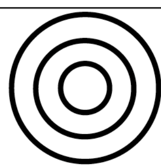
Parasitic-Mode Frequencies

FSR

- **Baseline Gaussian:** $\square\square = \square\square\square_{fund}$ = (integer) x 0.0614 x $\square c/L$
- **Fiducial MH:** ***[from cavity eigenequation, solved numerically]***

Azimuthal Nodes

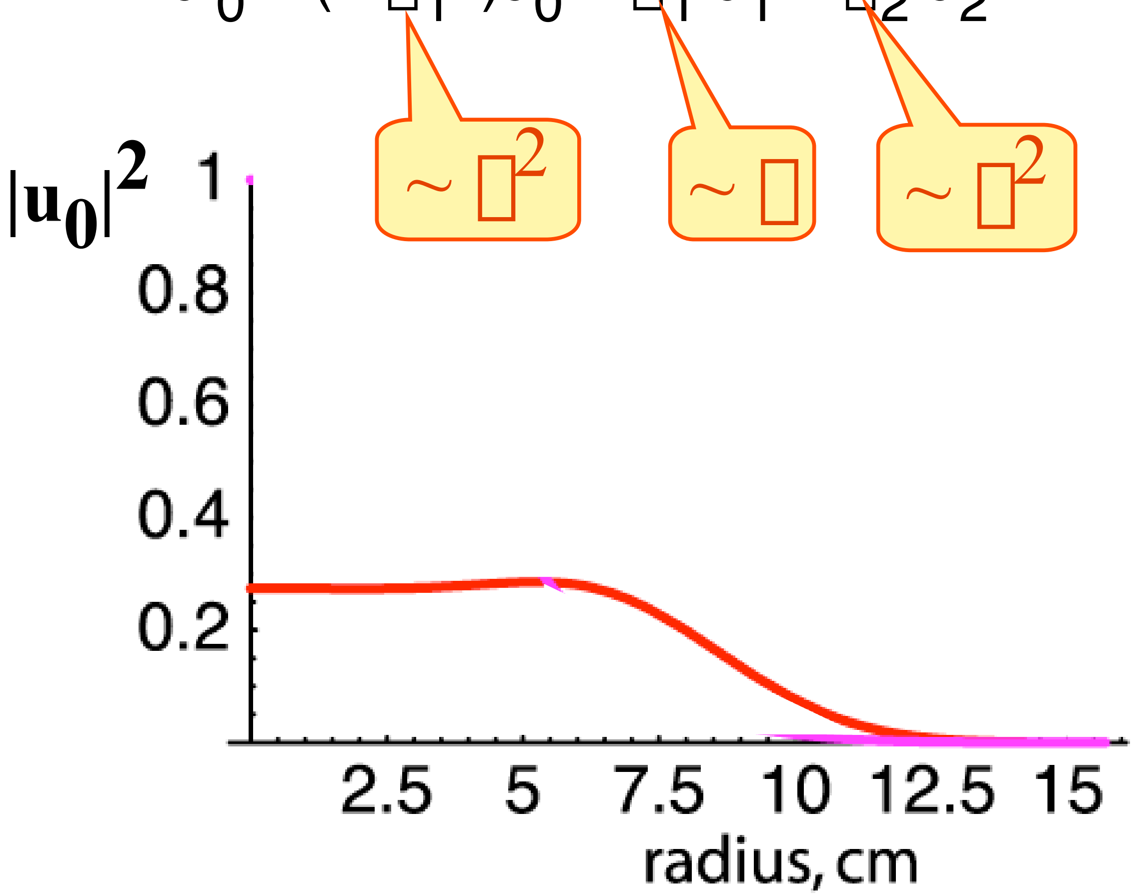
Radial Nodes

$\dfrac{\Delta\omega}{\pi c / L}$				
	0	**0.0404**	**0.1068**	**0.1943**
	**0.1614**	**0.2816**	**0.4077**	**-0.4581**
	**0.4303**	**-0.4140**	**-0.2570 (X)**	**-0.0812 (X)**
	**-0.2330 (X)**	**-0.0488 (X)**	**0.1406 (X)**	**(X)**

X → indicates diffraction losses per bounce > 1%

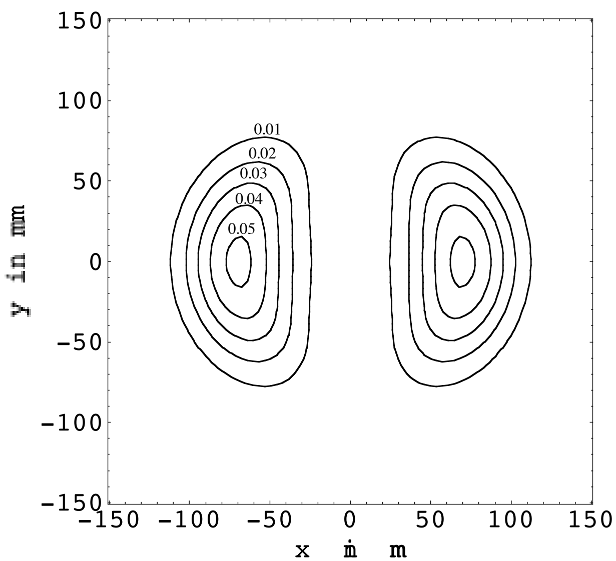
Tilt-Induced Mode Mixing

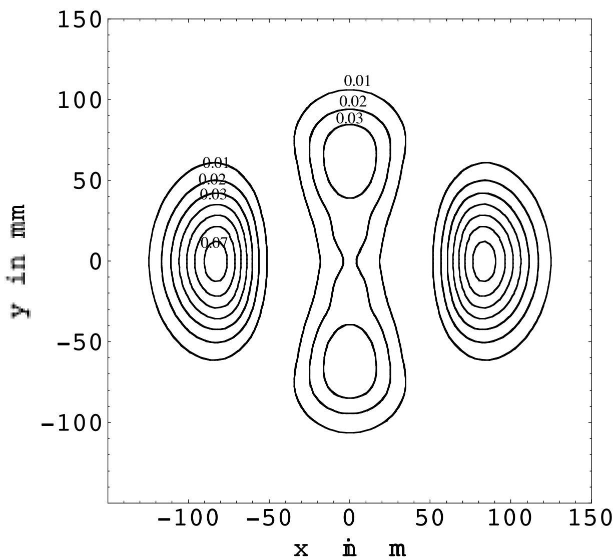
- Tilt arm-cavity ETM through an angle □
- Mode mixing:
  - $u'_0 = (1-□_1{}^2)u_0 + □_1\, u_1 + □_2\, u_2$



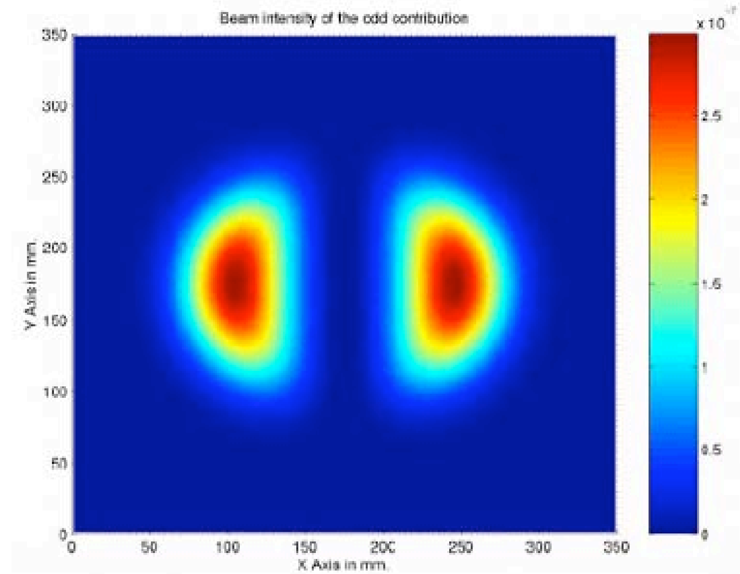
The Admixed Parasitic Modes

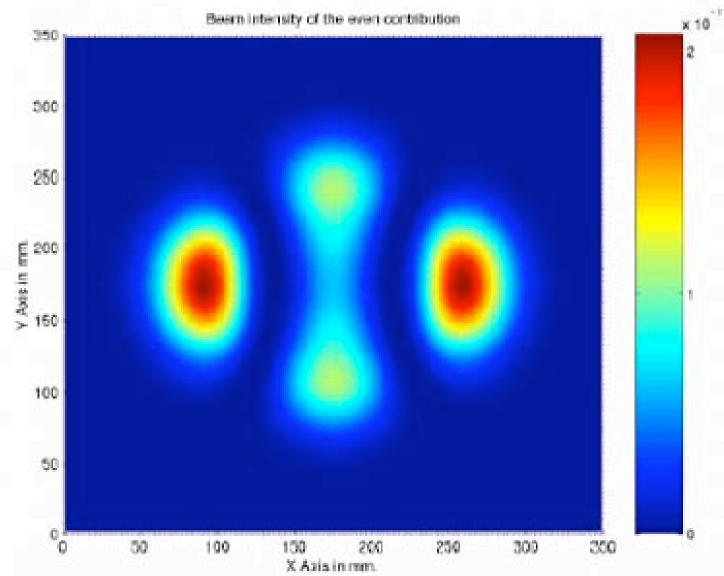
- From Eigeneqn + Pert'n Theory





- From FFT Simulations





Tilt-Induced Mode Mixing

- Tilt arm-cavity ETM through an angle □
  - $\Box_8 = \Box/10^{-8}$ rad
- Mode mixing:
  - $u'_0 = (1-\Box_1{}^2)u_0 + \Box_1\, u_1 + \Box_2\, u_2$

Fiducial MH Cavity

- **From Eigeneqn + Pert'n Theory**
- $\Box_1 = 0.02272\ \Box_8$
- $\Box_2 = 0.00016\ \Box_8{}^2$

Few/1000

~10%

- **From FFT Simulations**
- $\Box_1 = 0.0227\ \Box_8$
- $\Box_2 = 0.00018\ \Box_8{}^2$

Baseline Gaussian-Beam Cavity

- $\Box_1 = 0.00469\ \Box_8$

MH Cavity has same $\Box_1$ as Baseline Gaussian if tilt is controlled 5 times better

Influence of Tilt on Cavity Performance

- **Arm-Cavity Diffraction Losses**
  - Eigeneqn + Pert'n Theory: $L'_0 = (18. + 0.043\ \square_8{}^2)$ ppm
  - So small it has not been measured reliably in FFT simulations
- **Arm-Cavity Gain**
  - Eigeneqn + Pert'n Theory: $737\ (1 - 0.00055\ \square_8{}^2)$
  - FFT Simulations: $740\ (1 - 0.00059\ \square_8{}^2)$

5/1000

10 per cent

Influence of Tilt on Interferometer's Dark-Port Output Power

- Fraction of input power in dark-port dipolar parasitic mode u_1
  - Eigeneqn + Pert'n Theory: P1 = 478 $\square_8^2$ ppm
  - FFT Simulations: P1 = 482 $\square_8^2$ ppm

1 %

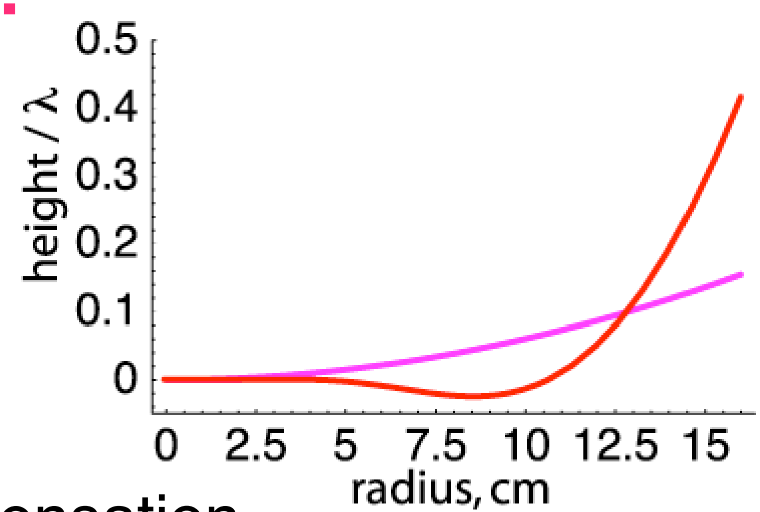
- Fraction of input power in fundamental mode u_0, and second-order parasitic mode (monopolar + quadrupolar) u_2
  - Eigeneqn + Pert'n Theory: P0 = 0.256 $\square_8^4$ ppm
  - P2 = 0.024 $\square_8^4$ ppm
  - (P0 + P2) = 0.28 $\square_8^4$ ppm
  - FFT Simulations: (P0 + P2) = 0.31 $\square_8^4$ ppm

10 %

- **Baseline Gaussian:** P1 = 22 $\square_8^2$ ppm;
  - P0 = 0.00048 $\square_8^4$ ppm
- ***MH is same as Gaussian if tilt is controlled 5 times better.***

SOME OTHER ISSUES THAT NEED STUDY

- **Theoretical Modeling issues:**
  - **Tolerances on mirror shapes**
    - Absolute tolerances
    - Tolerances in relative differences between mirrors
    - Thermal lensing and its compensation
  - **Possible dynamical instabilities**
    - e.g., rocking motion due to positive rigidity combined with time delay in response



- **Laboratory prototyping**