Analysis methods and template family for an efficient search for precessing binaries

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ASIS : Analysis Methods & Astrophysics/Source

P. Grandclément, V. Kalogera and A. Vecchio, Phys. Rev. D, **67**, 042003 (2003).
P. Grandclément and V. Kalogera, Phys. Rev. D, in press, gr-qc/0211075.

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ACTUAL SEARCHES

Matched-filtering : correlate output of the detectors with a family of templates.

Use only the non-precessing templates :

$$\tilde{h}(f) = \mathcal{A} f^{-7/6} \exp\left[i\phi(f)\right]$$

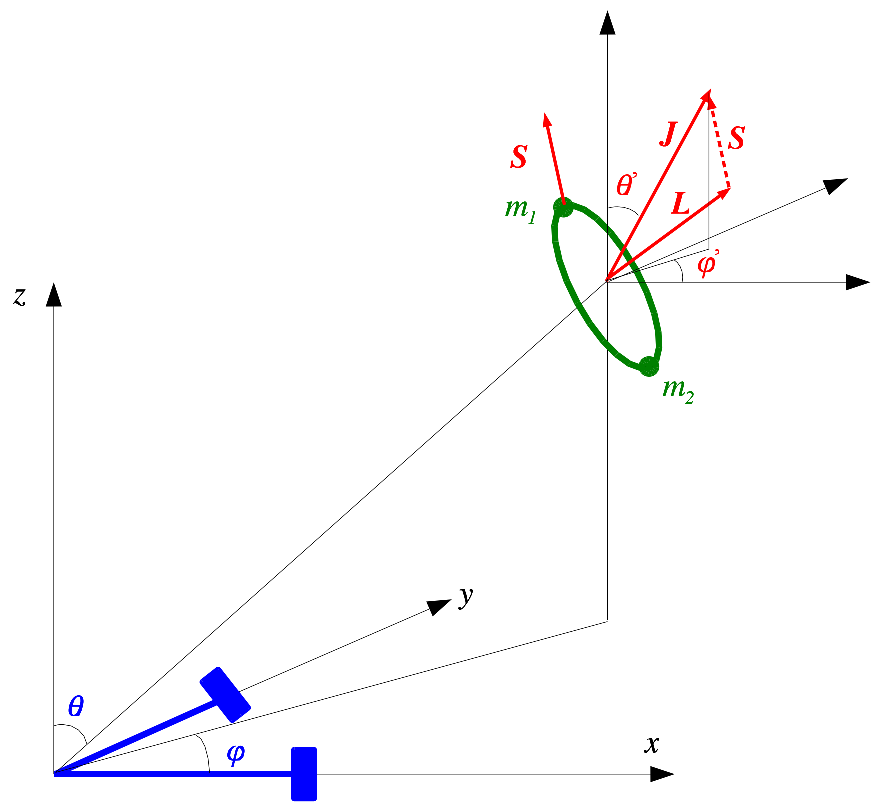
$$\phi(f) = \phi_{\rm const} + 2\pi f t_c + \frac{3}{128}(\pi\mathcal{M}f)^{-5/3}\left[1 + \frac{20}{9}\left(\frac{743}{336} + \frac{11\mu}{4M}\right)(\pi M f)^{2/3} - 16\pi(\pi M f) + 10\left(\frac{3058673}{1016064} + \frac{5429\mu}{1008M} + \frac{617\mu^2}{144M^2}\right)(\pi M f)^{4/3}\right]$$

Family with 2 effective parameters

- chirp mass : $\mathcal{M} = \left[(m_1 m_2)^3 / M\right]^{1/5}$
- total mass : $M = m_1 + m_2$

PRECESSION

- Spinning objects $\vec{J} = \vec{L} + \vec{S}$.
- If $\vec{S} \wedge \vec{L} \neq 0$ the system precesses due to *spin-orbit* and *spin-spin* couplings
- Direction of $\vec{J}$ remains almost constant.
- The misalignment angle given by $\kappa = \hat{\mathbf{S}} \cdot \hat{\mathbf{L}}$ also remains almost constant.
- Several cycles inside the LIGO-band (typically 10).
- Causes modulation of both the amplitude and phase of the signal.



PARAMETERS EVERYWHERE !

Even if the exact waveform was known, it depends on a lot of various parameters :

- the two masses m_1 and m_2.
- magnitude of the spins S_1 and S_2.
- initial orientations of the spins (4 angles).
- initial orientation of $\mathbf{L}$ (2 angles).
- position of the binary (2 angles).

Even if some parameters are not a concern (analytical maximization...), the remaining number is to high :

Precessing waveforms not practical as templates.

FITTING FACTOR

The FF measures the loss of SNR due to a mismatch between a signal W and a family of templates $T_{\vec{\lambda}}$ by :

$$\left(\frac{S}{N}\right) = \mathrm{FF} \times \left(\frac{S}{N}\right)_{\max}$$

By definition $0 \leq \mathrm{FF} \leq 1$ and we have

$$\mathrm{FF} = \max_{\vec{\lambda}} \left[\frac{(T_{\vec{\lambda}}|W)}{\sqrt{(T_{\vec{\lambda}}|T_{\vec{\lambda}})\,(W|W)}} \right]$$

The inner product of two waveforms is given by the optimum filter :

$$(h_1|h_2) = 2\int_0^{\infty} \frac{\tilde{h}_1^*(f)\,\tilde{h}_2(f) + \tilde{h}_1(f)\,\tilde{h}_2^*(f)}{S(f)} df$$

The detection rate decreases as FF^3.

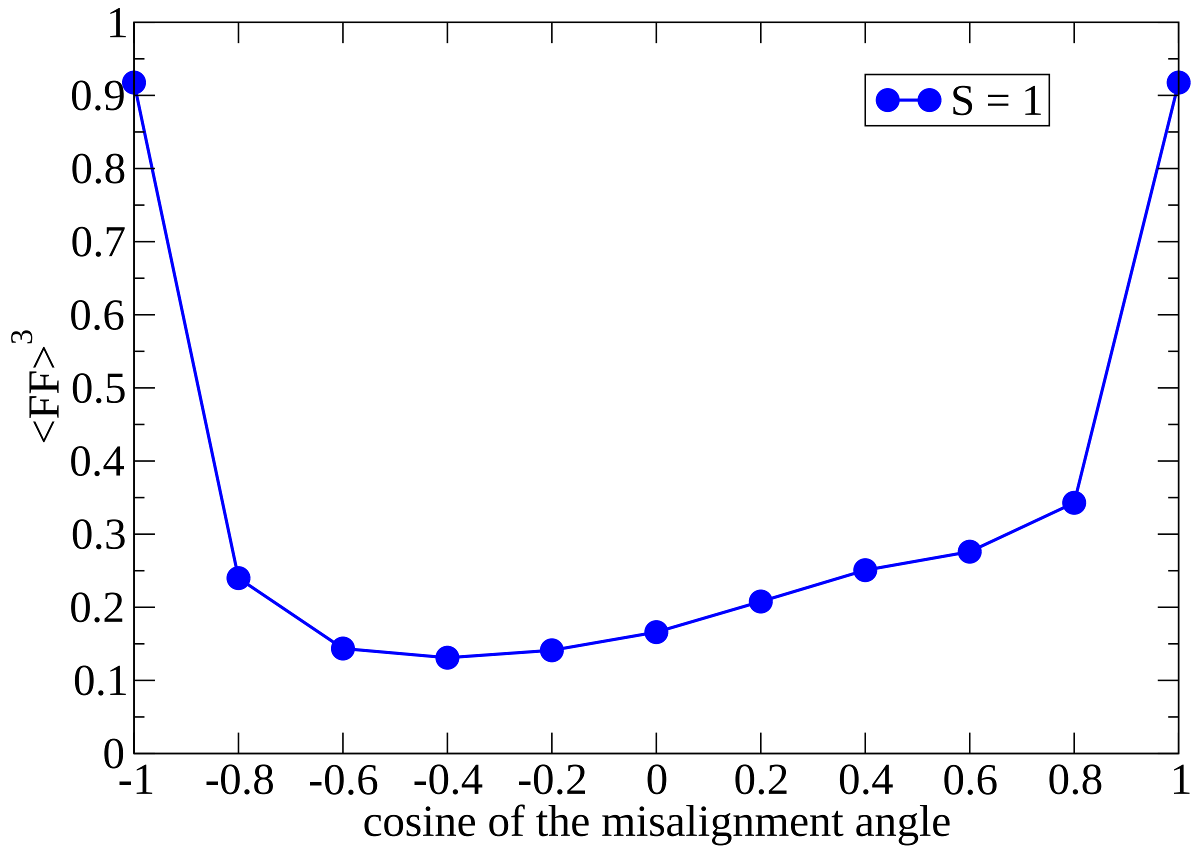
CAN WE DETECT PRECESSING BINARIES WITH NON-PRECESSING TEMPLATES ?

Technique :

- generate a ”true”, precessing signal (using the simple precession regime).
- calculate the FF using the non-precessing templates.
- average over the random orientation angles.

DECREASE OF THE DETECTION RATE

m_1=10 and m_2=1.4 solar masses



“MIMIC” TEMPLATES

ASSUMPTION : main effect of precession is to add an oscillatory term to the phase (the amplitude modulation is not crucial).

Apostolatos proposed to mimic the precession effect. Addition of the following phase :

$$\phi = C \cos\left(B f^{-2/3} + \delta\right)$$

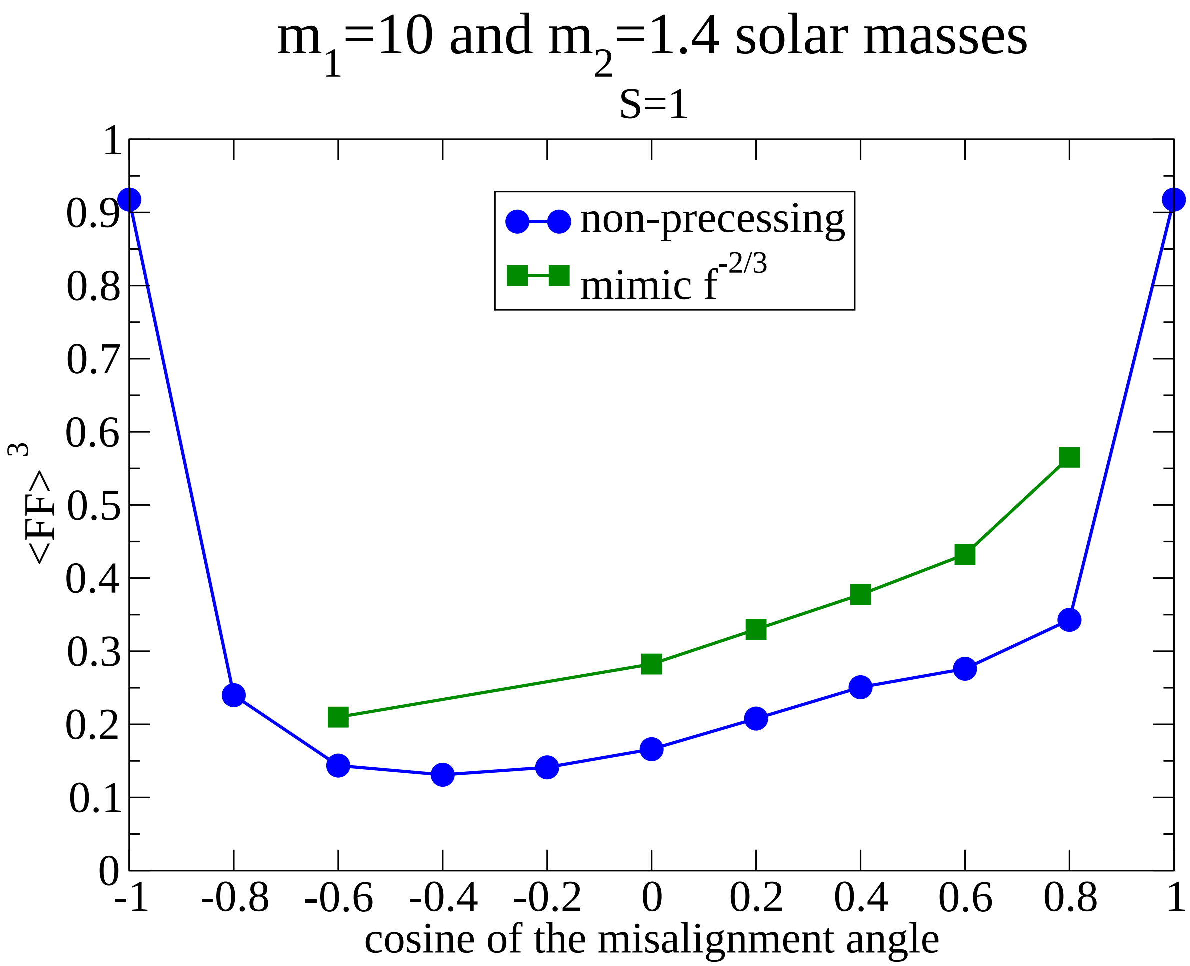
3 additional parameters C, B and δ.

Maximization over 5 parameters difficult...

Solution :

- First use the non-precessing templates.
- Maximization over the last three parameters using the “mimic”-family.

MODERATE IMPROVEMENT



RESIDUAL PHASE

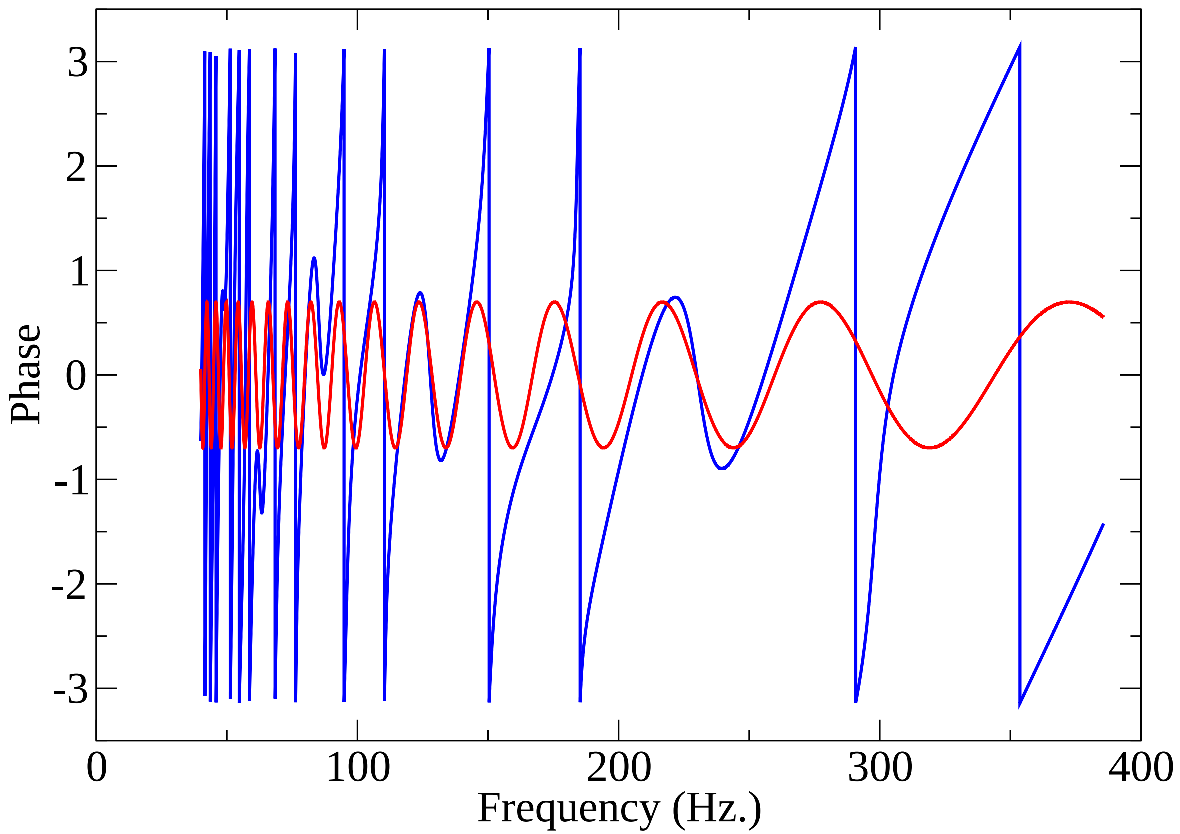
Goal : put the templates in phase with the signal.

Let ϕ^S be the phase of the signal and $\phi^T_{\rm PN}$ the phase of the non-precessing template corresponding to FF.

FF $< 1 \Longrightarrow$ the residual phase : $\Delta\phi = \phi^S - \phi^T_{\rm PN} \neq 0$

It is the **residual phase** that must be fit by any “mimic” phase.

TYPICAL RESIDUAL PHASE



SPIKES IN THE RESIDUAL PHASE

Presence of several jumps in the phase. We want to mimic them by a function having the following properties :

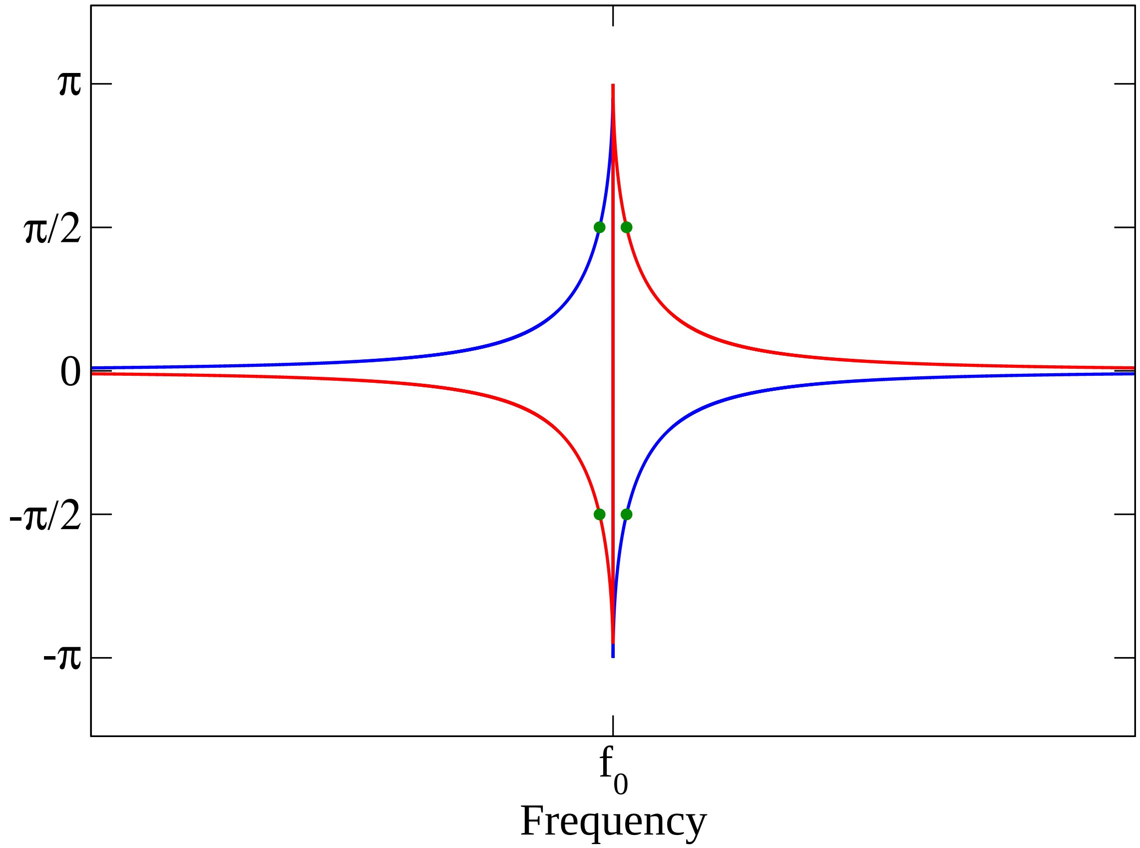
- is π and $-\pi$ at each side of the jump.
- has an infinite derivative at the jump.
- has a variable width.

$$\text{If} \quad f > f_0 \quad \text{then} \quad P\left(f_0, \sigma, \varepsilon\right) = \varepsilon\pi\left[\sqrt{\left(1 - \frac{1}{\left(\sigma\left(f - f_0\right) + 1\right)^2}\right)} - 1\right]$$

$$\text{If} \quad f < f_0 \quad \text{then} \quad P\left(f_0, \sigma, \varepsilon\right) = \varepsilon\pi\left[-\sqrt{\left(1 - \frac{1}{\left(\sigma\left(f - f_0\right) - 1\right)^2}\right)} + 1\right].$$

$\varepsilon = \pm 1$ is the orientation, σ gives the width and f_0 the position.

ONE SPIKE (2+1 parameters)



SEVERAL SPIKES

The spikes being localized, the observed behavior is well described by a sum of spikes, with various parameters : $\sum_{i=1}^{N} P\left(f_0\left(i\right), \sigma\left(i\right), \varepsilon\left(i\right)\right)$

How to choose to sequences of parameters ?

- Functions (like $f_0\left(i\right) = Ap^i$). Does not work because the location of the spikes must be very precise.
- Because of localization, the spikes are almost independent. **Hierarchical search** : spikes are found one by one, until the FF converges.

PROCEDURE

Sometimes oscillations are still present in the residual phase so we keep Apostolatos' correction, just before the spikes.

- **STEP I** : PN template

$$\phi_{\mathrm{Newt}} = 2\pi t_{\mathrm{max}} f + \frac{3}{128}\left(\pi \mathcal{M}_{\mathrm{max}}\right)^{-5/3} f^{-5/3} + \phi_{\mathrm{const}}$$

- **STEP II** : Apostolatos' correction

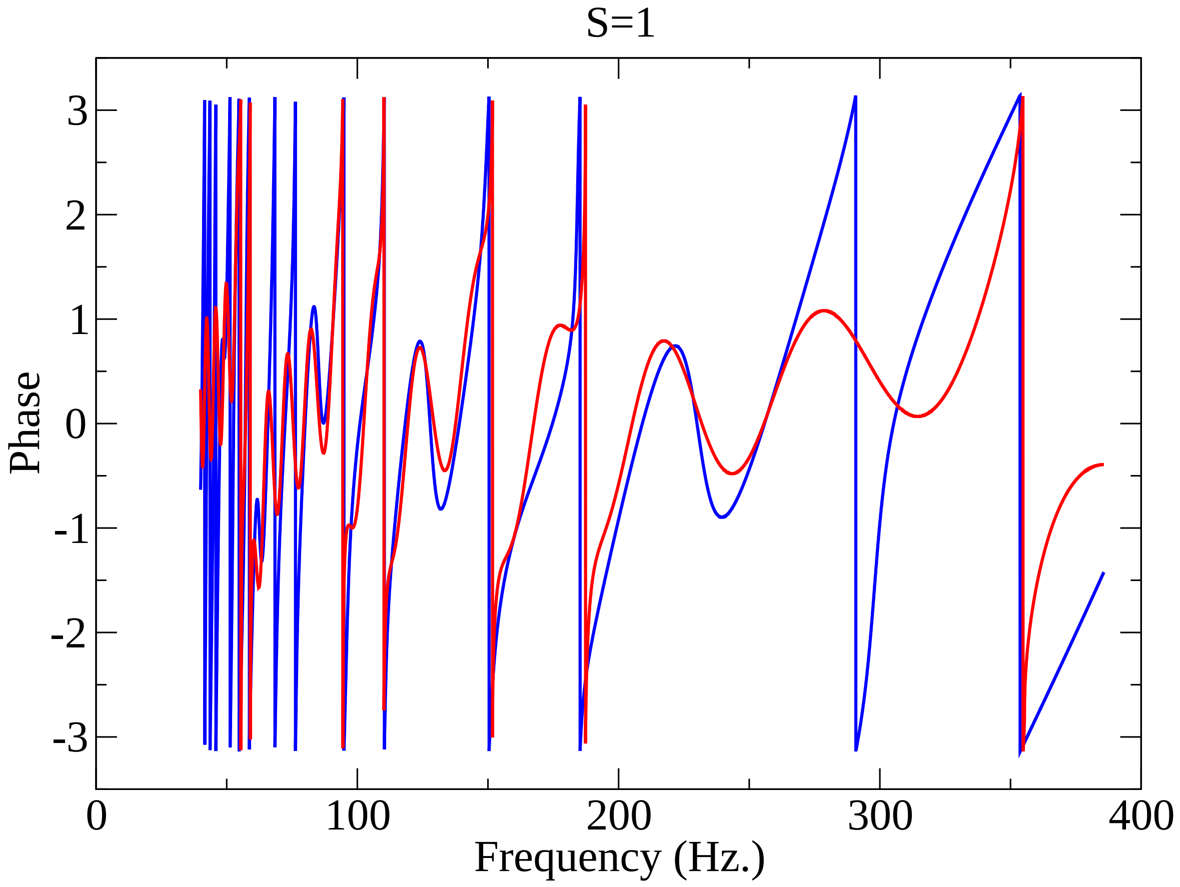
$$\phi_0 = 2\pi t_{\mathrm{max}} f + \frac{3}{128}\left(\pi \mathcal{M}_{\mathrm{max}}\right)^{-5/3} f^{-5/3} + \phi_{\mathrm{const}} + \mathcal{C}_{\mathrm{max}} \cos\left(\mathcal{B}_{\mathrm{max}} f^{-2/3} + \delta_{\mathrm{max}}\right)$$

- **STEP III** : Hierarchical search for the spikes

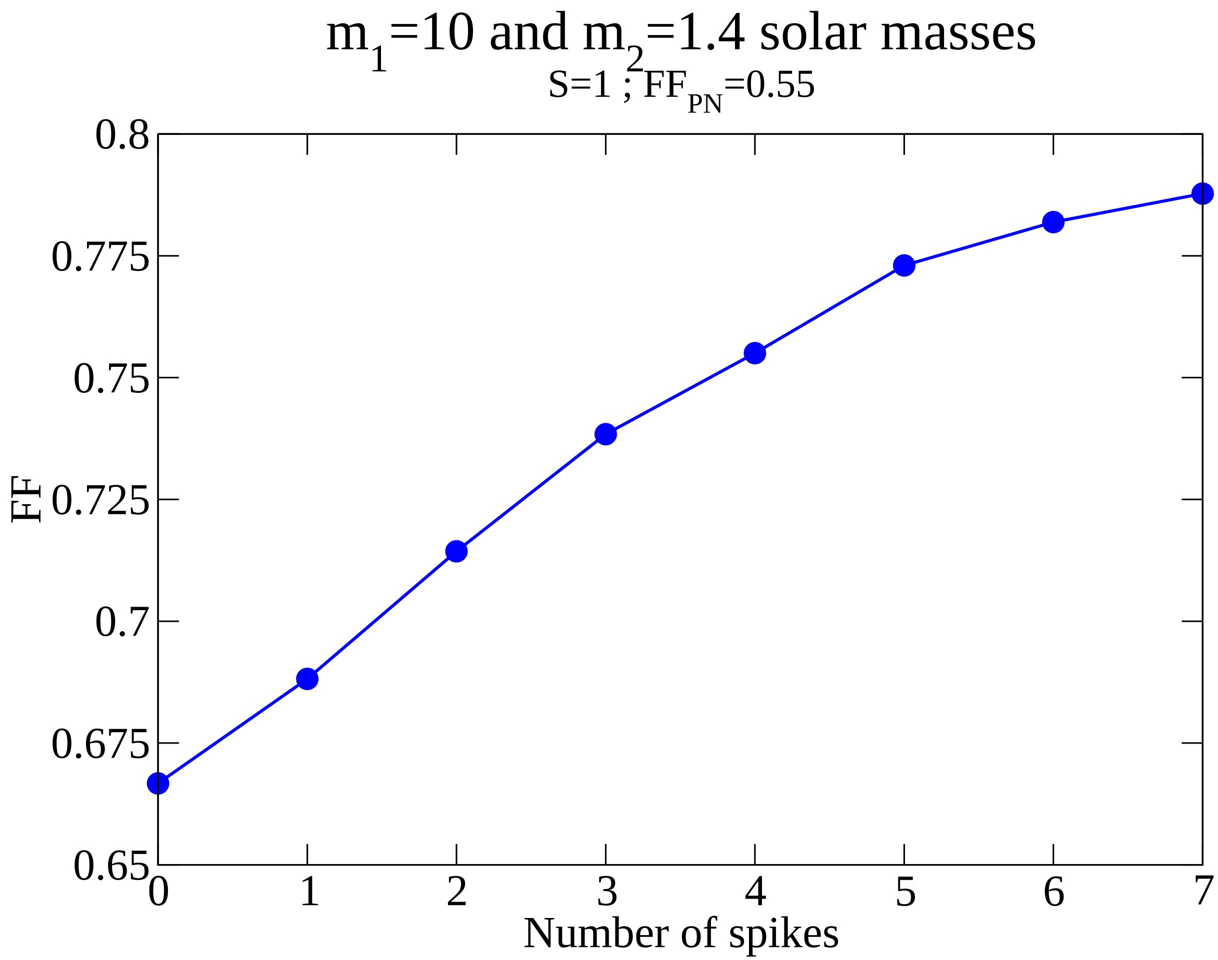
$$\phi_{N_{\mathrm{max}}} = 2\pi t_{\mathrm{max}} f + \frac{3}{128}\left(\pi \mathcal{M}_{\mathrm{max}}\right)^{-5/3} f^{-5/3} + \phi_{\mathrm{const}} + \mathcal{C}_{\mathrm{max}} \cos\left(\mathcal{B}_{\mathrm{max}} f^{-2/3} + \delta_{\mathrm{max}}\right) + \sum_{i=1}^{N_{\mathrm{max}}} P\left(f_{0\,\mathrm{max}}(i), \sigma_{\mathrm{max}}(i), \varepsilon_{\mathrm{max}}(i)\right)$$

TYPICAL EXAMPLE (7 SPIKES)

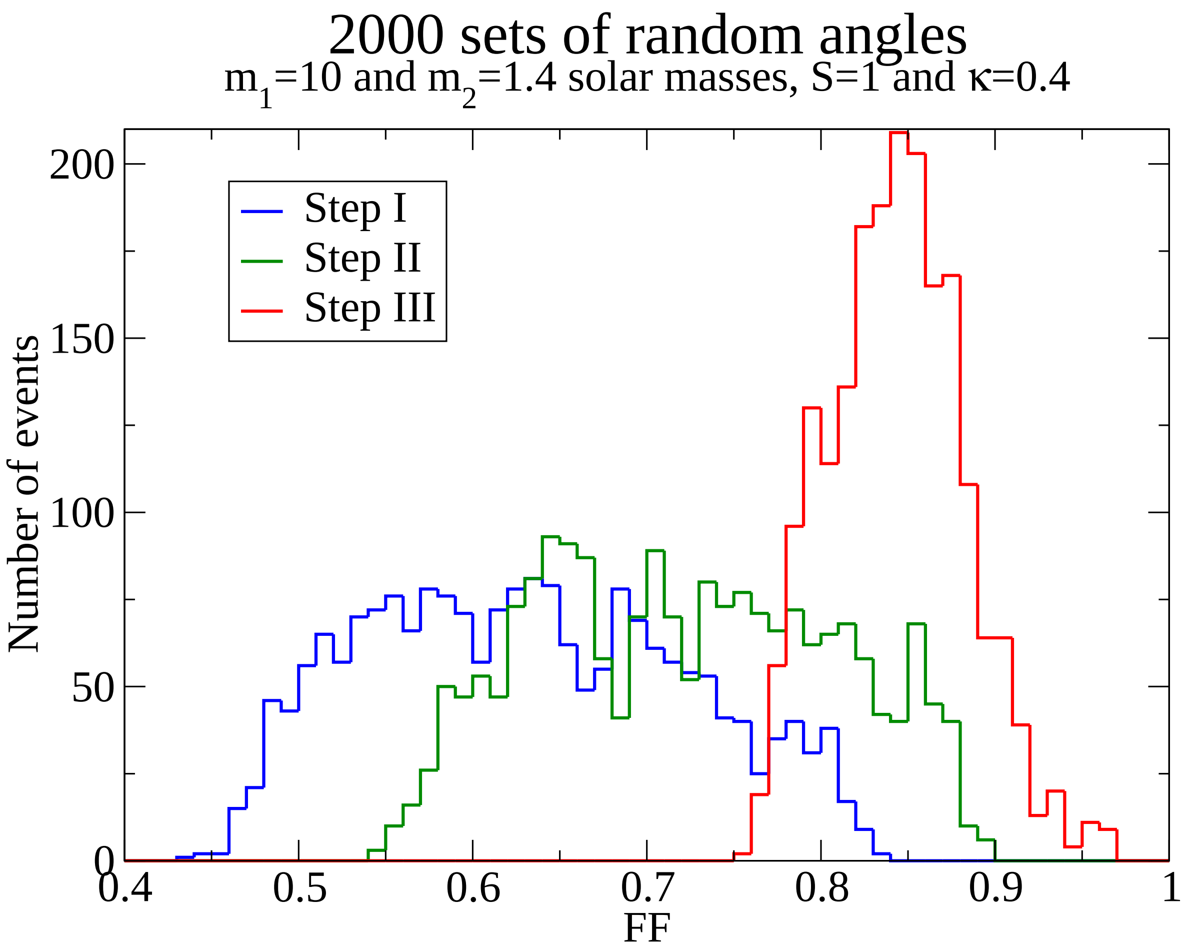
m_1=10 and m_2=1.4 solar masses



CONVERGENCE OF FF

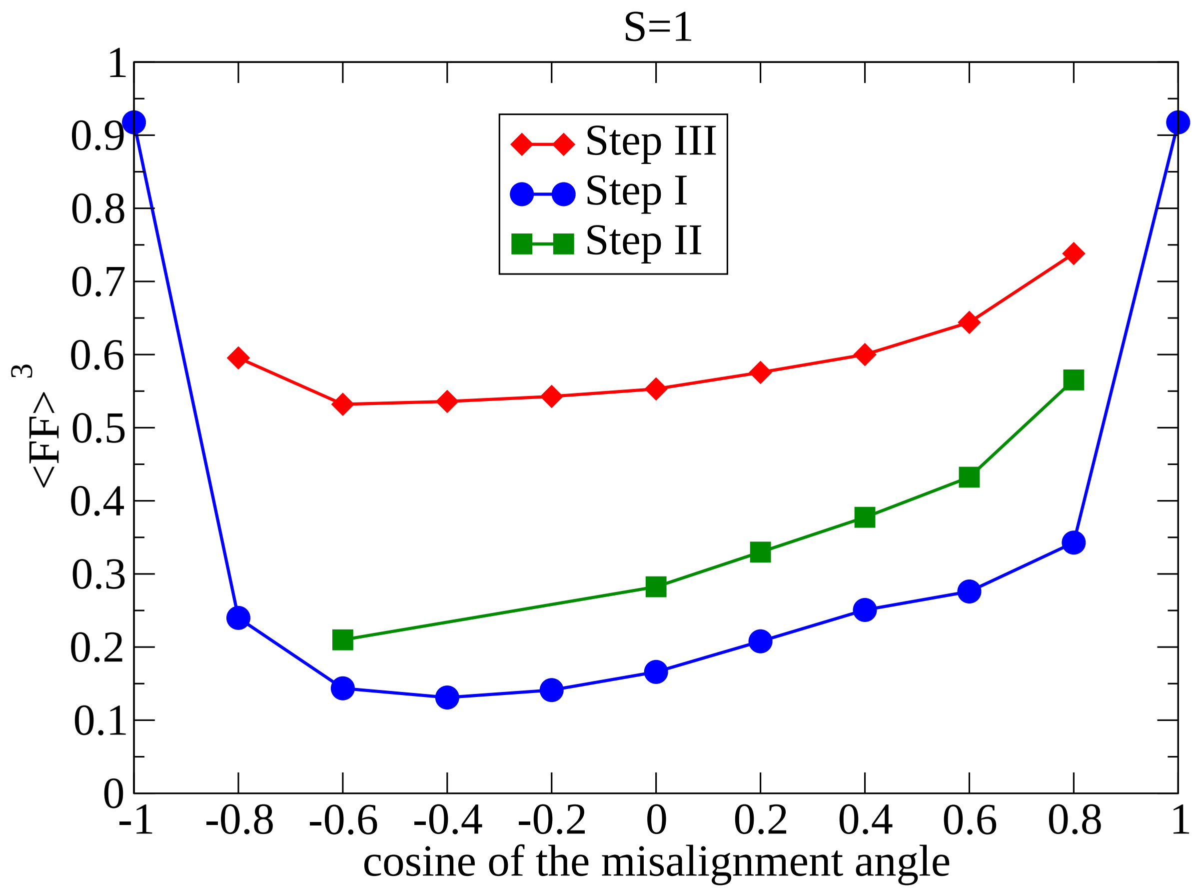


DISTRIBUTION OF FF

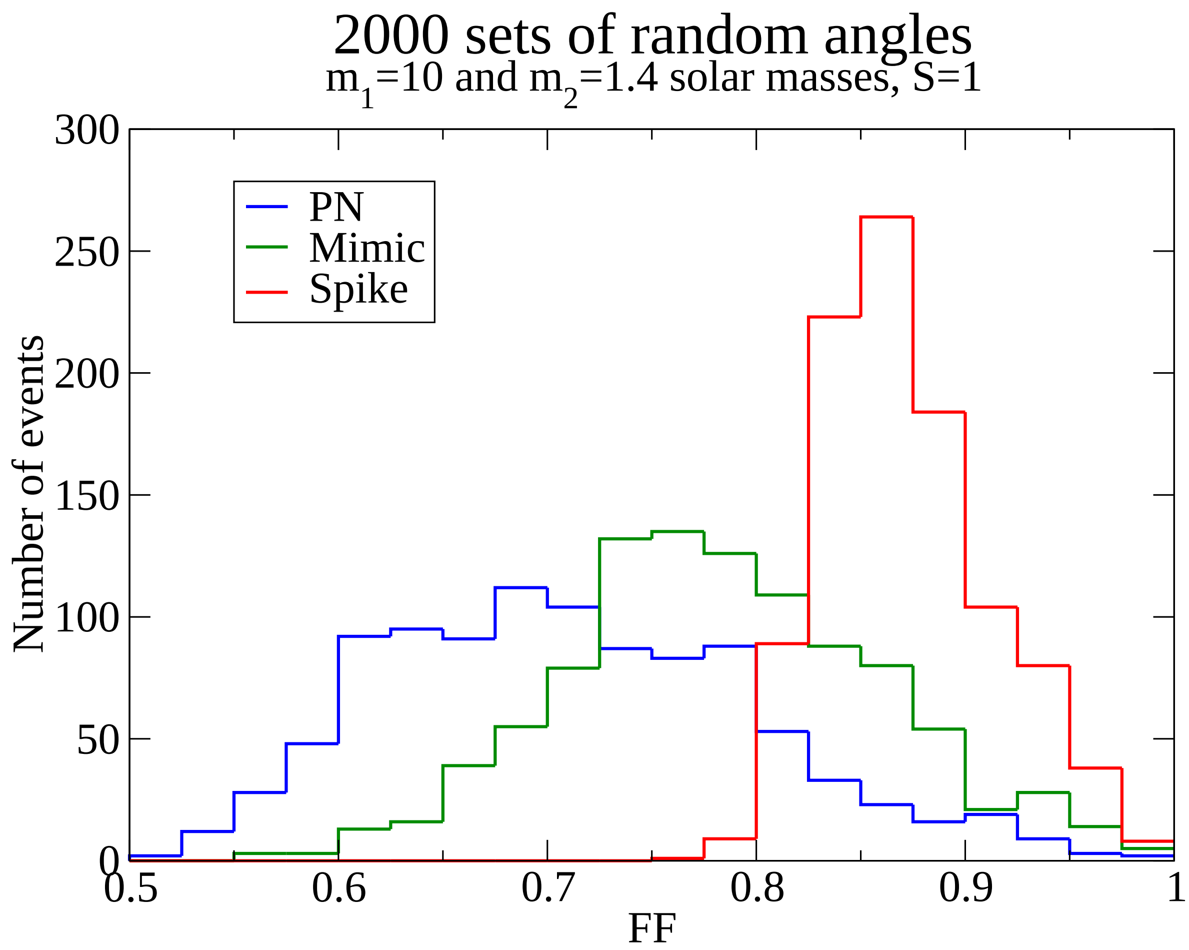


EFFICIENCY OF THE SPIKY TEMPLATES

m_1=10 and m_2=1.4 solar masses



ROBUSTNESS



2000 sets of random angles
m1=10 and m2=1.4 solar masses, S=1
PN
Mimic
Spike
Number of events
FF
0
50
100
150
200
250
300
0.5
0.6
0.7
0.8
0.9
1

CONCLUSIONS & PERSPECTIVES

- New family of templates : the "spiky"-family.
- Results promising.
- Robust to change in signal.
- Should "mimic" any modulation (not only precession).
- Tests in LAL ?