Comparing the ULs from the frequentist and Bayesian pipelines

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Bayesian Upper limit definition

- The 95% confidence upper limit is the value h_{95} satisfying

$$0.95 = \int_0^{h_{95}} p(h_0 \mid \{B_k\})\mathrm{d}h_0$$

- the posterior marginalized pdf for the data and a signal of amplitude h_0. This is a function of h_0, obtained by integrating out (marginalizing) the parameters ι, ψ, φ_0 that appear in the joint posterior probability distribution.

Parameter estimation 1

- For the joint posterior probability distribution of our unknown parameters, we use uniform priors on $\vec{a} = (h_0, \cos\iota, \psi$ and $\phi_0)$ over their accessible values, i.e.

$$p(\mathbf{a} \mid \{B_k\}) \propto p(\mathbf{a}) \cdot p(\{B_k\} \mid \mathbf{a})$$

posterior — prior — likelihood

$$p(\{B_k\} \mid \vec{a}) \propto \exp\left[-\sum_k \frac{\left|B_k - y(t_k; \vec{a})\right|^2}{2\sigma_k^2}\right] = \exp\left[-\chi^2/2\right]$$

very clean: it only depends on knowing the noise and on the assumption that the noise is additive.

F statistic UL

• measure a value of the optimal detection statistic: F*

• inject population of signals (fixed source position, spin-down parameter value, h_0; worst case nuisance parameters or uniform value population of nuisance parameters)

• do search for each injection ➔ derive a value of F for each injection. Get this way an ensemble of Nt values of F.

• count how many times F > F*, N* ➔ confidence = N*/Nt

$$0.95 = \int_{F^*}^{\infty} p(\mathrm{F} \,|\, \mathrm{h}_0)\, \mathrm{dF}$$

Upper limit definition

- One could think that it is enough to perform "a Bayesian" analysis on the F* value

$$0.95 = \int_0^{h_{95}} p(\mathrm{h}_0 \mid \mathrm{F}^*)\, \mathrm{d}h_0$$

with p(h_0|F) being the worst case (easier) or uniform prior case pdf.

Frequency domain analysis

• the F statistic is the maximum of the log of the likelihood ratio:

$$F = \max_{i,\psi,h_0,\varphi_0} \log\left(\frac{p(\{x\} \mid h_0)}{p(\{x\} \mid 0)} \right)$$

• we'd like to demonstrate that, in the Bayesian approach, when one marginalizes over the nuisance parameters with uniform priors one gets something like e^F*K, and K should tells us what differences we expect between the two approaches:

$$e^F K \overset{?}{=} p(h0 \mid \{B_k\}) = \iiint p(\{B_k\} \mid \vec{a})\, d\cos i \, d\psi \, d\varphi_0$$