White light Cavities

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Basics

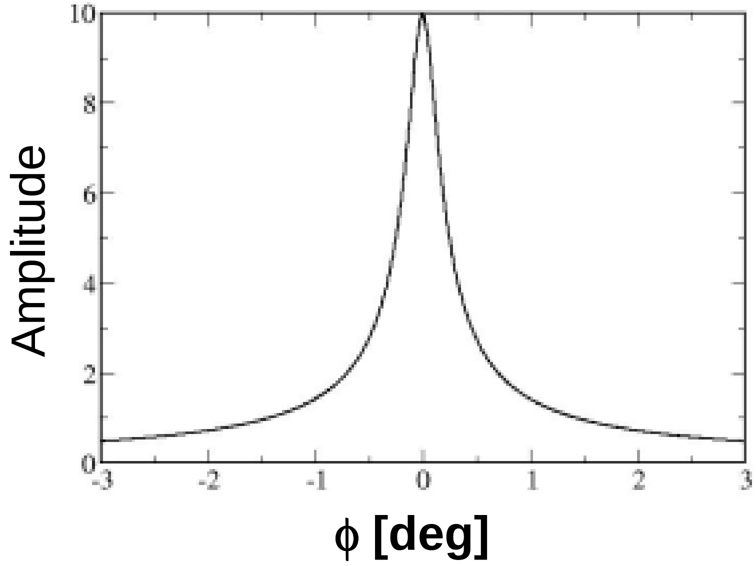
Transfer function of Cavity:

$$T(f) = \frac{X}{1 - r_1 r_2 e^{-i\phi}}$$

X depends on where we measure

$r_1 - r_2 e^{-i\phi}$ or $t_1 t_2$ or ...

ϕ : Round trip phase shift



Amplitude gives shot noise limit.

Bandwidth:

$\Delta\phi_{FWHM} = 2\pi/\text{Finesse}$

Idea

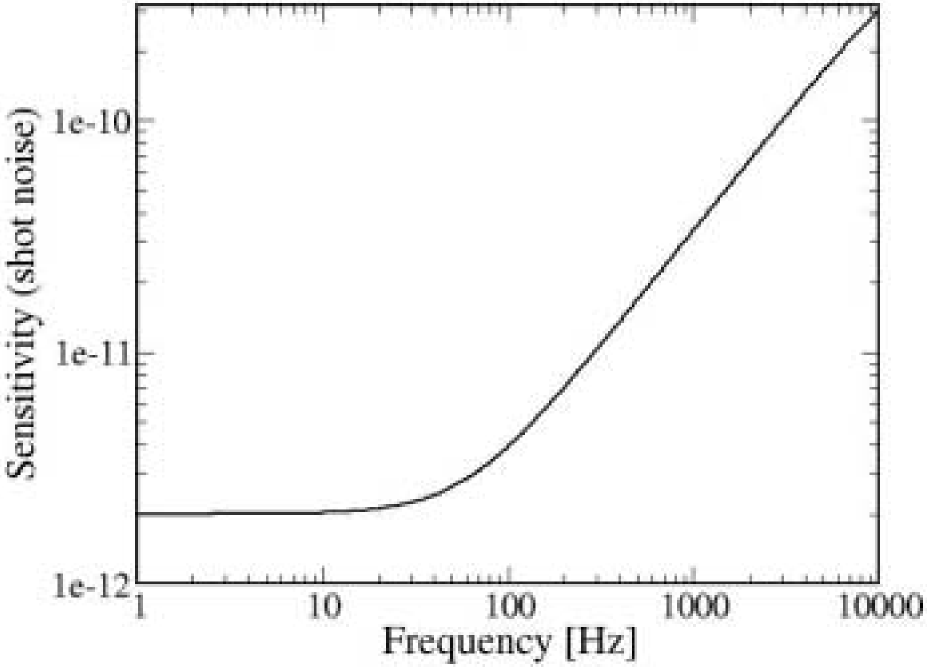
Round trip phase shift is proportional to frequency

$$\phi = 2\pi\,\nu\,L/c = N\,2\pi + 2\pi\,\Delta\nu\,L/c + 2\pi\,\nu\,\Delta L/c$$

which gives the standard cavity linewidth:

$$\Delta\nu_{FWHM} = \Delta\phi_{FWHM}\,FSR/\,2\pi$$

which limits
our bandwidth



Idea

Round trip phase shift is proportional to frequency

$$\phi = 2\pi \nu L/c = N 2\pi + 2\pi \Delta\nu L/c + 2\pi \nu \Delta L/c$$

which gives the standard cavity linewidth:

Solution:
Make round trip phase shift independent from frequency:

$\phi = N 2\pi + 2\pi \nu \Delta L/c$ **and get unlimited bandwidth !**

Original Idea published in: A. Wicht, K. Danzmann, M. Fleischhauer, M.O. Scully, G. Mueller, R.-H. Rinkleff Opt. Comm. 134 (1997), pg 431-439

How ?

Let L depend on the frequency (or wavelength) $L(\nu,\lambda)$

$$\phi = 2\pi\,(\nu_0+\Delta\nu)\,\left(L_0+\frac{\delta L}{\delta\nu}\Delta\nu\right)/c + 2\pi\,\nu\,\Delta L_0/c \overset{!}{=} N\,2\pi + 2\pi\,\nu\,\Delta L_0/c$$

$$L_0 = -\,\nu_0\,\frac{\delta L}{\delta\nu} \quad \text{or} \quad \frac{L(\lambda)}{\lambda} = \frac{\delta L(\lambda)}{\delta\lambda}$$

Round trip phase shift ϕ:

- **independent of laser frequency ν**
- **still depends on cavity length ΔL**

If Cavity resonant at ν_0 → resonant at all frequencies

How ?

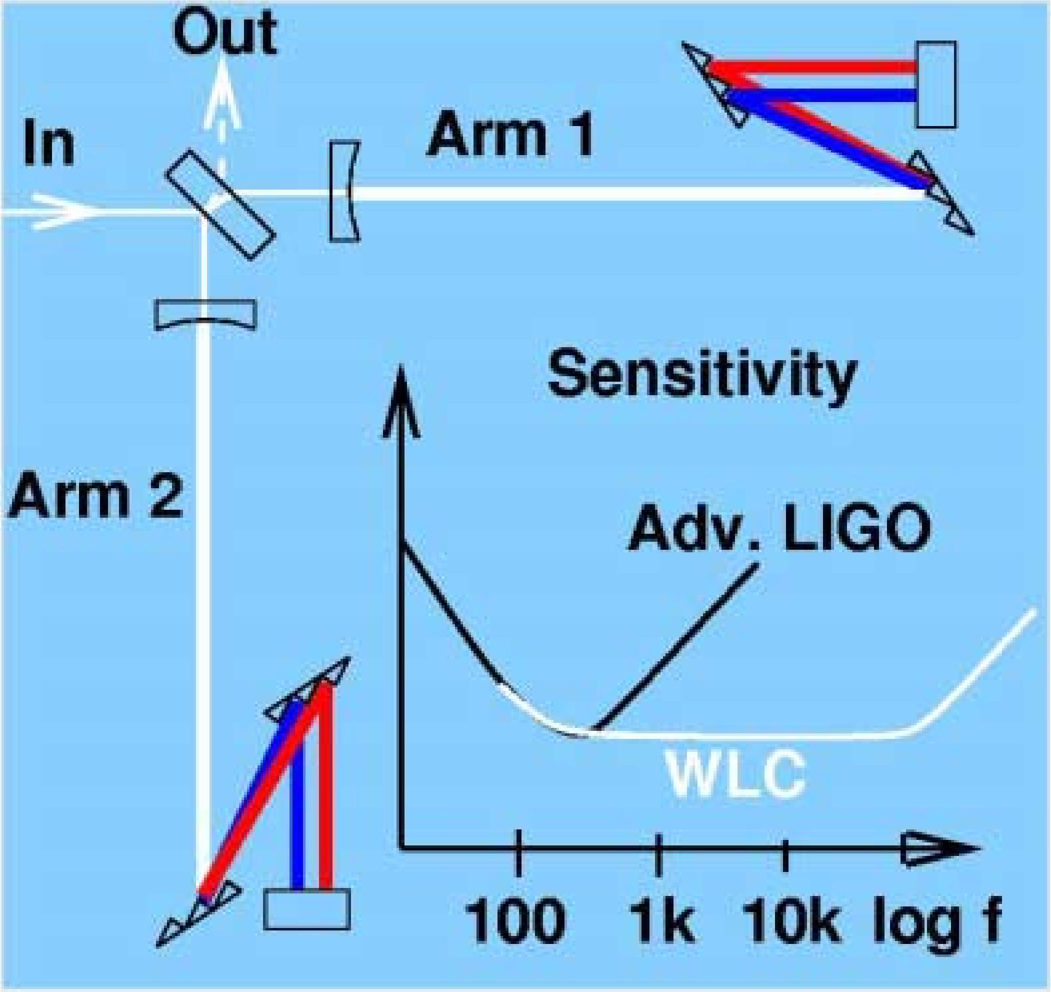
$$L_0(\nu) = -\nu_0 \frac{\delta L}{\delta \nu} \qquad \text{or} \qquad \frac{L(\lambda)}{\lambda} = \frac{\delta L(\lambda)}{\delta \lambda}$$

Several methods:

1. **Atomic resonances (Wicht-paper)**
   Index of refraction in resonantly pumped two level system (see *lasing without inversion*)
2. **Angular Dispersion**
   - **a) Prisms (not dispersive enough ?)**
   - **b) Gratings**
   - **c) misaligned triangular cavities (tricky)**

White light cavity

Grating in Compressor Configuration

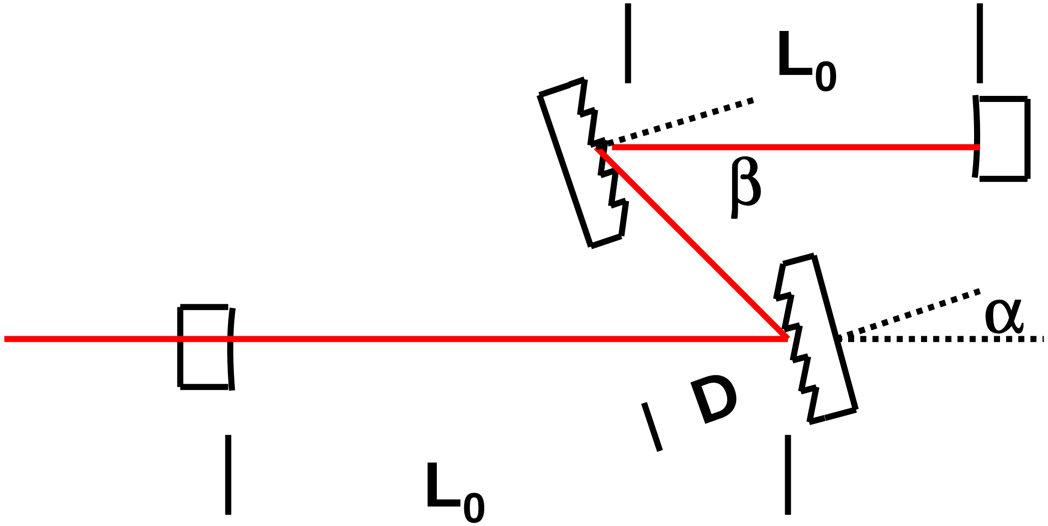


$$\frac{L(\lambda)}{\lambda} = \frac{\delta L(\lambda)}{\delta\lambda}$$

Or

Make Cavity longer for longer wavelength

White light cavity



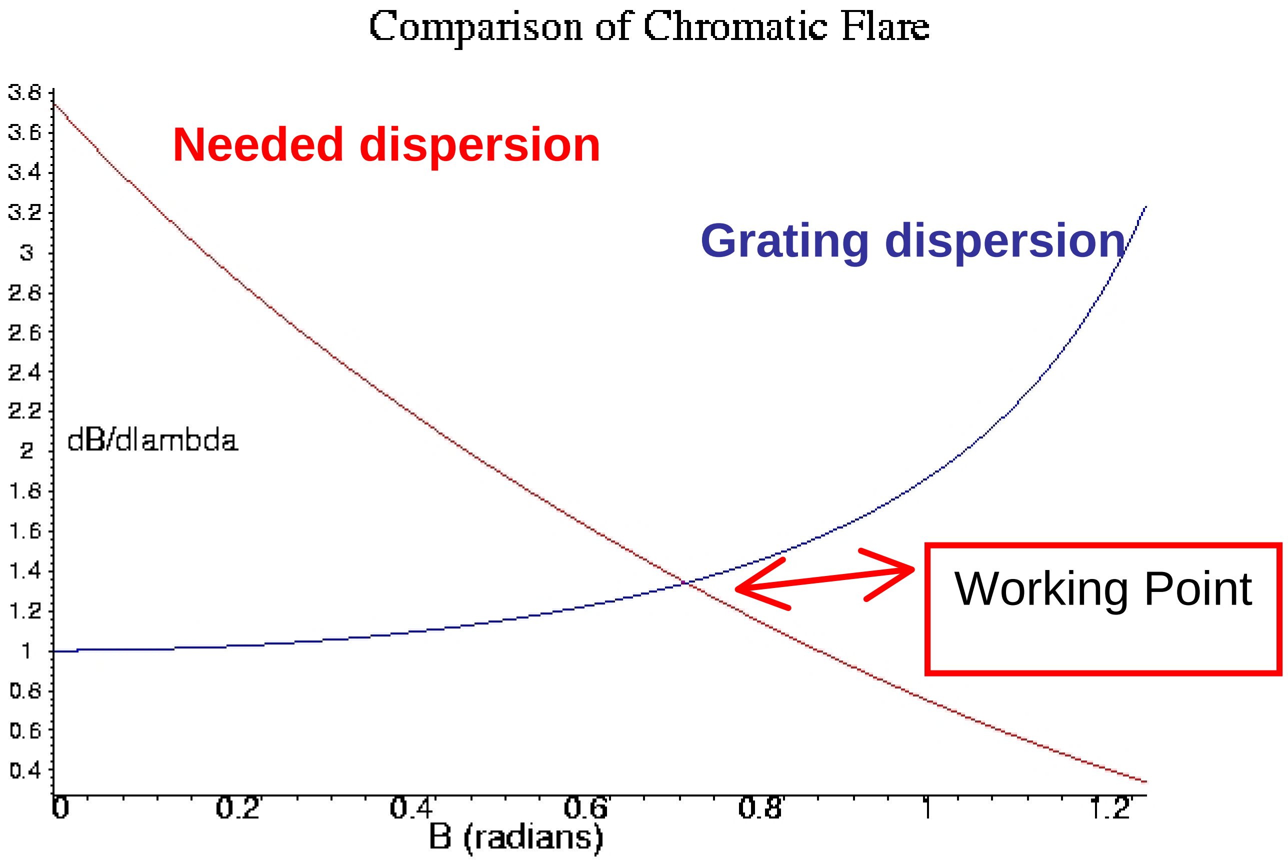
Cavity length: $$L(\lambda) = L_0 + \frac{D\,[1+\sin\alpha\,\sin\beta(\lambda)]}{\cos\beta(\lambda)}$$

WLC requires: $$\frac{L(\lambda)}{\lambda} \overset{!}{=} \frac{\delta L(\lambda)}{\delta\lambda} = \frac{\delta L}{\delta\beta}\,\frac{\delta\beta}{\delta\lambda}$$

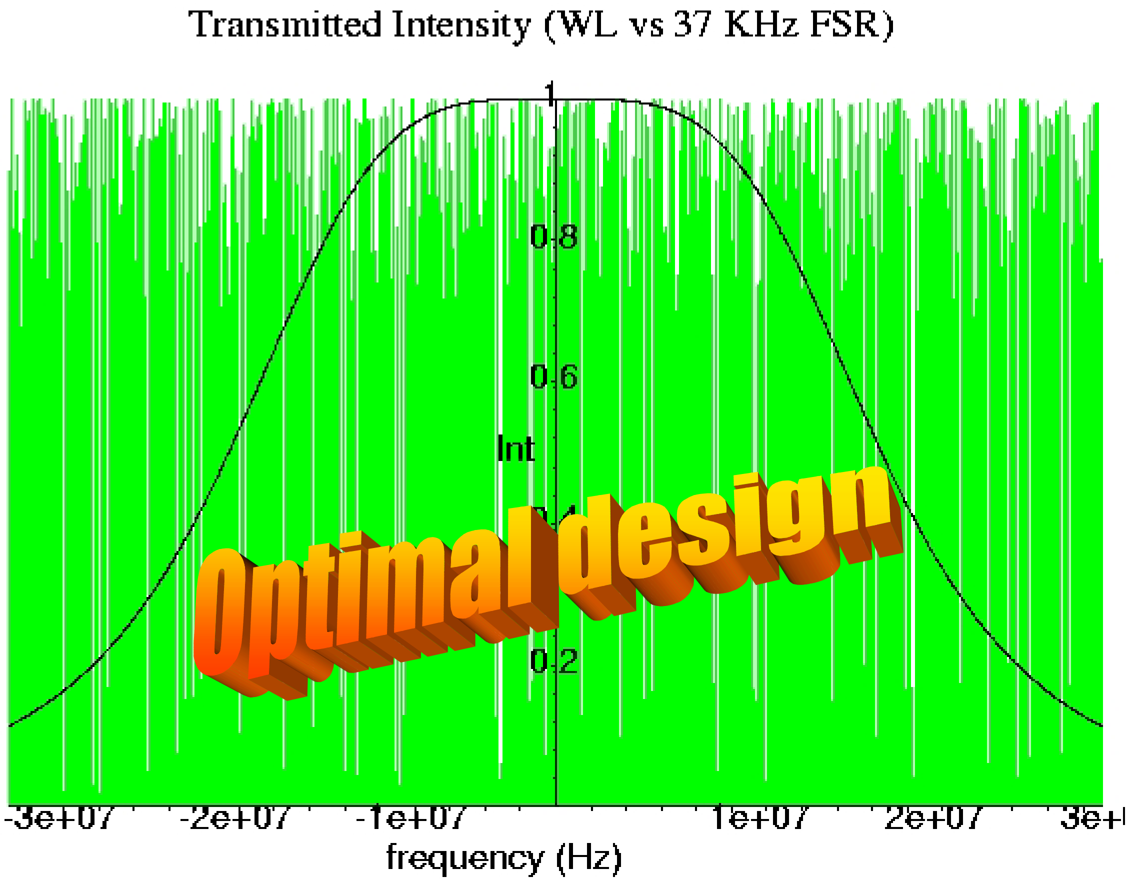
with: $$\frac{\delta L}{\delta\beta} = \frac{D}{\cos^2\beta}\,\frac{\lambda}{d} \qquad \frac{\delta\beta}{\delta\lambda} = \frac{m}{d\cos\beta}$$

Does this work ?

Is the dispersion in a grating large enough ?



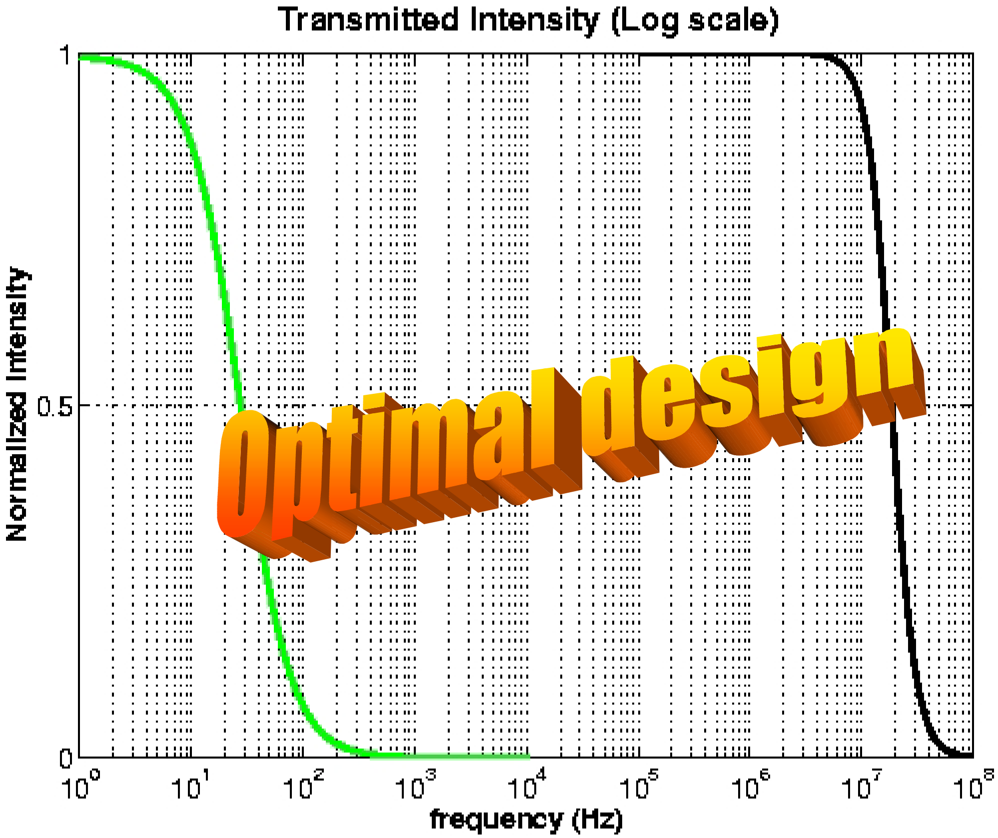
Final Bandwidth



Optimal design

Bandwidth increased from 60 Hz to 36 MHz

Final Bandwidth



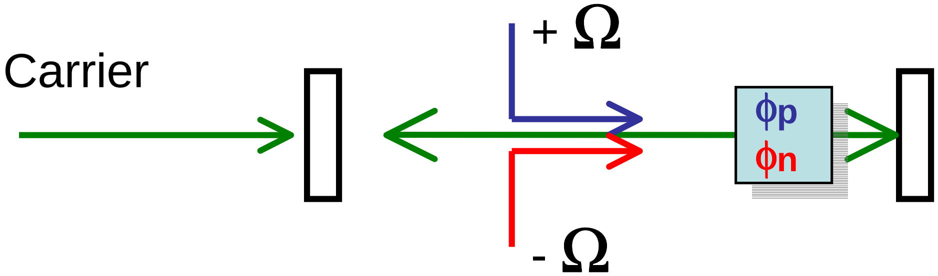
Bandwidth increased from 60 Hz to 36 MHz

But what about the storage time ?

Won’t we average out the signal ?

Gedankenexperiment

Assume GW-induced sidebands are added in one plane:



$$E_0(t) = E_C + E_+ e^{i\Omega t} - E_- e^{-i\Omega t}$$

First trip

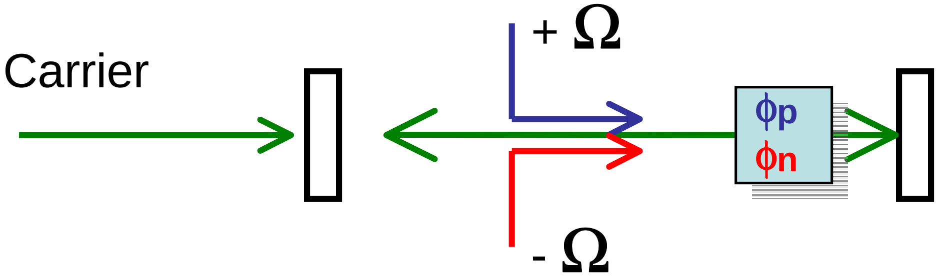
$$E_1(t+\tau) = r_1 r_2 \left(E_C\, e^{-i\Phi} + E_+\, e^{-i\phi_p}\, e^{i\Omega t} - E_- e^{-i\phi_n}\, e^{-i\Omega t} \right) + \left(E_C + E_+\, e^{i\Omega(t+\tau)} - E_-\, e^{-i\Omega(t+\tau)} \right)$$

$$\begin{aligned} E_2(t+2\tau) = (r_1 r_2)^2 &\left(E_C\, e^{-i2\Phi} + E_+\, e^{-i2\phi_p}\, e^{i\Omega t} - E_-\, e^{-i2\phi_n}\, e^{-i\Omega t} \right) \\ + r_1 r_2 &\left(E_C\, e^{-i\Phi} + E_+ e^{-i\phi_p}\, e^{i\Omega(t+\tau)} - E_-\, e^{-i\phi_n}\, e^{-i\Omega(t+\tau)} \right) \\ + &\left(E_C + E_+ e^{i\Omega(t+2\tau)} - E_- e^{-i\Omega(t+2\tau)} \right) \end{aligned}$$

…

Gedankenexperiment

Assume GW-induced sidebands
are added in one plane:



$$E_{tot} = E_C \sum (r_1 r_2)^j e^{-ij\Phi}$$

$$+ E_+ e^{i\Omega t} \sum (r_1 r_2)^j E_+ e^{ij(\Omega\tau - \phi_p)} - E_- e^{-i\Omega t} \sum (r_1 r_2)^j E_- e^{-ij(\Omega\tau + \phi_n)}$$

$$= E_C \frac{1}{(1 - r_1 r_2 e^{i\Phi})}$$

$$+ E_+ e^{+i\Omega t} \frac{1}{(1 - r_1 r_2 e^{i(\Omega\tau - \phi_p)})} - E_- e^{-i\Omega t} \frac{1}{(1 - r_1 r_2 e^{-i(\Omega\tau + \phi_n)})}$$

$\phi_p = \Omega\tau$ and $\phi_n = -\Omega\tau$ and every frequency is building up

<u>Storage time in Cavity has nothing to do with this</u>

Surprised ?

Think about Advanced LIGO !

1. **Build up GW-sidebands inside arm cavities**
2. **Turn off laser and GW**
3. **Measure decay time of GW-sidebands: T = 1s**
4. **Peak Sensitivity at 200 Hz !**

So we average over 200 cycles !

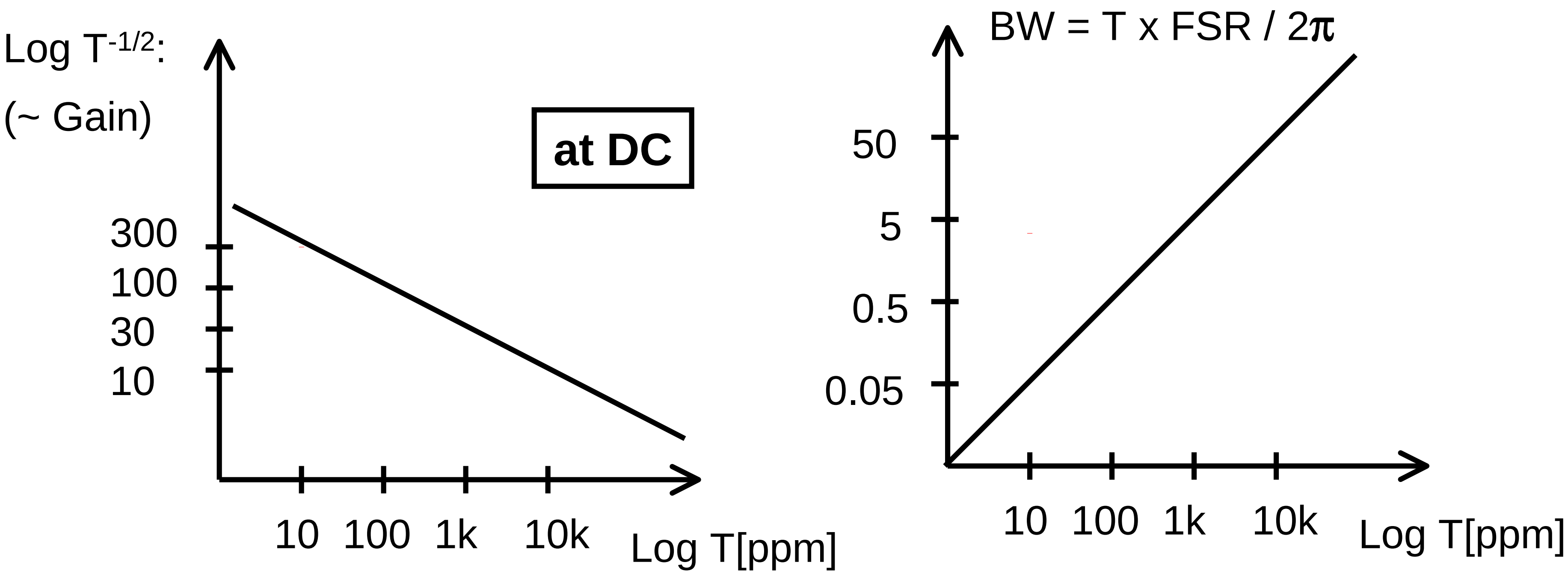
Is this a problem ?

No, because the detuned SR-cavity (compound mirror) ensures that we average always in phase with the signal.

Compare with Adv. LIGO

Assume: 1MW in each arm cavity (infinite mirror masses):

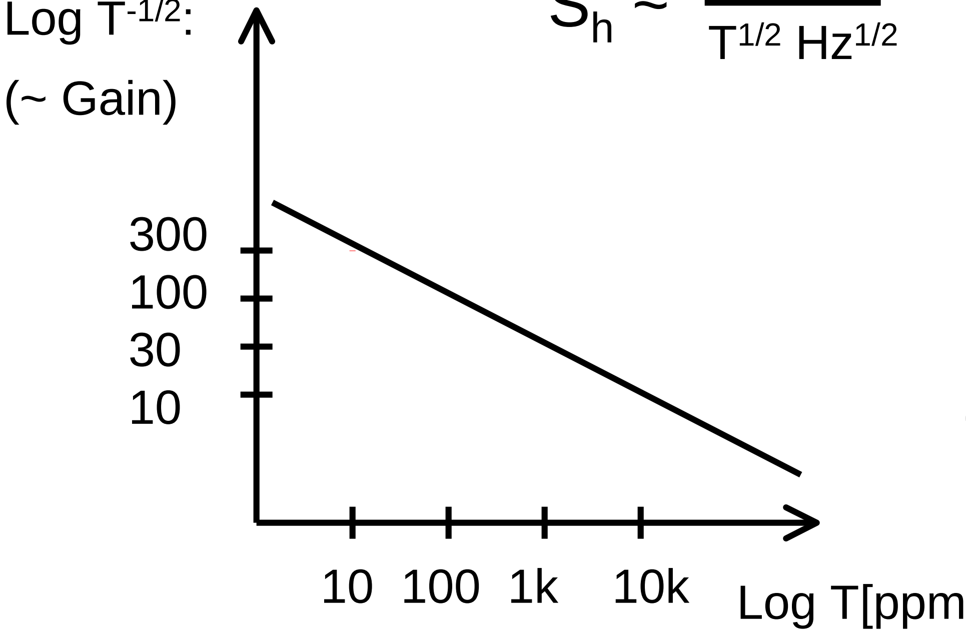
$S_h \sim \frac{5\text{x}10^{-23}}{T^{1/2}\ \text{Hz}^{1/2}}$ for a non recycled MI (T transmission of ITM)

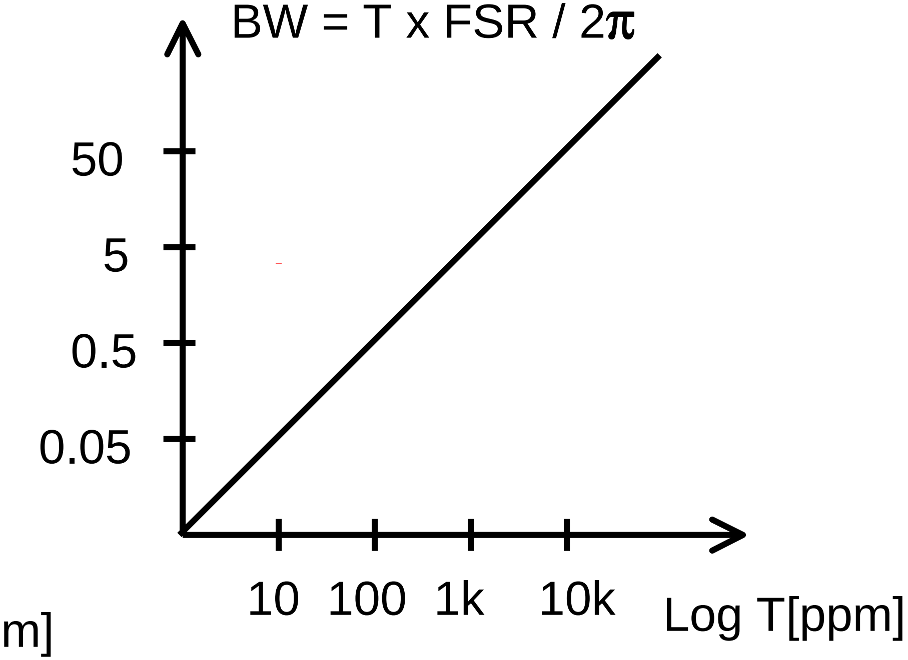


SR: shifts to other frequencies, Gain, BW: replace T by T_{eff}

Scaling

$$S_h \sim \frac{5\times10^{-23}}{T^{1/2}\ \mathrm{Hz}^{1/2}}$$





Choices: want $S_h \sim 5\times10^{-25}$ ⟶ BW ~ 0.5 Hz
want $S_h \sim 5\times10^{-24}$ ⟶ BW ~ 50 Hz

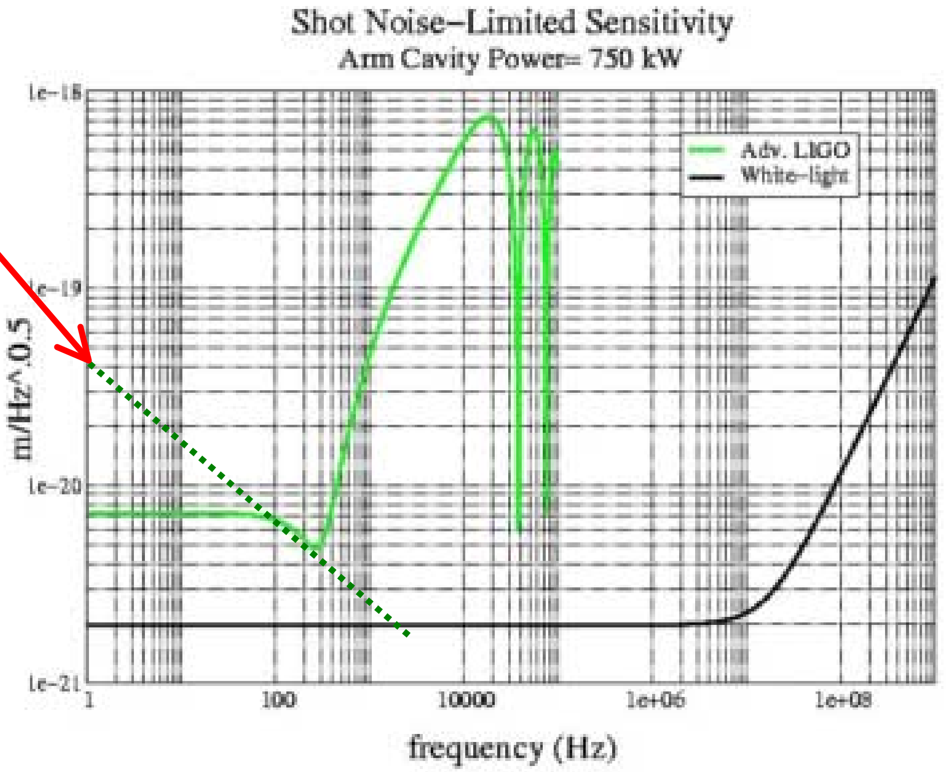
Example for Grating: L = 10k ppm $S_h \sim 5\times10^{-24}$, BW ~ MHz (P_{in} = 10kW)
(T = Losses = L) L = 1k ppm $S_h \sim 1.6\times10^{-24}$, BW ~ MHz ($P_{in}$ = 1kW)
L = 100 ppm $S_h \sim 5\times10^{-25}$, BW ~ MHz (Pin = 100W)

Remarks

- WLC reduces shot noise limit above cavity pol.
- Quadrature components of the quantum noise are uncoupled, no detuning.
- Radiation pressure noise will push on mirrors and noise will depend on mass of mirrors.
- Losses in gratings need to be below ~200ppm for grating, otherwise build up to low.
- Should be set up in an all reflective design.
- Nontransmissive materials for test masses possible: Silicon

Outlook

- **Gratings with 97% losses from Uni Jena will come**
- **They produced already gratings with >99% efficiency**
- **Stacy started to model gratings (preliminary: 99.6%)**
- **Designed tabletop with expected linewidth of 10GHz in 23cm cavity.**
- **…**



RPN+thermal noise

assumes
equal masses
in both cases.

All reflective optics
enables us to use new
materials (Silicon):
Larger masses,
better thermal properties
will reduce both noise
sources.